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DIAGRAMMATIC REPRESENTATION FOR A SPACE OF INTERVALS

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Abstract. The paper presents a two-dimensional graphical representation for the space of intervals and interval relations. It consists of the representation of the space of all intervals (the *IS-diagram*), and two diagrammatic representations of the space of relations between intervals: the *W-diagram* (based on the IS-diagram), and *lattice diagram* (for the neighbourhood lattice of the set of basic relations). Also, a set of new graphical symbols for the interval relations is proposed. Examples of application of the proposed representations to represent knowledge about interval algebra and certain interval problems are given, to illustrate the possibilities and advantages of the proposed representation.

Key words: diagrams, diagrammatic representation, diagrammatic reasoning, intervals, interval relations, interval algebra, knowledge representation, qualitative physics.

1. Introduction

As was stated by Simon [2]: "*...solving a problem simply means representing it so as to make the solution transparent.*" Not surprisingly, the problem of finding appropriate *representations* of various types of knowledge is a subject of continued research in the field of artificial intelligence and related fields since the beginning of these disciplines. In fact, much of the progress in science in general, and mathematics in particular, consisted of finding new representations of various phenomena or formal constructs: from diagrams of Euclid, through calculus notation of Newton and Leibnitz, to Feynman quantum particle diagrams, and many others. Devising a new way of representing knowledge about some phenomenon, formal system, or problem class offers new means of effective description of the domain objects and new possibilities of reasoning about them and solving problems involving them.

Since some time, so-called *diagrammatic representations* (and associated *diagrammatic reasoning* methods) gain considerable interest and popularity, as they often provide more effective means than other representations for storing, using and presenting complex information and knowledge, see, e.g., the survey paper [18]. The diagrammatic (visual) representation uses *diagrams* to represent data and knowledge. Diagrams are a kind of *analogical* (or *direct*) knowledge representation mechanism that is characterised by a parallel (though not necessarily isomorphic) correspondence between the structure of the representation and the structure of the represented. E.g., relative positions and distances of certain marks on a map are in direct correspondence to relative positions and distances of the cities they represent, whereas in a propositional representation, its parts or relationships between them need not correspond explicitly to any parts and relations within the thing denoted. The analogical representation can be said to *model* or *depict* the thing represented, whereas the propositional representation rather *describes* it. A similar distinction can be made regarding the method of retrieving information from the representation. The needed information can usually be simply *observed* (or *measured*) in the diagram, whereas it must be *inferred* from the descriptions of the facts and axioms comprising the propositional representation.

Despite objections to the effect that diagrammatic reasoning can be considered at most as an heuristic aid for the truly precise and formal thinking based on propositional logic, recent research has shown that it is quite possible to formalise diagrammatic reasoning so as to make it no less precise and formal than logical reasoning [14, 22, 23].

It should be noted, however, that rarely any single approach to knowledge representation solves all problems and is the best in all circumstances — hence the necessity of hybrid systems of representation and reasoning, with diagrammatic methods often constituting an important ingredient of the whole, see e.g. [20].

The research concerning the interval relations as described here originated in the field of reasoning about time (where the intervals were considered time intervals, hence the time-domain terminology appearing here sometimes) [3, 4, 5, 9, 11, 13, 15]. However, both the interval relations algebra and its proposed representation are useful in many other fields, among others spatial reasoning [7, 17], qualitative analysis of physical systems (qualitative physics), see e.g. [6, 16, 20], and research on reliability of numerical computations and optimisation [8, 12].

The paper starts with a short introduction to the basics of interval algebra and interval relations. Then, a two-dimensional, diagrammatic representation of the space of intervals, called an *IS-diagram* (introduced by Kulpa [21]), is elaborated. It constitutes an extension and refinement of the representation proposed by Rit [4] and used to illustrate the concept of *convex* interval relations by Nökel [11] and Ligozat [19]. Another diagrammatic notation based on it, called a *W-diagram*, is useful in investigations of interval relations and proving theorems about them, as was shown by Kulpa in [21, 24]. Additional diagrammatic notation, namely the *lattice diagram*, representing the neighbourhood structure of the set of relations [11, 13, 24], is also introduced. A set of new graphical symbols for the interval relations is proposed. Usefulness of these diagrams is illustrated with several examples of representing knowledge about interval algebra and certain interval problems. An application of the representation to prove the equivalence of different characterisations of convex interval relations has been described by the author elsewhere [24].

2. Intervals: Notation and Basic Definitions

The interval notation adopted for the purpose of this paper is essentially that of Ratschek and Rokne [12], with some minor simplifications.

An *interval* is an ordered pair $u = [e_1, e_2]$, where e_1 and e_2 (*endpoints* of the interval) are elements of some (at least partially) ordered set E (called the *base set*), such that $e_1 \leq e_2$. The interval is called *thick* if $e_1 \neq e_2$; *thin* (or *point*) interval if $e_1 = e_2$. The *beginning* and *end* of the interval u are denoted by lb u and ub u, respectively. Thus, u = [lb u, ub u]. To simplify notation, we will usually identify an interval with a set of elements lying between its endpoints: $u = \{e \mid \text{lb } u \leq e \leq \text{ub } u\}$.

In the paper we will be concerned mostly with *real intervals*, i.e. intervals defined over the set of real numbers. The set of all such intervals will be denoted by *IR* and called a (*real*) *interval space*. For real intervals, the *midpoint*, *radius* and *width* of the interval are defined, respectively, as follows (see also Fig. 1):

mid u = (lb u + ub u) / 2,

rad u = (ub u – lb u) / 2,

wid u = ub u – lb u = 2 rad u.

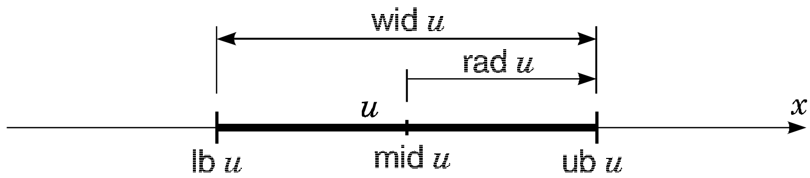

Fig 1. A (real) interval and its basic parameters

An interval i is called to lie *between* two intervals u and v, if the endpoints of i lie between the corresponding endpoints of u and v, i.e.:

min(lb u, lb v) ≤ lb i ≤ max(lb u, lb v), and

min(ub u, ub v) ≤ ub i ≤ max(ub u, ub v).

The set of all (proper) intervals lying between u and v, denoted by $\Diamond(u, v)$, will be called a *lozenge*. The concept of a lozenge is related to the concept of pairs of intervals called "*twins*" introduced in [1] — one of basic interpretations of a twin (as a set of intervals) is the lozenge defined by elements of the twin.

