MUTUAL INFORMATION AND REDUNDANCY IN CORTICAL SPIKE TRAINS DURING THE AWAKE STATES

J Szczepanski[1], MM Arnold[2], E Wajnryb[1], MV Sanchez-Vives[2]

1 Institute of Fundamental Technological Research,Polish Academy of Sciences, Warsaw, Poland

2 Instituto de Neurociencias de Alicante, Universidad Miguel Hernandez-CSIC, San Juan de Alicante,Spain.

An important problem to understand neural processing is neuronal cooperation on information transmission. In the past we calculated the information carried in spike trains by estimating the normalized Lempel-Ziv complexity, which measures the generation rate of new patterns along a spike sequence [Szczepanski et al., 2003; Amigo et al., 2004] and we also estimated the extent to which the information carried by one neuron varies with different brain states [Arnold et al., 2005]. Here we have been interested in the relative information carried by neighboring neurons. What is the relation between the information carried by closely located neurons? How much redundancy is there? Do neurons function in an independent or synergic way? To answer these questions we analyzed two parameters that are important to characterize cooperation: mutual information and redundancy in spike trains from different neurons. We quantified both parameters for the spontaneous spike discharge of nearby cortical neurons. Electrical activity of neuronal clusters was recorded by means of chronically implanted tetrodes in the visual cortex of freely moving rats. Spike discharge was spontaneous, the recordings done in the dark and in the absence of visual stimulation. To analyze the information conveyed by a given neuron in relation to the one carried by a different neuron we introduce the concept of relative mutual information (*RMI*) [Borst, 1999]. *RMI(X,Y)* between information sources corresponding to a pair of neurons *X* and *Y* results from the comparison between mutual information *I(X,Y)* and average information transmitted by *X* and by *Y*. Redundancy*(R)* was estimated using the definition in [Reich et al., 2001]. *R* results from comparing the sum of information rates transmitted by neurons, each treated separately, with the information rate transmitted by the whole group. Evaluation of both *R* and *RMI* need an entropy estimator. To estimate the entropy rate at a given moment of time, we use normalized Lempel-Ziv complexity [Szczepanski et al., 2003; Amigo et al., 2004]. The results revealed an almost invariable *RMI* within all the different analyzed clusters of neurons. At variance from *RMI*, *R* was found to be very different for different neuronal clusters. Our results show that neurons can collaborate in a flexible way (both synergistically and with a leading neuron). The values of *RMI* obtained show that during information transmission, the mutual information between each pair of neurons remains at the same level in relation to the amount of information being transmitted by both neurons.

METHODOLOGY

Three aspects of the Methodology will be reported here:

1) The realization of single cell recordings in chronically implanted behaving rats: surgery, recordings and cluster cutting.
2) The estimation of the information content (entropy rate) in the spike trains.
3) The calculation of the relative Mutual Information and Redundancy estimation (*via* entropy rate)

1) Single cell recordings in chronically implanted behaving rats

We recorded from Lister Hooded rats chronically implanted with tetrodes in primary visual cortex (AP: -6, ML: -4 from bregma) during awake, drowsy and sleep periods. Each tetrode was made from four twisted strands of 17 or 25μm diameter HM-L-coated platinum-iridium wire and the four tetrodes were held by a cannula which was attached to a microdrive(Fig.2).

Figure 1. Implanted rat.

After one week of post-operative recovery we started the recording. The electrodes were advanced by up to 50-75μm daily in 25-50μm steps.

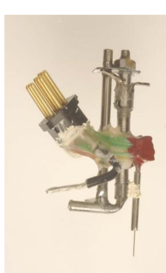

Figure 2. Microdrive mounted with two tetrodes.

