On the data model for dynamic hierarchical scheduling problem

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Abstract:: In practical applications, the scheduling problem should enable taking into account dynamic changes of tasks and resources. Very often, the task must be considered as a complex object possesing its own resources. Thus, we obtain a hierarchy of cooperating units, e.g., computer network, multiprocesor computer unit, matrix/vector processor, and scalar processor, all together making possible to distribute scheduling procedure into the hierarchy of levels and into several parallel processes at each level. This way we have an opportunity to increase the scheduling system performance. But from the point of view of a single unit, the higher its position in the hierarchy, the larger search space for its optimal schedule and, what it follows, more difficoult or practically impossible exact (analytic) solution of the problem.

In this paper we propose a method of scheduling based on this hierarchical structure with its origin in dynamic distributed production process. Our model applies the Prioritized Fuzzy Coinstraints Scheduling Problem approach (PFCSP) [7]. This makes the obtained schedules robust against small fluctuations of the model variables what makes possible not to carry through all the optimization procedure every time the parameter change. The basis of the model is a specific structure of the data (hiperdata) displaying the structure of the functional units of the schedule. Due to an autonomy of the units, we apply protocols to adjust optimal overall schedules. Every supervising unit negotiates the local schedule with subordinate units, that prepare propositions according to their own optimality criteria (e.g., cost, full employment, quality, etc.).

Key words: Fuzzy theories, hierarchical scheduling, data models.

1. INTRODUCTION

In classical scheduling problem one resource cannot make more than one task at the same time, see [5]. If the project is detailed enough, such a scheduling model is sufficient to obtain the solution. However, in hierarchical projects, where the supervising unit do not have complete information about the subordinate participants possibilities. There is some confidential information like technology know-how (effecting in certain time regime within the company) or management and organization details of the company. Such autonomic, distributed functional units constitute the hierarchical network The purpose of this structure is to perform some distributed process in an optimal way. Like in other autonomic distributed systems (e.g. Meeting Scheduling Problem in multiagent systems, see [8]), we propose the protocol of negotiation of the optimal schedules. Such a protocol makes it possible to decompose the overall schedule into local schedules within individual units on each hierarchy level. This is especially useful if there are several similar units performing analogous sub-processes, what makes possible to parallelize the elements of the schedule. The classical scheduling process takes into account the schedule time limitations, possibilities of the resources and the constraints caused by the immediate succession of the tasks, see [1], [5]. It constructing the optimal schedule is natural taking into account additional quality measures of the schedule, e.g. overall cost of the process (due to different costs of individual resources), stability of the system (including all the participants of the network into the schedule during temporarily low loading), costs of the component processes, etc. We introduce measures describing the component quality factors and, to obtain the global optimal solution, the measures aggregation methods, see [10], [11].

In our further considerations we increase the scheduling flexibility by application of Prioritized Fuzzy Constraint Satisfaction Problems (PFCSP) approach [5], [7], [10], which gives an additional possibility of prioritizing tasks sharing over resources. This procedure let us possible to generate different strategies, e.g., to increase the reliability of the schedule, to support some partners or to stabilize the consortium structure in dynamic environment, etc.

The structure of the paper will be the following. In the first section we formulate the problem, give the definition of a certain functional unit at a level and the hierarchy of levels. Next, we introduce the measures of quality of the possible solutions. Section 3 gives a proposition of the protocol of negotiating the schedule acceptable both from the overall optimality criteria and individual partners limitations point of view. Finally, we discuss some related possible extensions of the model. In the Appendix we present the definitions of specific mathematical operations used in the paper.

2. HIERARCHICAL SCHEDULING MODEL DEFINITION

The hierarchical scheduling problem can be considered as a case of the constraints problem. Before we give the complete definition of the hierarchical scheduling model, we present some fundamental definitions and elementary examples.

Example 1. Consider a model of the factory of some mass-produced electronic components. Each component is produced in n steps, $\{e_1,...,e_n\}$. The schedule of the production of such an element is the sequence of pairs $\{[t_i^s, t_i^w]: \forall i \in \{1,...,n\}\}$, where t_i^s and t_i^w are the start time and the end time of the step e_i, respectively. The time instants can satisfy some relations, e.g., $t_{10}^w \le t_{12}^s$ (what means that step e_{10} should be finalized before step e_{12} starts). The ensamble of all relations defined on the set of steps constitutes the constraints satisfied by a certain schedule.