There are many possible definitions of the *in-between relation* in the space of intervals. The choice of the particular definition determines also the definition of corresponding metric in the space of intervals (not used here) as well as the definition of convexity of interval sets and relations (see Sec. 5). The definition above has been chosen to conform with the structure of the space of arrangement interval relations introduced in the next Section, and used extensively in reasoning about time and space in AI and Qualitative Physics [3-7, 9-11, 13, 15-17, 20].

3. Interval Relations

The basic notation for algebra of relations used in the paper is as follows. Let X, Y, Z, W be sets. Then $\mathbf{R} \subseteq X \times Y$, $\mathbf{Q} \subseteq Y \times Z$ are (*binary*) *relations*; $x\mathbf{R}y$ means $(x, y) \in \mathbf{R}$. As relations are sets, the *union* $\mathbf{R}\cup\mathbf{Q}$ and *intersection* $\mathbf{R}\cap\mathbf{Q}$ of relations are defined straightforwardly:

$x(\mathbf{R}\cup\mathbf{Q})y \Leftrightarrow x\mathbf{R}y \vee x\mathbf{Q}y$, and

$x(\mathbf{R}\cap\mathbf{Q})y \Leftrightarrow x\mathbf{R}y \;\&\; x\mathbf{Q}y$.

Notation $\mathbf{R}\circ\mathbf{Q}$ (shortly: $\mathbf{RQ}$) is used for a *composition of relations*:

$x\mathbf{RQ}y \Leftrightarrow (\exists z)\; x\mathbf{R}z \;\&\; z\mathbf{Q}y$.

A relation $\mathbf{R}^{-1}$ such that $y\mathbf{R}^{-1}x \Leftrightarrow x\mathbf{R}y$ is called an *inverse* of the relation $\mathbf{R}$. Obviously, $(\mathbf{R}^{-1})^{-1} = \mathbf{R}$. Note that the notion of inverse relations is in most part a notational convention — only positions of the arguments are exchanged, without real change of the underlying relationship between them.

The notation:

$W\mathbf{R} = \{y \in Y \mid w\mathbf{R}y \;\&\; w \in W\}$, and

$\mathbf{R}W = \{x \in X \mid x\mathbf{R}w \;\&\; w \in W\}$

denotes an *image* and *coimage*, respectively, of the set W under the relation $\mathbf{R}$. An image of a set under $\mathbf{R}$ coincides with its coimage under $\mathbf{R}$'s inverse, i.e. $W\mathbf{R} = \mathbf{R}^{-1}W$. We will simplify $\{w\}\mathbf{R}$ and $\mathbf{R}\{w\}$ to $w\mathbf{R}$ and $\mathbf{R}w$, respectively.

The following distribution laws are also useful:

$W(\mathbf{R}\cup\mathbf{Q}) = W\mathbf{R}\cup W\mathbf{Q}$, $\quad (\mathbf{R}\cup\mathbf{Q})W = \mathbf{R}W\cup\mathbf{Q}W$,

$W(\mathbf{R}\cap\mathbf{Q}) = W\mathbf{R}\cap W\mathbf{Q}$, $\quad (\mathbf{R}\cap\mathbf{Q})W = \mathbf{R}W\cap\mathbf{Q}W$.

An *arrangement interval relation* (*AIR*) is any relation between intervals (i.e., a subset of $IR \times IR$) that can be defined using only the order relation (and equality) defined in their base set between interval endpoints, and logical connectives. Note that some simple relations between intervals, e.g. the "*equal width*" relation, are not arrangement relations.

Thirteen of the *AIR*s (minimal under the union of relations) constitute the set of *basic* (or *simple*) *interval relations* (*BIR*) [3, 9, 13], see Fig. 2. New proposed graphical symbols for the relations are also shown there. The symbols were chosen to conform with the graphical arrangement of the intervals that belong to the given relation — compare them with the diagrams in the second column of Fig. 2. However, some of the symbols, as a result, violate the natural convention that symbols for mutually inverse relations are usually made to be reflections of each other with respect to the vertical axis (as it holds for the first three pairs of relations, like "**<**" and "**>**" in Fig. 2). The remedy for this would be to rotate the symbols by 90 degrees, preferably counterclockwise. But this would in turn degrade the intuitive similarity of the relation symbols with corresponding arrangements of intervals — the very reason for this particular choice of the symbols. However, the reflection with respect to horizontal axis does produce inverse relations for all the new symbols (but now the pair “<” and “>” becomes an exception). Also, the proposed symbols better conform with the structure of the W-diagram, see Fig. 6, hence the final choice.