Multiple single units recordings and data processing

Electrical spontaneous activity was recorded during sleep, drowsy and awake states while the animal was in its home cage. All the activity included in the analysis was spontaneous and recorded from the visual cortex in the absence of visual stimuli. The recordings were done in darkness and we supervised the animal´s behavior with an infrared camera. Single and multi unit recording were made with a 16 channel data acquisition system from Axona Ltd (London, UK). After amplification (1000x) and filtering, EEG field potentials (sampled at 250Hz, low-pass filtered at 500Hz) and extracellular action potentials (spikes, sampled at 48 KHz, high-pass filtered at 360 Hz) were recorded from supragranullar layers. Recording sessions lasted 30 minutes. After each recording session, the data were transferred to a Pentium IV personal computer and were analyzed off-line.

Spike Sorting

Cluster cutting was done using the Offline Sorter software (Plexon Inc). The waveforms are sorted into units by selecting an automatic method: the valley - seeking algorithm (Fukunaga, 1972, *Intro_ duction to Statistical Pattern Recognition, Chapter 11, Ac. Press.*). The 3D cluster view option was used, with Principal Component (PC) 1, PC2, PC3 as X, Y, and Z axis. (Fig.3)

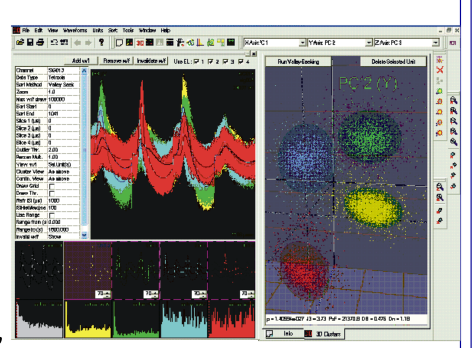

Figure 3. Offline Sorter software showing the isolation of 4 clearly different single units.

2) Analysis of the information content in the spike trains

Encoding Procedure

Each second is divided into *f* (encoding frequency) bins (each bin=12.5 ms). With each bin we associate 1 or 0 depending on whether or not there has been at least one spike in this bin (Fig. 5).

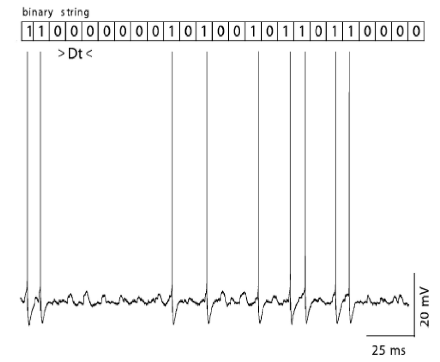

Figure 4. Illustration of the encoding procedure (from Amigo et al, 2004).

Lempel-Ziv Complexity (1976)

For a given sequence S_n of bins Lempel-Ziv76 complexity *denotes the number of new patterns that appear when we go along the sequence.* Normalized complexity is defined as

$$c_2(S_n) = \frac{C_2(S_n)}{n/\log_2 n}$$

Entropy Estimation

In the case of stochastic processes, ***entropy*** *H* can be expressed as entropy per symbol (*average information carried by symbol = ENTROPY RATE, the symbol being 0 or 1).* Assuming ergodicity of stochastic processes (generating sequences) it was proven that $c_{norm}(S_n) \to H$

Information carried by a spike-train

Information carried by event *A* in Shannon sense is $I(A) := -\log_2 P(A)$ where *P(A)* is probability of *A*; we see that less probable events carry more information. For ergodic sources we have

$$\frac{I(S_n)}{n} \to H$$

In further considerations we estimate the information rate carried by a given spike train taking into account what precedes that spike train. Here the noise information is not extracted.

Thus, for sufficiently long sequences, information carried by single spike-trains is close to entropy. Consequently, normalized complexity for sufficiently long lasting sequences approximates the information carried by single encoded spike-trains.

Assumptions for analysis with sliding windows

We consider normalized complexity within sliding windows along encoded spike-trains with a length of 5 sec. This is due to trade off between:

- Keeping conditions like ergodicity of the process (thus we assume also local stationarity) for which normalized complexity works, and
- Considering sufficiently long windows (sequences) to get a good approximation of information *via* normalized complexity.