To define constraints, one can use fuzzy relations, see [5], [7], [10].

Definition 1. The fuzzy constraints problem is defined as the triple (X, D, C^f), where:

- $X = \{x_1, x_2, \ldots, x_n\}$ is the set of variables.
- $D = \{d_1, d_2, \ldots, d_n\}$ is the set of domains, where d_i is the domain of x_i.
- $C^f = \left\{ R_i^f : \left(\prod_{x_j \in \mathrm{var}(R_i^f)} d_j \right) \to [0,1] \right\}$, where $\mathrm{var}(R_i^f)$ is the set of variables of the fuzzy relation R_i^f. This means that C^f is the set of fuzzy relations defined for the variables belonging to the set X.

Example 1 (continuation). We can define the fuzzy relation between steps e_1 and e_2 in the following way: $t_i^w - t_j^s$ is approximately 120 minutes and $i, j \in \{1,2\}$ and $i \neq j$. Such a relation can be considered as a fuzzy set A over the domains of the variables t_i^w and t_j^s by the following correspondence:

$$\chi_A(t_i^w, t_j^s) = \begin{cases} \dfrac{|t_i^w - t_j^s|}{20} - 5 & for \quad |t_i^w - t_j^s| \in [100,120], \\ -\dfrac{|t_i^w - t_j^s|}{20} + 7 & for \quad |t_i^w - t_j^s| \in [120,140], \\ 0 & otherwise. \end{cases}$$

The above relation plays a fundamental role in the technological process. It is the most important one in the set of all relations. To make possible gradation of the relations between tasks one can introduce the priority functions for the fuzzy constraints problem, see, e.g. [5], [7], [10].

Definition 2 The *prioritized fuzzy constraints problem* is the quadruple (X, D, C^f, ρ), where the first triple constitutes the fuzzy constraints problem and

$\rho : C^f \to [0,\infty)$ is the *priority function*.

Example 1 (continuation). We can extend the production scheme presented in the example. Now, consider the company h which consists of three factories $\{f_1, f_2, f_3\}$. Each factory is able to produce certain class of electronic details. The company can produce electronic elements containing components coming from factories $\{f_1, f_2, f_3\}$.

This way we obtain the hierarchical production structure. The supervising unit distributes tasks in such a way that the resources of all factories are used in an optimal way. To formalize this procedure we start from the precise definition of the unit.

Definition 3. The *functional unit* is an object which can run and distribute tasks. The functional unit is represented by the six elements $(J, \mathcal{T}, \mathcal{X}, \mathcal{D}, \mathcal{M}, \mathcal{C})$:

1. $J = \{J_j\}_{j=1}^{n}$ is the set, which elements are the jobs defined for the unit.
2. $\mathcal{T} = \{T_j\}_{j=1}^{n}$, where $T_j = \{t_{j,l}\}_{l=1}^{n}$ is the set, which elements are the operations defining the jobs J_j, $j = 1,2,\dots,n$.
3. $\mathcal{X} = \{\mathcal{X}_\mathcal{T}, \mathcal{X}'_\mathcal{T}\}$, $X_{j,i} \in \mathcal{X}_\mathcal{T}$ is the set of arguments of $i-th$ operation in $j-th$ job and, analogously for $X'_{j,i} \in \mathcal{X}'_\mathcal{T}$;[1]
4. $\mathcal{D} = \{\mathcal{D}_\mathcal{T}, \mathcal{D}'_\mathcal{T}\}$ is the set of the domains $D_{j,i}$ and $D'_{j,i}$ of the variables belonging to the sets $X_{j,i} \in \mathcal{X}_\mathcal{T}$ and $X'_{j,i} \in \mathcal{X}'_\mathcal{T}$, respectively.
5. $\mathcal{M} = \left\{ M_j^k : \left(\prod_{x_j \in \mathrm{var}(M_j^k)} d_j \right) \to d'_k, d_k^i \in D'_{j,i} \right\}$ is the set of measures.
6. $\mathcal{C}$ is the set of prioritized fuzzy constraints problems C_T^f, C'^f_T over the variables $\bigcup_i X_{j,i}$ and $\bigcup (X_{j,i} \cap X'_{j,i})$, respectively.