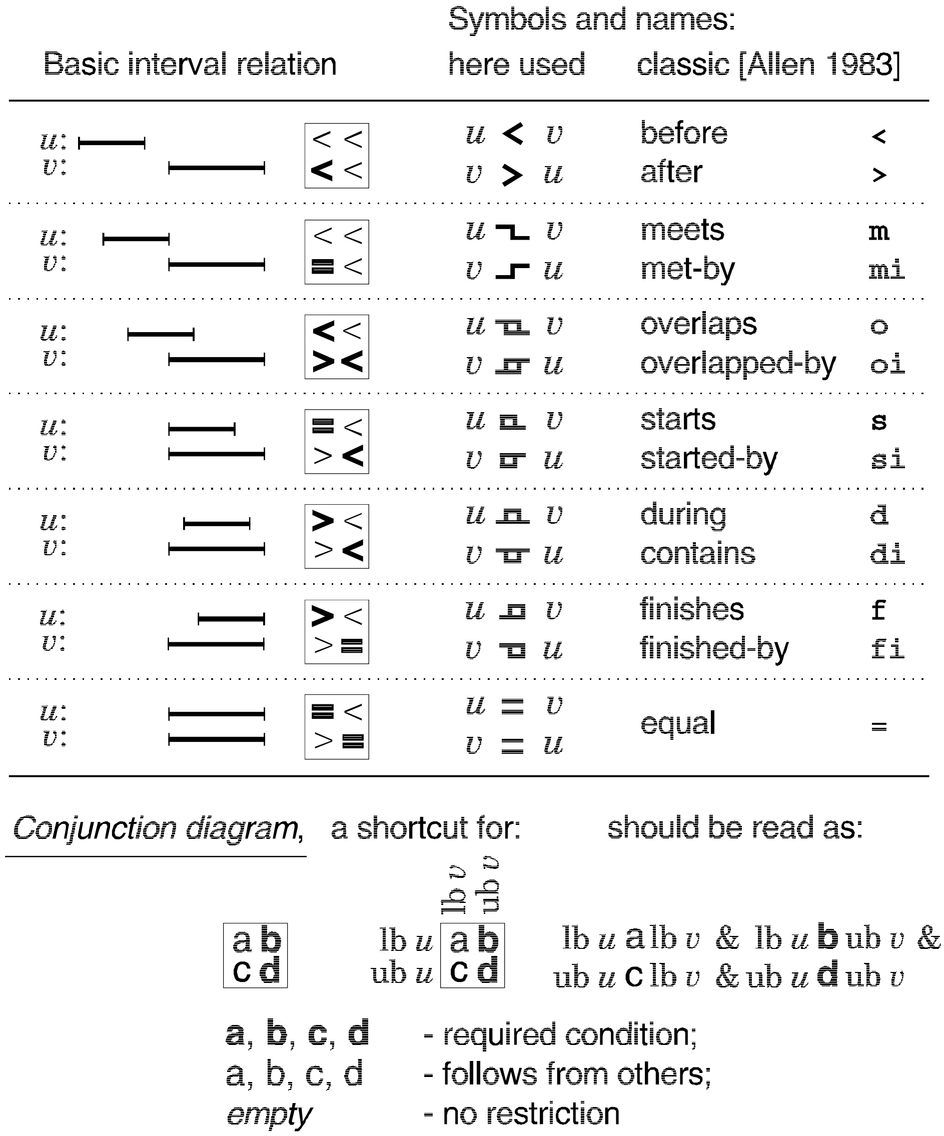

Fig 2. Basic interval relations and the conjunction diagram

The relations are defined with classic interval diagrams and with the help of square *conjunction diagrams* denoting the conjunction of relations between interval endpoints. Relations shown in boldface are required; the other follow from the interval definition (lb $u \leq$ ub u) and transitivity of ordering relation. Contrary to the initial claim by Freksa [13, p. 202] that all basic relations can be defined by at most two relations between the interval endpoints, two of them (the overlap relations) require the specification of at least three such relations.

All *AIR*s are unions of *BIR*s, and the set *AIR* is closed under union, intersection and composition of relations. There are some other subclasses of *AIR*s, wider than the *BIR* class. Two of them are worth mentioning here:

- the *convex interval relations* (*CIR*s), important in applications [5, 9, 10, 11, 13], and discussed in more details in Sec. 5 below,
- the *pointisable interval relations* (*PIR*s), a little wider class than *CIR*, sharing with it some useful properties (see Sec. 5) [9, 15], though not discussed further in this paper.

The inclusions $BIR \subset CIR \subset PIR \subset AIR$ hold for these classes.

4. Interval Space Diagrams

4.1. The IS-diagram

To uniquely define an interval, two numerical parameters are necessary, hence the space of intervals is two-dimensional. Different pairs of parameters can be used as co-ordinates of the interval space. The most obvious is the *endpoints space* (e_1, e_2) [4]. It is not very convenient, however, as the set of (proper) intervals occupies the "skewed" half-plane above the diagonal. Other natural choices of co-ordinates are better in this respect. The *centred* space (mid u, rad u) has particularly convenient properties for our purposes, and thus constitutes the basis of the IS-diagram representation of interval space, see Fig. 3. A very similar (mid u, wid u) space has been considered by Kulpa in [21].

As rad $u \geq 0$ always, the space of (thick) intervals occupies the half-plane above the m = mid u axis (with thin intervals on the axis). Points below this axis may be considered as representations of "improper" intervals (for which $e_1 > e_2$). This interesting extension will not be further pursued in this paper.

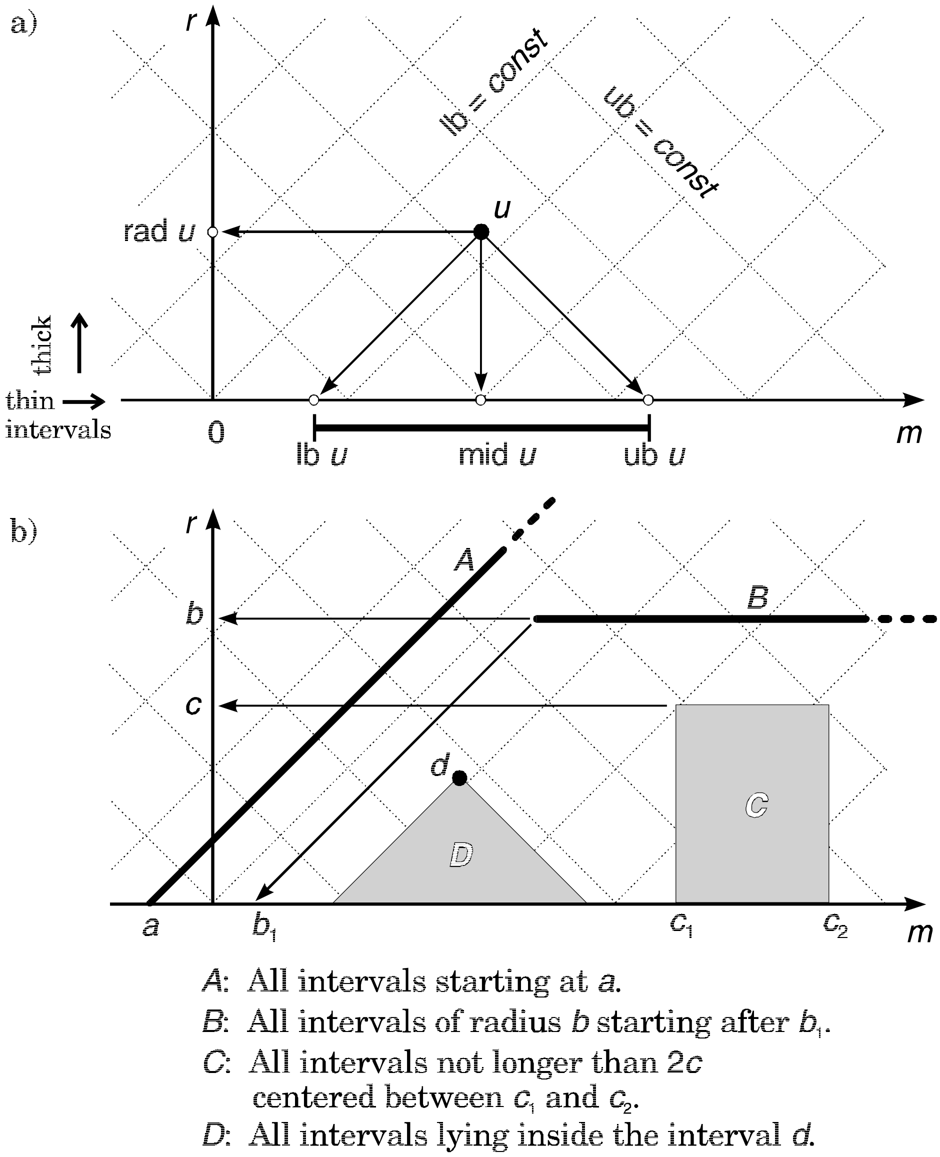

Fig 3. The *IS-diagram* of the space of relations (a), and representations of various sets of intervals (b)

Sets of intervals can be conveniently represented on the IS-diagram. They correspond to *lines* (one-dimensional sets) or *regions* on the (m, r) plane. Several illustrative examples are shown in Fig. 3b. Note that the set D (without the boundary) represents an image of one of the basic interval relations (the relation "contains"). If we consider the boundary of the region as belonging to the set, the closed set D will now represent the non-basic interval relation equal to the union of four *BIR*s (contains ∪ started-by ∪ finished-by ∪ equal) which might be named "covers", cf. Fig. 2 and the **P** relation of Figs. 7a and 8a.