The choice of parameters will be crucial. This work is based on our earlier results (J. Szczepanski, 2003) describing that an increase of encoding frequency above 80 Hz does not have an effect on the normalized complexity value obtained for a given experimental data. Therefore that encoding frequency was applied.

Entropy Rate Estimators

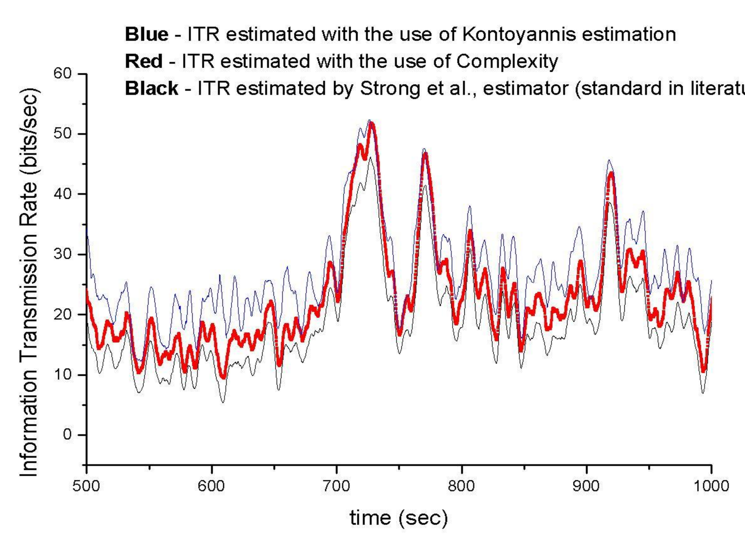

Figure 5. Comparison of different Entropy Rate estimators. Qualitatively, we detect similar behavior. They were applied to 500 sec long recording.

3) Relative Mutual Information and Redundancy

3A) RELATIVE MUTUAL INFORMATION

We introduce the definition of Relative Mutual Information RI that measures Mutual information I(X,Y) between random variables X and Y in comparison to the average information carried by these random variables. The precise formula is

$$RI(X,Y) := \frac{I(X,Y)}{[H(X)+H(Y)]/2}$$

$$\frac{H(X)+H(Y)-H(X,Y)}{[H(X)+H(Y)]/2}$$

We have shown that :

THEOREM (related to the RI interpretation)

A) $0 \le RI(X,Y) \le 1.$

B1) RI(X,Y) =0 is equivalent to the fact that X, Y are independent.

B2) RI((X,Y)=1 is equivalent to the fact that the relation between X and Y is deterministic.

3B) REDUNDANCY

The concept of redundancy that we apply was introduced by DS. Reich, F. Mechler and JD. Victor (2001) in order to measure the cooperation of a group of neurons. The definition is as follows.

Let assume we have group of neurons neighboring *N1, N2,… , Nk.* Denote by: $l_c,\ l_s,\ l_i$

The information rate for the encoding spike-train coming from: the whole group, the separate sum of information rates for each cell alone, each cell alone, respectively.

The redundancy is defined by $$R := \frac{l_s - l_c}{l_s - \max\limits_{1\le i\le k}\{l_i\}}$$

Redundancy interpretation

The redundancy index is greater than 1 means that $\max\limits_{1\le i\le k}\{l_i\} > l_c$

i.e. there is a single neuron for which the information rate is greater than the information rate of the spike-train of the whole ensemble (the result of the group cooperation is contradistory and confusing).

The redundancy is less than 0 means that $l_s < l_c$

i.e. the sum of information rate coming from all neurons is smaller than information rate of the spike-train of the whole ensemble (neurons cooperate synergetically).