Example 1. (continuation). The factory f_1 defined above is a simplified example of the object of Definition 3. The structure of the functional unit makes possible to describe tasks themselves and their costs. Assume that the factory must run several jobs of the same type J_3, described by the set of operations T_3. Each sample job can have different schedule and, what follows, different cost dependent of costs of run operations. If the factory produces some final products than it can happen that

[1] For simplicity of the reasoning we assume that the first set of variables $(\mathcal{X}_\mathcal{T})$ describes time variables (start and end time) while the second one $(\mathcal{X}'_\mathcal{T})$ describes the other measures of the schedule's quality (e.g. costs, product quality, etc.).

some components (certain chips) should be done in other units (members of the company). Assume that the operation $T_{3,5}$ is run by factory f_2 as its job J_{10}. The factory f_2 calculates the exact schedule for this job, that is time of beginning and time of end of individual operations. This way, it fixes the resultant time of the job J_{10} and the production costs. This information is sent to the factory f_1 and it can be used for preparation of the schedules in the factory f_1.

The task described in the above example can be realized in the following hierarchical functional model.

Definition 4. Let $\mathcal{F}$ be the set of functional units. The *functional unit in the hierarchical model* is the object, which is the functional unit in the sense of Definition 3 extended by the three elements:

7. $\mathcal{E} = \{E_{j,i} : E_{j,i} \in \mathcal{F}\}$ is the set of allowances for orders (fixing which superior unit can order its operations at a subordinate unit).

8. $C_E^f = \{R_{j,i} : E_{j,i} \to [0,1]\}_{i,j}$ is the set of fuzzy relations.

9. $\rho : C_E^f \to [0,\infty)$ is the priority function.

If the functional unit f_i can run the operation $t_{j,l}$ itself, then $f_i \in E_{j,l}$. Consider the graph $G = (V, \mathcal{E})$, with the set of vertices $\mathcal{E}$ and the set of edges V. If $v_{i,j} \in V$ then one knows that the unit f_i orders some tasks at the unit f_j. To create an optimal schedule one should define graph G without cycles. The models of technological processes where graphs have cycles lead to redundant loading of the system.

Theorem. The object described by Definition 4 can be introduced, in the equivalent way, by two following statements:

1.The functional unit in the hierarchical model is the functional unit in the sense of Definition 3, where the operations (of the superior unit) are run by (subordinate) functional units as their jobs, both selected with certain preferences.

2.The functional unit in the hierarchical model is the functional unit in the sense of Definition 3, where the jobs (of subordinate units) correspond to operations of some (superior) functional units, both selected with certain preferences.

Proof.

I. Let $f \in F$ be the functional unit according to Definition 4. Assume a certain operation $T_{j,i}$. It has its well-defined set $F \supseteq E_{j,i} \neq \emptyset$. $\forall f \in E_{j,i} \exists\, l : T_{j,i}$ is J_l^f. This means, that the functional units $E_{j,i}$ can run operations $T_{j,i}$ as their jobs. As the importance of the operations $T_{j,i}$ we take the values of $\rho(R_{j,i})$, where corresponds to $E_{j,i}$, while as the preferences of units we take the values of relations $R_{j,i}$. Thus, the functional unit f satisfies conditions of the definition given in point 1). Inversely, let $f \in F$ be the functional unit according to definition given in point 1). To show that it satisfies conditions of Definition 4 it suffices to define, for each operation $T_{j,i}$, the set $E_{j,i}$. It contains the units which can run the operation as their jobs. The relations $R_{j,i}$ are defined as

$$R_{j,i}(f) = \frac{p_{j,i}(f)}{p_{\max}}$$

where $p_{j,i}$ is the function of a subordinate unit selection preference and $p_{\max} = \max_{f \in E_{j,i}} (p_{j,i}(f))$. The priority function $\rho(R_{j,i})$ now takes the value of the importance of the operation $T_{j,i}$.

II. Let $f \in F$ be the functional unit according to Definition 4. Fix the job J_k. Create the set $E'_{f,j}$:

$$E'_{f,j} = \left\{ f' \in F : f' \in F \wedge f \in f'(E_{j,i}) \wedge T_{j,i} \text{ is } J_k \right\}.$$

It contains the functional units having an operation which corresponds to the job J_k. The value of the importance of the operation corresponding to the job J_k is defined as $\rho(R)$ for $f' \in E'_{f,j}$. The preference of selecting the unit f to run the operation T is now equal to $R(f)$. Thus, the unit f satisfies conditions of definition given in point 2). Inversely, let $f \in F$. Then one should define sets $E_{j,i}$. For any j,i

$$E_{j,i} = \left\{ f' \in F : f' \in F \wedge \exists\, l : J_l \text{ is } T_{j,i} \right\}$$

The relations $R_{j,i}$ and the function ρ are defined as in point I. Thus, f satisfies conditions of Definition 4.