Various interval relations and transformations have also convenient representations on the IS-diagram. In particular, the horizontal and vertical moves on the diagram correspond to *translation* (constant width) and *zooming* (constant position) transformations, respectively, while the diagonal moves correspond to interval transformations leaving one endpoint unchanged (single-end *stretching*), see Fig. 4.

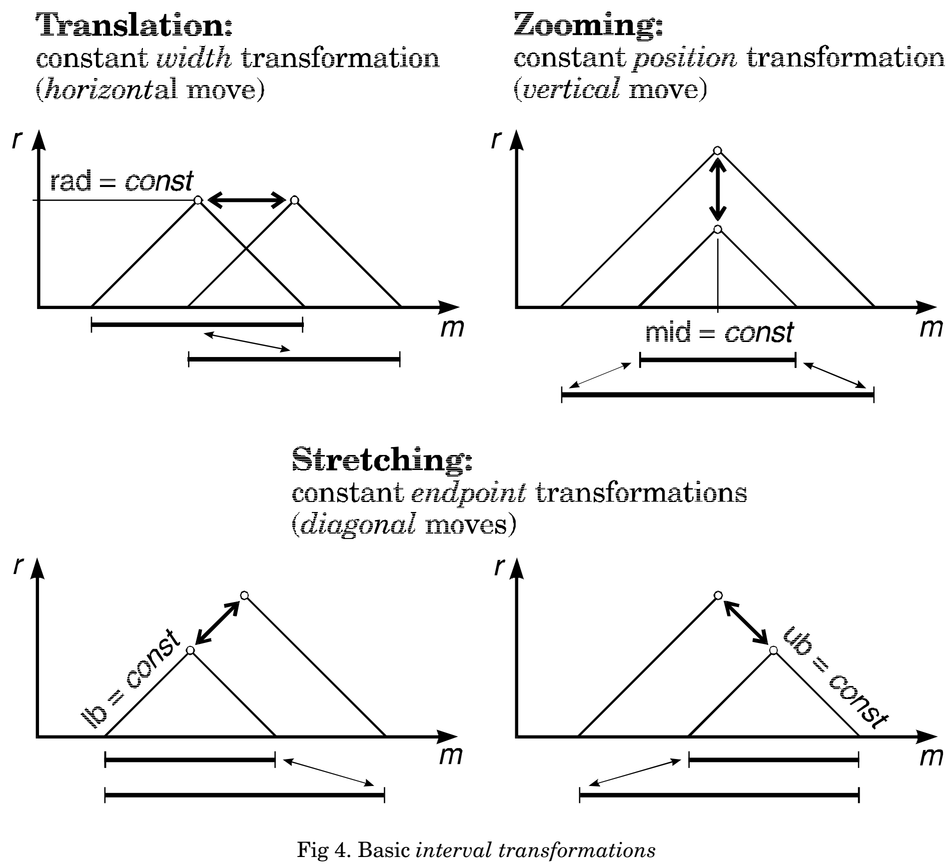

Fig 4. Basic *interval transformations* as represented on the IS-diagram

As shown in Fig. 5, representation of various cases of the in-between relation causes no problems on the IS-diagram — the lozenges are simply rectangles tilted by 45 degrees (in some cases, cut off by the m axis: remember that we do not take into consideration the "improper" intervals located below the axis). Classic one-dimensional representation of intervals is far less convenient in this respect, especially in some cases (e.g., non-overlapping intervals). Note also, that a diagonal line obtained as a trace of the single-end stretching transformation is also a (one-dimensional) lozenge defined by its endpoints, namely the original interval and the result of the transformation.

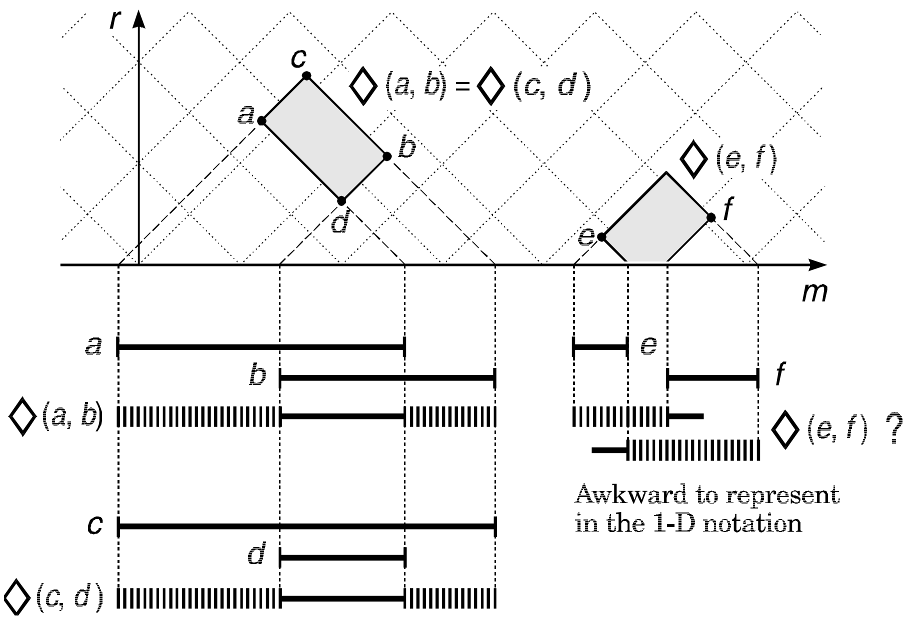

Fig 5. Representation of the *in-between* relation:
lozenges on the IS-diagram versus classic representation

4.2. The W-diagram

Sets of intervals are easily representable on the IS-diagram, and so are the *images* and *coimages* of interval relations. Representing images (or coimages) of all *BIR*s on the IS-diagram leads to the so-called *W-diagram*. The lines and regions in the *image W-diagram* (Fig. 6a) constitute the images (on the IS-diagram) of an arbitrary (thick) interval u under all 13 *BIR*s. The images are labelled by graphical symbols of corresponding relations. Since $u\mathbf{R} = \mathbf{R}^{-1}u$, the coimage diagram is essentially the same, only with relations changed to their inverses (inverse relations are placed on the diagram symmetrically with respect to the "=" relation). Hence, in the sequel the term W-diagram will usually mean "*any of the two*", whichever fits the current context best (which usually means the *image* W-diagram).