Proof of the THEOREM

A) By the well known facts (Ash) we have (*)

$$H(X,Y) = H(X) + H(Y\,|\,X)$$
$$H(X,Y) = H(Y) + H(X\,|\,Y)$$

where $H(X\,|\,Y),\ H(Y\,|\,X)$ are conditional entropies that cannot (Ash) be negative! Thus:

(**) $$H(X) \le H(X,Y),\ H(Y) \le H(X,Y)$$

Now, adding the sides of the above inequalities, we receive $\frac{H(X)+H(Y)}{2} - H(X,Y) \le 0.$

Adding $\frac{H(X)+H(Y)}{2}$ to both sides and next dividing both sides by $[H(X)+H(Y)]/2$

we receive $\frac{H(X)+H(Y)-H(X,Y)}{[H(X)+H(Y)]/2} \le 1$ What means that $RI(X,Y) \le 1!$ Property

$0 \le RI(X,Y)$ follows from the fact that standard Mutual Information index cannot be negative.

B1) follows from the fact that *X, Y* are independent iff standard Mutual Information is 0

B2) By (*) the inequality (**) becames equality only in the case when $H(X\,|\,Y)=0,\ H(Y\,|\,X)=0$ what implies (Ash, 1965; page 51) that the relation between *X* and *Y* must be deterministic for $RI(X,Y)=1$

RESULTS

REDUNDANCY

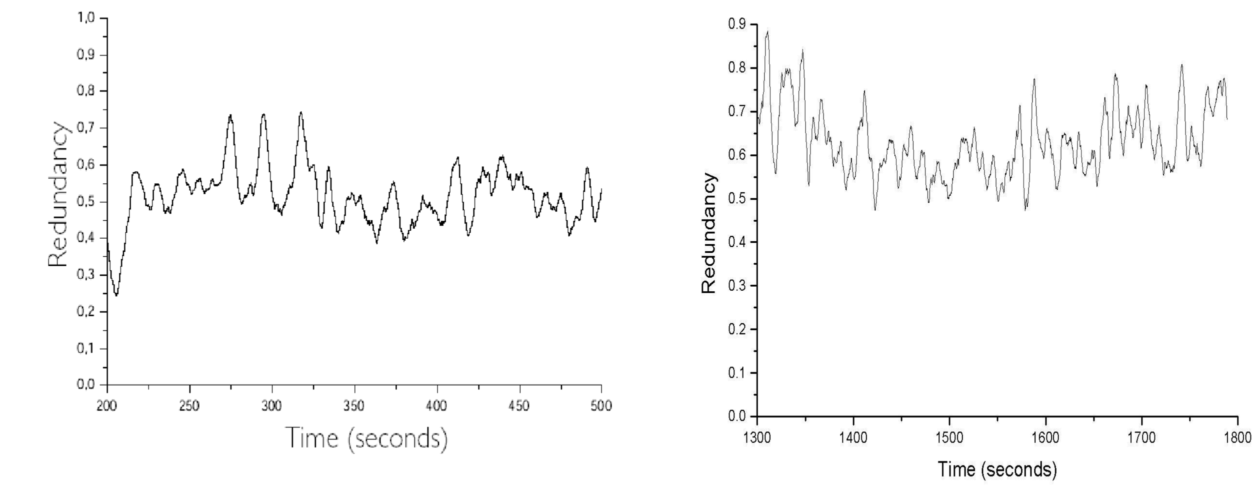

Figure 6. Typical plots of Redundancy for 300 s and 500 s of recording during the awake state.

The typical behaviour of redundancy is characterized by a large variability, suggesting that neurons can collaborate in a flexible way.