3. THE ASSESSMENT OF THE SCHEDULE QUALITY

In the previous chapter we defined the hierarchical scheduling model. In this model the functional units can run the operations themselves or order them at some subordinate unit. All decisions of such a kind lead to some schedule. The final schedule must be then evaluated, that is one must verify if all relations are satisfied and, finally, calculate the values of measures on the relations. In the case when relations are without priorities it can be easily done. The general satisfaction degree $\alpha(\mathbf{x})$ is defined in the following way, see [5], [7].

Definition 5. The *general satisfaction degree* is measured by

$$\alpha(\mathbf{x}) = \Delta \left\{ \mu\left(R^f \left(\mathbf{x}_{\operatorname{var}(R^f)} \right) \right) : R^f \in C^f \right\},$$

where Δ is $T-norm$.

We decide that the value of $\mathbf{x}$ is acceptable it $\alpha(\mathbf{x}) \geq \alpha_0$ for some assumed threshold value α_0.

In the case when the relations R^f have priorities $\rho\left(R^f\right)$ the Definition 5 can be generalized to consider also such a situation, see [7].

Definition 6. The *global satisfaction degree* for the relations $R^f \in C^f$ is measured by

$$\alpha(\mathbf{x}) = \oplus\left\{g\left(\rho\left(R^f\left(\mathbf{x}_{\mathrm{var}(R^f)}\right)\right), \mu_{R^f}\left(\mathbf{x}_{\mathrm{var}(R^f)}\right)\right) : R^f \in C^f\right\},$$

where

$$\oplus : [0,1]^n \to [0,1]$$

and

$$g : [0,\infty) \to [0,1].$$

The operator $\oplus$ and the function g should satisfy the following conditions:

1. If for a certain $\mathbf{x}$ the relation with the greatest priority function has the realization degree equal to 0 then $\alpha(\mathbf{x}) = 0$.

2. If every relation takes the same value of the priority function then $\alpha(\mathbf{x})$ is calculated according to Definition 5.

3. If for $R_i^f, R_j^f \in C^f$, where $\rho\left(R_i^f\right) \geq \rho\left(R_j^f\right)$ and for any $\mathbf{x}$ and $\mathbf{x}'$ the following conditions are satisfied:

a) $\forall R^f \in C^f \setminus \left\{R_i^f, R_j^f\right\},\ \mu_{R^f}\left(\mathbf{x}_{\mathrm{var}(R^f)}\right) = \mu_{R^f}\left(\mathbf{x}'_{\mathrm{var}(R^f)}\right)$,

b) $a_i = \mu_{R_i^f}\left(\mathbf{x}_{\mathrm{var}(R_i^f)}\right) = \mu_{R_i^f}\left(\mathbf{x}'_{\mathrm{var}(R_i^f)}\right) + \delta$

c) $a_j = \mu_{R_j^f}\left(\mathbf{x}_{\mathrm{var}(R_j^f)}\right) = \mu_{R_j^f}\left(\mathbf{x}'_{\mathrm{var}(R_j^f)}\right) - \delta$

then

$$g\left(\rho\left(R_i^f\right), a_i\right) \leq g\left(\rho\left(R_j^f\right), a_j\right) \Rightarrow \alpha(\mathbf{x}) \geq \alpha\left(\mathbf{x}'\right)$$

4. If for any $\mathbf{x}$ and $\mathbf{x}'$ the following condition is satisfied

$$\forall R^f \in C^f\ \mu_{R^f}\left(\mathbf{x}_{\mathrm{var}(R^f)}\right) \geq \mu_{R^f}\left(\mathbf{x}'_{\mathrm{var}(R^f)}\right),$$

then

$$\alpha(\mathbf{x}) \geq \alpha\left(\mathbf{x}'\right)$$

5. If for some $\mathbf{x}$ all the relations have the realization degree equal to 1 then $\alpha(\mathbf{x}) = 1$.

The function g calculates the local degree of satisfaction of the relations R^f, under the condition that $\rho\left(R^f\right)$ takes a certain value. The operator $\oplus$ is taken here instead of the $T - norm$ used in the definition 5.