As can be easily seen from the diagram, all *BIR*s are disjoint and cover the whole space of intervals, i.e. every pair of intervals belongs to exactly one of these relations. Strictly speaking, this holds exactly only for thick intervals. E.g., if one of the intervals is thin and coincides with the endpoint of the other, there is ambiguity as to the proper relation holding between them. It can be resolved at the cost of introducing two additional basic relations, or else we may restrict the analysis to thick intervals only. Unless otherwise specified, the latter approach will be assumed in the following, as being simpler and sufficient for the purposes of this paper (as well as for most practical purposes [3, 13]).

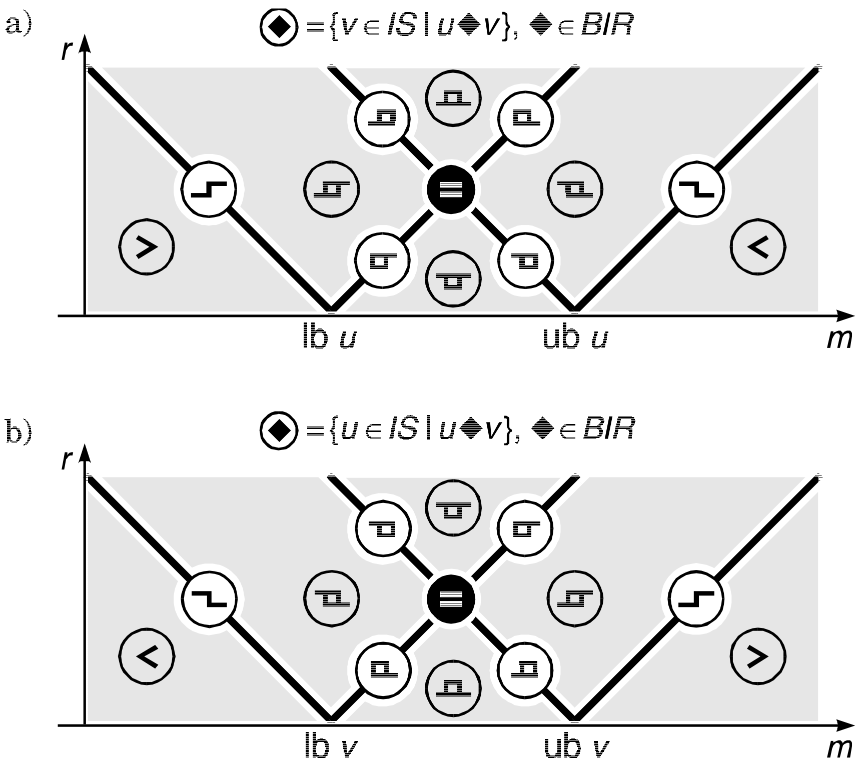

Fig 6. The *image* (a) and *coimage* (b) W-diagram of basic interval relations

The diagram shows that the basic relations fall into three distinct classes according to the dimensionality of their images: *0-dimensional* (a point; the "=" relation only), *1-dimensional* (lines), and *2-dimensional* (regions). Not surprisingly, the dimensionality corresponds (inversely) to the number of "**=**" conditions in the conjunction of terms relating the interval endpoints in the definition of the relation (see Fig. 2). Note also that all images of *BIR*s are open, i.e. regions do not contain their borders and lines their endpoints.

The W-diagram can be used as a basis of convenient graphical representation of arrangement interval relations and operations on them, see examples below.

4.3. The lattice of basic relations

Two interval relations are considered *neighbours* if their images can be linked by a diagonal move (i.e., by changing continuously only one endpoint of one interval, see Fig. 4) which does not cross an image of any other relation. Alternative definition says that two relations are neighbours if there exists a pair of intervals, one belonging to the image of the first relation, the other to the image of the second relation, such that the lozenge defined by them is fully contained within the (union of) images of these two relations.

Linking two basic relations with the edge when the two relations are neighbours produces a graph (Fig. 7a, called the *A-neighbourhood graph* in [13]) which, when turned into vertical position (with the right-hand node of the diagram on top), can be considered as a Hasse diagram of the *lattice of basic relations* $\boldsymbol{L}_{BIR}$ [11]. Freksa [13] used simplified diagrams of this kind as iconic symbols for *AIR*s (see also examples in Fig. 8 below). The nodes of the lattice graph are colour-coded in Fig. 7a according to the dimensionality of the corresponding relations; note that neighbour nodes differ in dimensionality by exactly one.

The structure of the lattice mirrors closely the structure of the W-diagram (compare Figs. 7a and 6a). An isomorphic *dual lattice* (with the lattice diagram put upside-down) corresponds to the coimage W-diagram. As a lattice defines partial order in a set, we can define intervals (*quotient sublattices*) over the set *BIR*. Two examples of such intervals are shown in Fig. 7; unions of relations belonging to such intervals (like the relations **P** and **Q** shown here, see also Fig. 8a) have some special properties, as described below. The construction of such relations can be made more clear when nodes of the lattice are replaced with the W-diagrams of corresponding relations, see Fig. 7b.

4.4. Example relations in diagrammatic representations

Four examples of *AIR*s, defined by means of formulas in the algebra of relations, the W-diagram notation, logical formulas depicted as conjunction diagrams, and as subsets of the lattice of basic relations (similar to icons used by Freksa [13]) are shown in Fig. 8. The relations are also described by the semantically meaningful English names.