RELATIVE MUTUAL INFORMATION

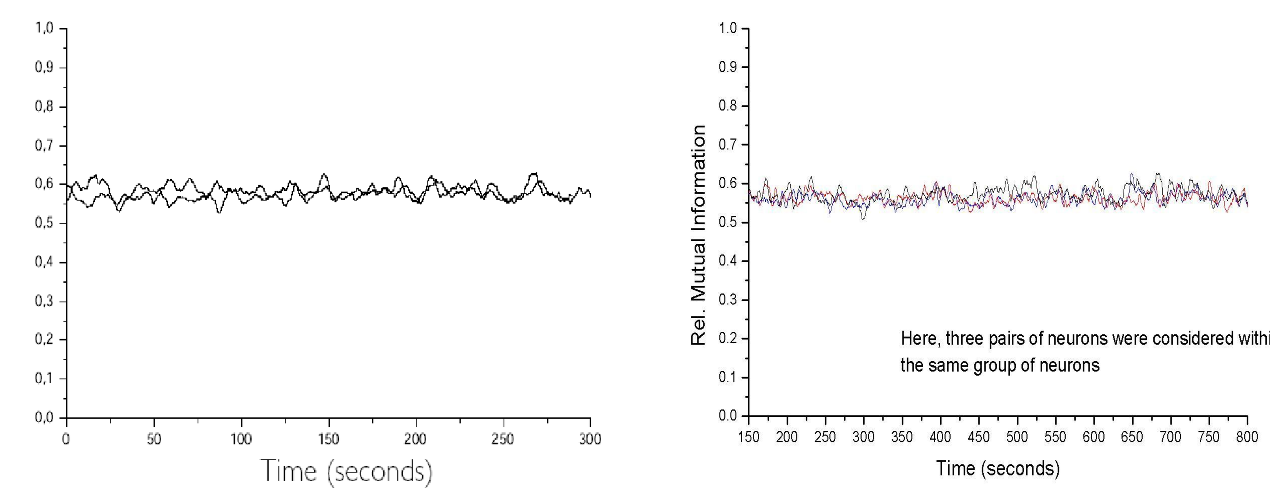

Figure 7. Typical plots of Relative Mutual Information in two different clusters recorded for 300 s and for 650 s.

Typical behaviour of Relative Mutual Information in awake state is characterized by relatively high level but small variability

CONCLUSIONS

- We introduce the Method of Estimating Information Transmission Rate (ITR) for Neurons.
- We propose a new quantity (RMI) to characterize mutual information between neurons
- Information Transmission by nearby neurons occurs in the mid-regime (Redundancy and Relative Mutual Information).
- Redundancy is characterized by large fluctuations what suggests that during information transmission neurons can cooperate in a flexible way (synergetically as well as with an advantage of a single neuron).
- Relative Mutual Information between neurons is characterized by a small variability.
- The high value of Relative Mutual Information suggests that the transmission is resistant against disturbances, a natural phenomenon in biological systems.

REFERENCES

J. M. Amigo, J. Szczepanski, E. Wajnryb, M. V. Sanchez-Vives, „Estimating the Entropy Rate of Spike Trains via Lempel-Ziv Complexity", Neural Computation, Vol. 16, I. 4, pp. 717 - 736, (2004)

M.M. Arnold, J. Szczepanski, E. Wajnryb, MV. Sanchez-Vives, Information Kontent of Cortical Spike Trans During Different Brain States. Society for Neuroscience, Abstract 856.6 (2005)

R. B. Ash, Information Theory, John Wiley & Sons, 1965

A. Borst, F.E. Theunissen, "Information Theory and Neural Coding", Nature Neuroscience, 2, 947-957, (1999)

A. Lempel, J. Ziv, On the Complexity of Individual Sequence, IEEE Trans. Inform. Theory, IT-22, 75-88, 1976

D.S. Reich, F. Mechler, J.D. Victor, Independent and Redundant Information in Nearby Cortical Neurons, Science, Vol. 294, 2566-2568, December 2001

J. Szczepanski, M. Amigo, E. Wajnryb, M.V. Sanchez-Vives, „Application of Lempel-Ziv Complexity to the Analysis of Neural Discharges", Network: Computation in Neural Systems 14 (May 2003), 335-350

•Acknowledgements: **Research supported by the EU (PRESENCIA project) and Spanish/Polish grant 5009/R04/R05. Thank you to John O´Keefe, C. Lever and F. Cacucci from UCL and to J. Donnett from Axona Ltd for their help on the set up of the recording technique.**