Following the paper [7] we can present how to generate the function g. This function can be expressed by the generalized division operator (see Appendix)

$$\varnothing : (0,\infty)\times(0,\infty)\to[0,1]$$

and the priority operator

$$\Diamond : [0,1]\times[0,1]\to[0,1]$$

The function g is defined as

$$g\left(\rho\left(R^f\right),\mu_{R^f}\left(\mathbf{x}_{\mathrm{var}(R^f)}\right)\right)=\left(\rho\left(R^f\right)\varnothing\ \rho_{\max}\right)\Diamond\ \mu_{R^f}\left(\mathbf{x}_{\mathrm{var}(R^f)}\right)$$

This function lets us to generate new functions defining values of $\mu^{\rho}_{R^f}\left(\mathbf{x}_{\mathrm{var}(R^f)}\right)$ and being the local degree of satisfying the relations R^f. All the definitions of the operators used in the above are given in the Appendix.

The simplest example of generalized division operator $\varnothing$ is

$$a_1 \varnothing a_2 = \frac{a_1}{a_2}.$$

The example of the priority operator $\Diamond$ is

$$a_1 \Diamond a_2 = \min\left(1-a_1, a_2\right).$$

Some example of the operator $\oplus$ is every the priority $T-norm$, defined as:

Definition 7. The two-argument function Δ is the *priority* $T-norm$ if it is a $T-norm$ and if it additionally satisfies the following condition:

$$0\le a_1\le a_2\wedge\rho>0\wedge a_1+\rho\le 1\wedge a_2+\rho\le 1\ \Rightarrow\ \left(a_1+\rho\right)\Delta\ a_2\ge a_1\ \Delta\left(a_2+\rho\right)$$

The simple examples of the priority $T-norm$ are:

1. $\Delta\left(a_1,a_2\right)=\min\left(a_1,a_2\right)$
2. $\Delta\left(a_1,a_2\right)=a_1\cdot a_2$

The results presented in this chapter can be applied in the hierarchical scheduling models. Assume that a certain schedule $\mathbf{h}$ of the job J_j is given in the form of the values of the variables belonging to the sets $X_{j,i}, X^{'}_{j,i}$. The next step is fixing the two sets of relations $C^f_T\left(\mathbf{h}\right)\subset C^f_T$ and $C^{'f}_T\left(\mathbf{h}\right)\subset C^{'f}_T$, that will be verified for the schedule $\mathbf{h}$.

1. $C^f_T\left(\mathbf{h}\right)=\left\{R^f : R^f\in C^f_T\cup C^f_E\wedge\mathrm{var}(R^f)\subset X_{j,i}\right\}$,
2. $C^{'f}_T\left(\mathbf{h}\right)=\left\{R^f : R^f\in C^{'f}_T\cup C^f_E\wedge\mathrm{var}(R^f)\subset X_{j,i}\right\}$.

These relations are those, whose arguments have been selected by the schedule $\mathbf{h}$. Now we calculate the local degree of satisfaction for the relations $R^f\in C^f_T\cup C^f_E$ according to

$$\alpha^{\rho}_{R^f}\left(\mathbf{h}\right)=g\left(\rho\left(R^f\right),\mu_{R^f}\left(\mathbf{h}\right)\right)$$

Next, one should calculate the values of $\alpha\left(\mathbf{h}\right)$ and $\alpha^{'}\left(\mathbf{h}\right)$. Both calculated numbers must exceed the threshold values α_0 and $\alpha^{'}_0$.

4. PROTOCOL OF THE SCHEDULE AGREEMENT

In the previous chapter we introduced methods letting us to verify the correctness of the schedule. However, to take all the advantages of the hierarchical model of scheduling one should to propose an adequate communication protocol between the functional units on every level in the hierarchy. This protocol makes possible to transmit the data from unit to unit. The proper scheduling needs sending orders containing all required data to let subordinate units prepare themselves the schedule of their jobs. As a return message the units send to their superior unit the immediate data describing their optimal schedule. The orderer accepts the schedule or asks to prepare a new one satisfying updated requirements. This way the schedule is calculated iteratively. The initial required quality of the schedule is high. Then, in the case of problems with calculation of the high-quality schedule, the threshold quality must be decreased to enable finding solution.