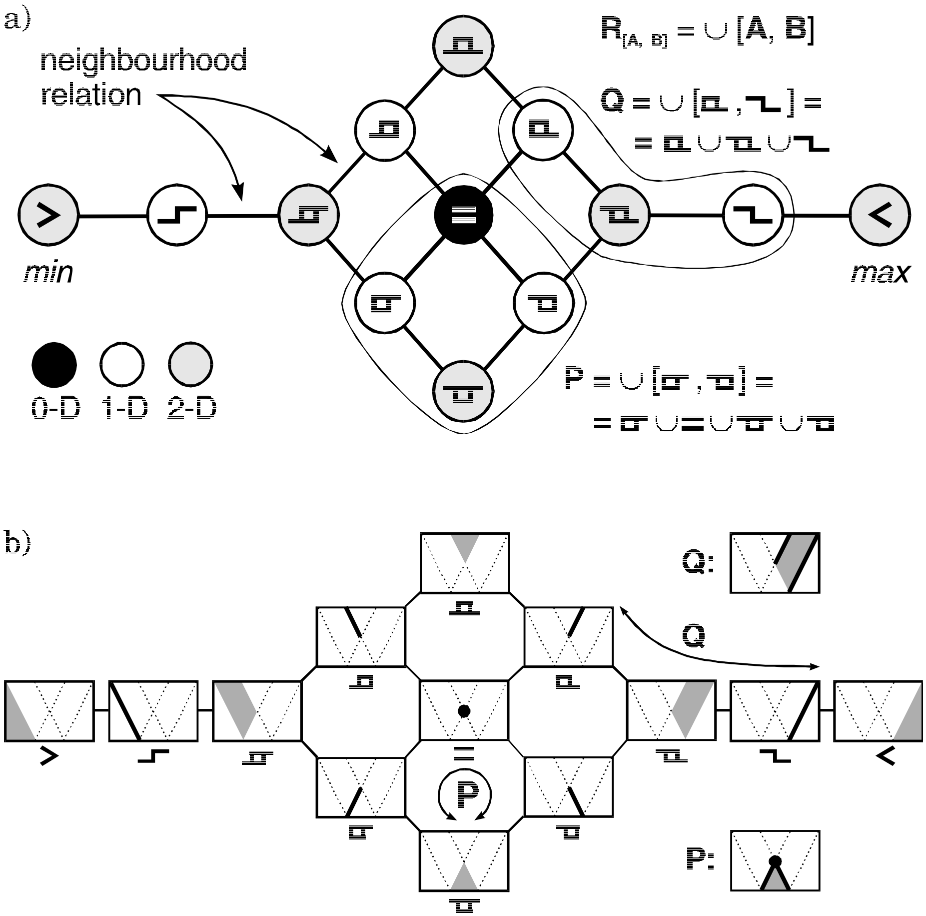

Fig 7. The lattice L_{BIR} of basic interval relations:
with relation symbols (a), and their W-diagrams (b) as nodes;
the example relations **P** and **Q** defined as intervals in the lattice

Note that the relations **Y** and **V** (Fig. 8b) have a different "look" than **P** and **Q** (Fig. 8a): they seem more complicated, require disjunctive logical formulas to define, and their W-diagrams and lattice diagrams look differently too — one may say, they look *less compact*. The observation is indeed significant — the **P** and **Q** relations belong to the important subclass of *convex relations*, as discussed in Sec. 5 below, whereas the relations **Y** and **V** are *not* convex.

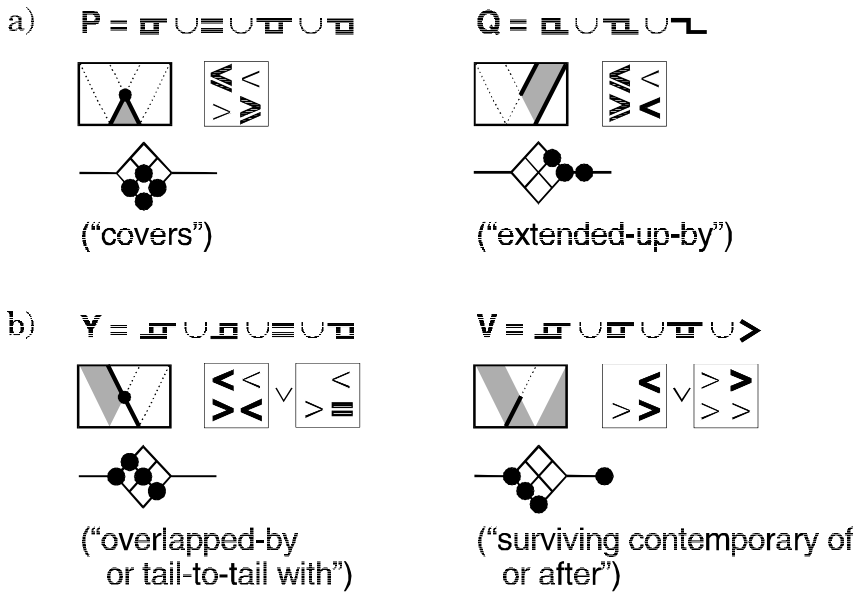

Fig 8. Some example relations and their diagrammatic representations

5. Convex interval sets and relations

The property of *convexity* of sets (and relations) is important in many respects and has a lot of uses, both in theoretical considerations and practical applications. The importance of *convex interval relations* (*CIR*s) comes from at least two facts:

- They constitute a class of relations coinciding more or less exactly with those that are common and natural in everyday reasoning involving intervals, especially reasoning about time and space [5, 7, 9, 11, 13, 17].
- Algorithms for solving several important problems involving networks of constraints between intervals are tractable (of polynomial complexity) within the *CIR* class (strictly speaking, this holds also for a wider class *PIR* of *pointisable* relations [9, 15]), whereas these problems often become *NP*-complete when other relations are also allowed [3, 9, 10, 11].

5.1. Definition and diagrammatic examples

Once the *in-between* relation is defined, the concept of a *convex set* follows naturally. The set S of intervals is *convex*, iff for every pair of intervals $u, v \in S$, also

$w \in S$ for every interval w lying between u and v. It is easy to formulate the definition in diagrammatic terms also: The set S of intervals is *convex*, iff any lozenge with opposite corners in S is fully contained in S. Fig. 9 illustrates the concept with examples of some non-convex sets of intervals.

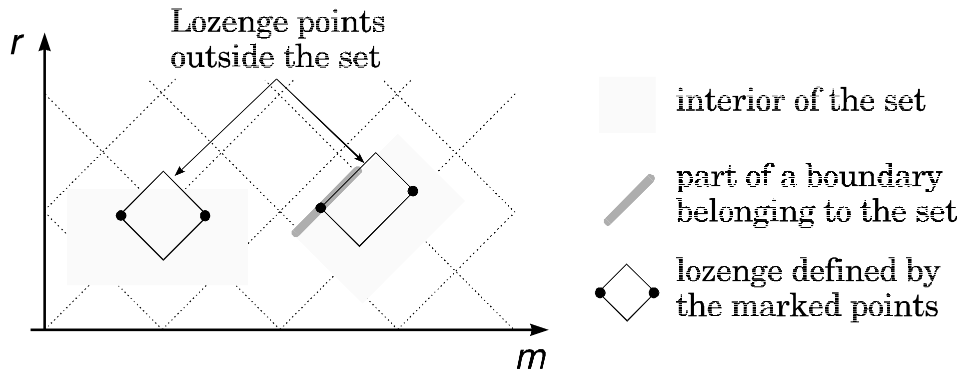

Fig 9. Examples of non-convex sets of intervals

The convexity of (arrangement) interval relations is defined analogously. The interval relation **R** is *convex*, iff for any intervals i, u, v such that i**R**u and i**R**v, also i**R**w for every interval w lying between u and v. Again, in more diagrammatic terms, the interval relation **R** is *convex*, iff any lozenge with opposite corners in the image (or coimage) of **R** is fully contained in this image (or coimage) — that is, if its image (or coimage) in the IS-diagram is a convex set of intervals. Relations **P** and **Q** in Fig. 8a are convex, whereas relations **Y** and **V** in Fig. 8b are non-convex. We can easily verify this diagrammatically, using the W-diagram representations of these relations in Fig. 8 and the definition given above.