Now we propose the protocol of negotiations of the schedule of the job $J_j \in J$ for the functional unit $f \in F$. The protocol runs in several steps:

1) Initiation.
   a) Determine the temporal limitations of the job J_j running
   b) Estimate (with some traditional methods) the temporal limits for the individual operations $t_{j,i}$
   c) Determine of the reply time $t > 0$
   d) Initially assume the threshold values for the quality α and the cost $\alpha^{'}$ of the schedule
   e) Assume the value of $\alpha_0, \alpha_o^{'}, \Delta\alpha > 0$
   f) Assume the maximal number of iterations i_1, i_2, i_3 .
2) Send a tender to every unit $f \in E_{j,i}$ concerning running operation $t_{j,i}$ together with the limiting values of the variables form the file $X_{j,i}$ and the set of variables $X_{j,i}$ and $X^{'}_{j,i}$ that must be set.
3) The functional units $f \in E_{j,i}$ process the query and not later than the reply time $t > 0$ return to the unit f the values of the variables and the values of the measures. The units that do not reply in time are excluded from the negotiation.
4) The functional unit f calculates the schedule $\mathbf{h}$
5) Evaluation of the schedule: calculate $\alpha(\mathbf{h}), \alpha^{'}(\mathbf{h})$ and calculate the satisfaction degree of the relations, $\left\{\mu_{R^f}\left(\mathbf{h}_{\mathrm{var}(R^f)}\right) : R^f \in C^f_{T_j}(\mathbf{h})\right\}$.

   5a) If $\alpha(\mathbf{h}) \geq \alpha$ then go to 6) else go to 5b)

   5b) Check the arguments $\mathrm{var}(R_{\min})$ of

$$R_{\min} = \left\{R \in C^f_{T_j}(\mathbf{h}) : \forall R^f \in C^f_{T_j}(\mathbf{h}),\ \mu^{\rho}_{R}\left(\mathbf{h}_{\mathrm{var}(R)}\right) \leq \mu^{\rho}_{R^f}\left(\mathbf{h}_{\mathrm{var}(R^f)}\right)\right\}.$$

The schedules for the operations dependent on $\mathrm{var}(R_{\min})$ must be modified.

5c) Iterate Step 5) until going to Step 7) or until the iteration limit i_1 is achieved; then go to Step 6).

5d) Decrease the value α with $\Delta\alpha$ and go to 6).

6) For the operations selected in Step 5b) repeat sending tenders to the subordinate units. After their reply and obtaining the new values of the variables and measures go to 5b). Iterate this step maximally i_2 times. After reaching this limit:

   6a) If $\alpha > \alpha_0$ then decrease the value α with $\Delta\alpha$ and go to Step 2).

   6b) Else the protocol fails.

7) If $\alpha'(\mathbf{h}) \geq \alpha'$ go to 8) else go to 7a).

   7a) Check the arguments $\mathrm{var}\left(R'_{\min}\right)$ of

   $$R'_{\min} = \left\{ R \in C'^{f}_{T_j} : \forall R^f \in C'^{f}_{T_j},\ \mu^{\rho}_{R}\left(\mathbf{h}_{\mathrm{var}(R^f)}\right) \leq \mu^{\rho}_{R}\left(\mathbf{h}_{\mathrm{var}(R^f)}\right)\right\}.$$

   The schedules for the operations dependent on $\mathrm{var}\left(R'_{\min}\right)$ must be modified in such a way that the satisfaction degree $\alpha\left(\mathbf{h}'\right) \geq \alpha$, where $\mathbf{h}'$ is the new schedule.

   7b) If the target of Step 7a) cannot be achieved or the number of iterations exceeded i_3 then:

   If $\alpha' \geq \alpha'_0$ then decrease α' with $\Delta\alpha$ and go to 7) else go to 6a).

8) Optional: Select the operations with the highest measures and such that their relations $R \in C'^{f}_{T}$ have the smallest satisfaction degree; for such operations send the query concerning the measures reduction.

Comments.

Add 4) The initial choice of the schedule can be random in the worst case, however it is the easiest solution to use classical scheduling methodology. In such a case, at first one should change the fuzzy relations to the sharp relations by $\mu_{R^c}(\mathbf{x}) = \lfloor \mu_{R^f}(\mathbf{x}) \rfloor$. Then, one can use the As-Soon-As-Possible (ASAP) scheduling method or As-Late-As-Possible (ALAP) scheduling method, see [1], as the first approximation.

Add 5c), 7b) The improvement of the schedule is the searching the whole space of schedules that can be generated on the basis of information obtained from the subordinate units. As a heuristic search method one can use, for example, Tabu Search, Simulated Annealing, see [3], [6], [9].

Add 6) Assume that the relations calculated by the data proposed by subordinate unit do not satisfy the required criteria. Then the superior unit can locally search the relations domain to find reasonable solutions (for which the relations are satisfied) and propose the subordinate unit to tune its schedule to obtain similar output.

The protocol of the schedule negotiation presented in the above always ends its action. It returns the schedule which is acceptable according to the assumed requirements or says that it cannot generate such a protocol. The working time of the protocol depends on the assumed parameters i_1, i_2, i_3 limiting the number of iterations at each step of the algorithm, the thresholds α, α', and the increment $\Delta\alpha$ of the thresholds reduction. Initially, the superior unit looks for the solution in the space of all allowed schedules (for the known data); if this procedure fails asks the subordinate units for the data modified according to the new constraints. Usually the modification is made only for the operations with the lowest satisfaction degree of the relations. To avoid falling into a local extreme trap (breaking the optimal schedule searching process), one restarts the protocol with new values of thresholds α and α'. Slow decreasing of the thresholds lets us approaching the optimal solution.

Exploiting the protocol, the superior unit should collect information about efficiency of the subordinate units' work. Every mistake (exceeding cost limits, crossing deadlines, etc.) by the unit f should result in decreasing the value of the measure $\mu_R(f)$, where $R \in C_E^f$ see Definition 4. Systematic reliable work should result in increasing the measure. Thus, it is seen that the relations make possible generation of the cooperation strategy in a distributed environment, either tending to equilibrium (corporative model) or increasing preferences (competitive model).

The additional problem connected with distributed scheduling is the protocol security. During the protocol, the participants can cheat, e.g., generate some false schedules to block the agreement. One could propose some solutions, e.g., to introduce the Trust Authority (very useful in the competitive model), with some tools to verify the honesty of schedules. Since there are various possible attacks on protocols, the Trust Authority should have an appropriate methodology to detect them. For example, to verify if the units do not hide production resources it must use measures of resources utilization (utilization factor). The problems of protocols security is now extensively studied in the literature, see. e.g. [2]. However, more detailed discussion of the problem exceeds the frames of this presentation.

5. APPENDIX

Two following definitions can be found in [4]:

Definition A1. The operator Δ,

$$\Delta : [0,1] \times [0,1] \to [0,1],$$

is $T-norm$ if it satisfies the following conditions:

1. $\forall x, y \in [0,1] : \mathrm{x}\ \Delta\ \mathrm{y} = \mathrm{y}\ \Delta\ \mathrm{x}$,
2. $\forall x, y, z \in [0,1] : (\mathrm{x}\ \Delta\ \mathrm{y})\ \Delta\ \mathrm{z} = \mathrm{x}\ \Delta\ (\mathrm{y}\ \Delta\ \mathrm{z})$,
3. $\forall x_1, x_2, y_1, y_2 \in [0,1]:\ x_1 \le x_2 \wedge y_1 \le y_2 \Rightarrow x_1\ \Delta\ y_1 \le x_2\ \Delta\ y_2$,
4. $\forall x \in [0,1] : x\ \Delta\ 1 = x.$

Definition A2. The operator ∇ :

$$\nabla : [0,1] \times [0,1] \to [0,1],$$

is $S-norm$ if it satisfies conditions 1)-3) of the Definition A1 of $T-norm$ and, moreover,

4. $\forall x \in [0,1]: x \,\Delta\, 0 = x$.

Definition A3. The operator $\varnothing$:

$$\varnothing : [0,\infty)\times(0,\infty) \to [0,1],$$

is *generalized division operator* if it satisfies the following conditions:

1. $\forall x \in (0,\infty): x \varnothing x = 1$
2. $\forall x \in (0,\infty): 0 \varnothing x = 0$
3. $\forall x_1, x_2, y \in (0,\infty): x_1 \le x_2 \Rightarrow x_1 \varnothing y \le x_2 \varnothing y$
4. $\forall x_1, x_2, y \in (0,\infty): x_1 \le x_2 \Rightarrow y \varnothing x_1 \ge y \varnothing x_2$.

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