5.2. Equivalence theorem

As we have seen in the previous Section, there are at least two different definitions of convex interval relations. In fact, other definitions are also possible, as stated by the following:

Theorem 1. For arrangement interval relations, all the definitions (listed below) of convex interval relations are equivalent. An interval relation **R** is in *CIR* iff:

[*Primary characterisation*] — for any interval i and every pair of intervals u, v such that i**R**u and i**R**v, also i**R**w for every interval w lying between u and v.

[*W-diagram characterisation*] — its image (or coimage) in the W-diagram includes together with any two intervals also the lozenge defined by these intervals.

[*Lattice characterisation*] — it is a union of all relations belonging to some interval (including thin intervals) over a lattice $\boldsymbol{L}_{BIR}$ of basic interval relations [11].

[*Term characterisation*] — it can be defined as a conjunction of simple terms involving only relations in the set {≤, ≥, <, >, =} defined in the base set between interval endpoints, (i.e., the order relation, its inverse and their negations, plus equality) [9].

Diagrammatic reasoning using the diagrams described here has been shown to be very useful as a tool for proving the above theorem, see another paper by the author [24] for the diagram-aided proof of the theorem. The reader is encouraged to verify the various characterisations given in the theorem on the example interval relations **P**, **Q**, **Y**, and **V** (Figs. 7 and 8).

5.3. Decomposition of convex relations

In the course of proving some of the equivalencies in the above theorem (see [24] for details), two other lattice diagrams turned out to be useful. First, note that any interval l in $\boldsymbol{L}_{BIR}$ can be obtained as an intersection of two other intervals. One of them is defined by the lowest element of the lattice and the end of the interval l, and the other defined by the beginning of the interval l and the uppermost element of the lattice (see Figs. 7 and 10ab). Such “lower-end” and “upper-end” intervals are called *ideals* and *filters* in the lattice theory terminology. Every element of a lattice defines its ideal and its filter, and the sets of ideals and filters constitute their own lattices, isomorphic with the original lattice.

Figures 10a and 10b give these lattices for $\boldsymbol{L}_{BIR}$, labelled by W-diagrams of relations defined by appropriate ideal and filter. The W-diagrams for the ideal and filter relations are easily obtained as unions of W-diagrams associated with nodes of $\boldsymbol{L}_{BIR}$ (Fig. 7b) included in the given ideal or filter. Note that every relation corresponding to an ideal is an inverse of a relation corresponding to a filter that is placed symmetrically to it in the appropriate lattice, compare Figs. 10a and 10b. Also, the ideal and filter relations are obviously convex (as being defined by certain intervals in $\boldsymbol{L}_{BIR}$).

Hence, as it was shown in [24], every convex interval relation can be decomposed into an intersection of two (also convex) interval relations corresponding to the ideal and filter whose intersection gives the interval l, see the construction for the relations **P** and **Q** in Fig. 10c.

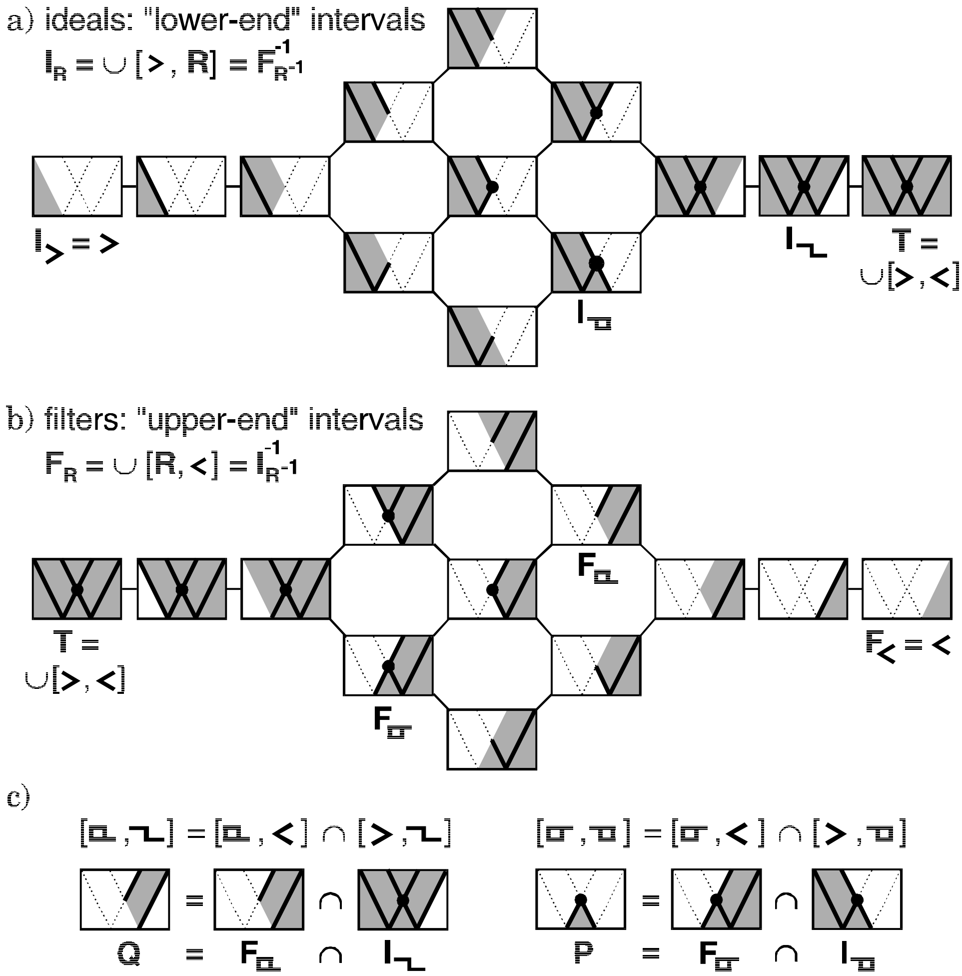

Fig 10. The lattices of filters (a) and ideals (b) of the lattice L_{BIR},
and construction of example relations **P** and **Q** (c)

6. "*Meeting at Lunch*" example

Since the IS-diagram is convenient as a tool to represent various sets of intervals, it can be also used for solving various problems involving specifying such sets and reasoning about them. An example problem of this type, let us call it the "*Meeting at Lunch*" problem, may be formulated as follows:

> *"I want to meet someone **during my lunch break**. I will need **at least 20 minutes** to discuss the matter with the person, and my partner **must depart***

***before 12:45pm**. My lunch break may start **not earlier than 11:30am** and must **end at most at 1pm** and may take **from half an hour to an hour**. When can I arrange the meeting?"*

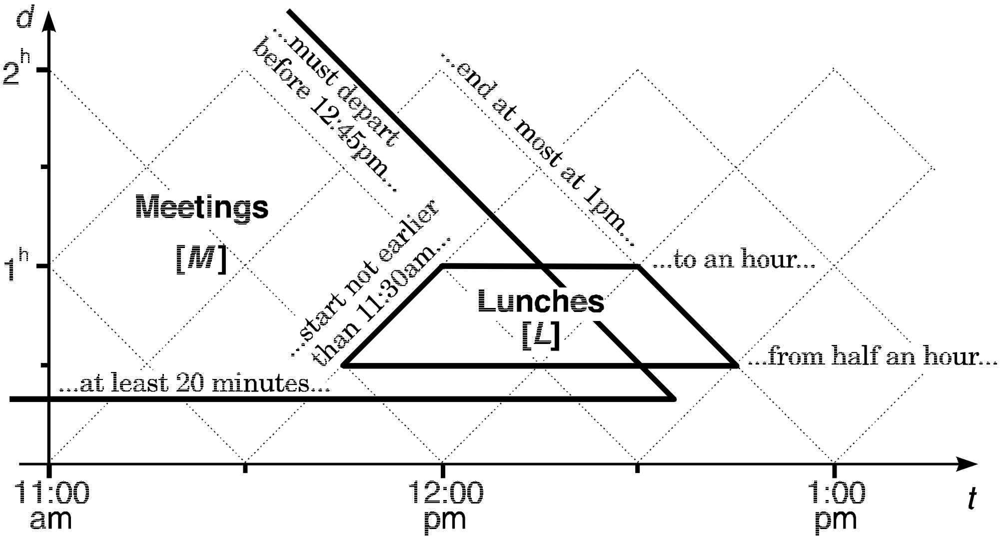

Fig 11. Diagrammatic translation of the "meeting at lunch" problem

As can be seen, various conditions highlighted in the problem formulation above define certain sets of intervals, in particular the set of possible meetings M and the set of possible lunches L. These conditions can be easily translated into diagrammatic form, as shown in Fig. 11, where the conditions are written along or next to the corresponding line segments forming the boundaries of the sets M and L.

After translating definitions of these sets into diagrammatic form, the problem can be easily solved diagrammatically, as shown in Fig. 12. First, as the formulation uses the condition "*during my lunch break*", we must generate a *coimage* of the set L of possible lunches under the interval relation "during". This is easily done, with the help of the W-diagram showing the coimage of a single interval under this relation (Figs. 2 and 6b, see also the set D in Fig. 3b), as shown by the shaded area (both light and dark) in Fig. 12. Why we must generate a coimage rather than an image? We are looking for the set $\{x \mid x$ during $L\}$, i.e., a set ⊓ L, and this means a coimage, according to the definitions in Sec. 3. Note also, that "*during the lunch break*" means different thing than "*within the region L*" in Fig. 11: The latter contains all possible *full* lunches (i.e., intervals at least half an hour long, according to the problem formulation), whereas the former includes also time intervals shorter than that but contained in some possible lunch (hence the shaded area extends also *below* the region representing lunches).

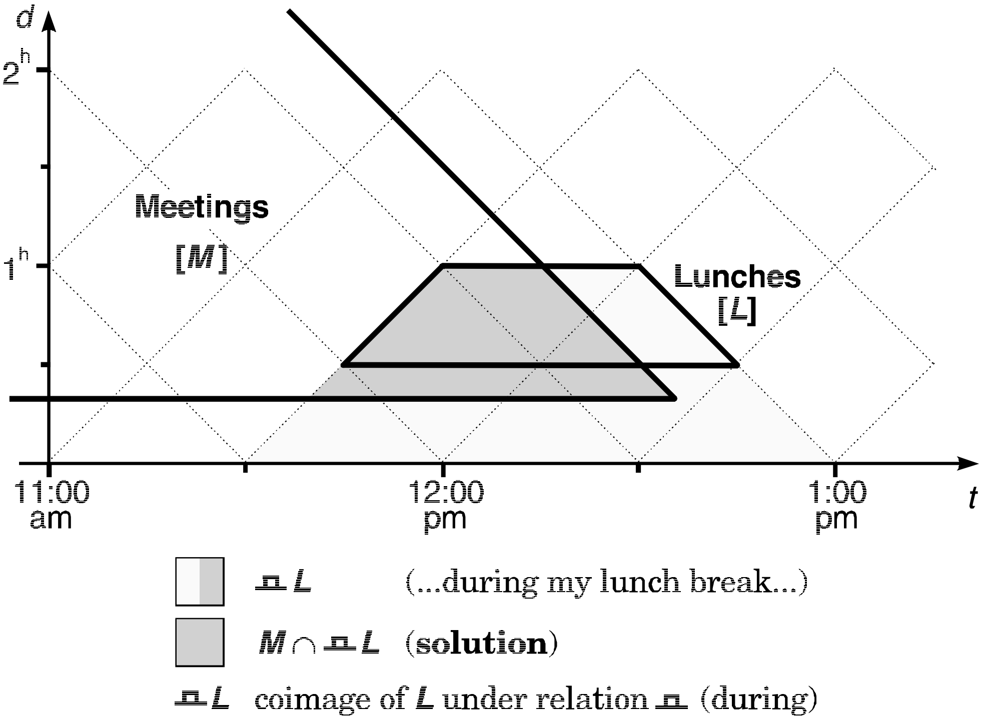

Fig 12. Diagrammatic solution of the “meeting at lunch” problem

Then, we are interested in all intervals that can represent a valid meeting (according to the constraints imposed on the set *M*) and at the same time are occurring “*during the lunch break*”, i.e., are members of the coimage ⊓*L*. This is easily obtainable as an intersection of the set ⊓*L* with the set *M*, as shown by the dark shaded area in Fig. 12.

The obtained diagrammatic representation of the solution set can now be easily translated back into textual form (with the help of correspondences shown in Fig. 11), as follows:

> *“The meeting may start between 11:30am and 12:25pm, and will end between 11:50am and 12:45pm, taking from 20 minutes to an hour.”*

From the diagram, without much further manipulations, we can also easily read a number of other pieces of information about the situation described by the problem. E.g., we can find that the longest, one-hour meeting might end not earlier than 12:30 (the end of the interval at upper left corner of the solution set), or that to be sure that our "someone" will not waste more than half an hour of our precious lunch time we should rather avoid scheduling it for earlier than 12:15.

7. Conclusions

A two-dimensional, diagrammatic representation of the space of intervals has been elaborated and shown to be useful for representing knowledge and making inferences about intervals and interval relations. The representation comprises several diagrammatic tools, namely the interval space diagram (the *IS-diagram*), the diagrammatic representation of interval relations (the *W-diagram*), and auxiliary diagrams: the *conjunction diagram* (for compact representation of logical definitions of interval relations) and *lattice diagram* (of several lattices of interval relations).

Further avenues of research include, among others, various possible extensions of the representation (e.g., to handle multidimensional intervals (interval vectors), improper intervals, and various areas of interval arithmetic), as well as formalization of the representation and diagrammatic inference rules [14, 22, 23] needed for use of the representation in formal reasoning within the algebra of intervals.

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