Jerzy Rojek, Michal Kleiber, Antoni Piela, Rafal Stocki and Jaroslaw Knabel:

Deterministic and stochastic analysis of failure in sheet metal forming operations

Numerical simulation can be successfully applied to evaluate product manufacturability and predict possible defects. Material breakage, wrinkling and shape defects due to springback are most frequent defects in sheet metal forming operations. In this paper we shall deal with prediction of sheet breakage during stamping process. The breakage possibility in our study is evaluated using Forming Limit Diagrams (FLD) commonly used in industrial practice. Typically deterministic analysis of sheet forming process is carried out. The first part of the paper includes an example of such simulation. Sheet forming operations, however, are characterised with a significant scatter of the results. This can be caused by differences that can occur in forming of each part. The second part of the paper presents a stochastic approach to assessment of sheet metal failure during forming operation. Methodology developed is based on the application of reliability analysis of structures to estimate probability of sheet breakage in metal forming operations. Numerical examples illustrate the stochastic approach to failure analysis in sheet forming processes.

In recent years a substantial progress has been made in sheet metal forming technology to meet higher technical and economical requirements. Technical advances have been possible in great part thanks to development in the area of computer simulation methods for the forming operations - a number of specific techniques and simulation programs are now in widespread use. As a rule they allow for the determination of deformation and stresses at every point in the simulated sheet at any stage of the forming process. It is very rare, however, that the numerical simulations accounted in a truly rational way for the inherent variability of various parameters governing the response of the sheet under consideration. Process characteristics may be affected essentially by the stochastic nature of the problem.

This paper shows the possibility of sheet breakage prediction by typical deterministic analysis as well as more advanced stochastic analysis of sheet failure. The latter is carried out by employing system reliability assessment techniques in sheet forming simulation.

Finite element modelling of sheet forming

Different formulations can be used in the simulation of sheet forming operations. Within the finite element method the analysis of these processes can be performed employing either dynamic or quasistatic models, cf. [1]. Considering the solution method we have either implicit or explicit formulation. Because of its efficiency in the analysis of large-scale systems the explicitly integrated dynamic approach has become very popular in sheet stamping simulation. In the present work the explicit dynamic program Stampack has been used [2].

Sheet was discretized using a simple triangular shell element with translational degrees of freedom only, known as the BST (Basic Shell Triangle) element [3]. The BST shell element has only three displacement variables at each node which makes the element computationally efficient and suitable for large scale analysis such as the simulation of industrial sheet stamping problems.

Elasto-plastic constitutive models implemented in the program Stampack for simulation of sheet metal forming employ the Huber-Mises yield criterion for isotropic materials and the Hill'48 [4], Hill'79 [5] or Hill'90 [6] criteria for anisotropic plastic behaviour. The stress-strain curve will be defined analytically by the following equation

$$\sigma_Y = K\left(a + \bar{\varepsilon}^p\right)^n \tag{1}$$

where σ_Y is the yield stress and K, a and n are the material constants.

Deterministic analysis of fracture in sheet forming

Possibility of material fracture in sheet metal forming operations is usually estimated in practice using forming limit diagrams (FLD), in which major principal strain values are plotted against minor principal strain values. Points representing strain states all over the deformed sheet are confronted with the forming limit curve (FLC). FLC is supposed to represent the boundary between the strain combinations which produce instability (above the curve) and/or fracture and those that are permissible in forming operations (below the curve) as shown in **Fig. 1**.

Formability of the part shown in **Fig. 2** has been studied experimentally and numerically. The initial tailor welded blank was made by welding two similar steel sheets of grade H260YD+Z 1.2 mm thick. Figure 2a shows the fracture occurring in the part under excessive biaxial tension, which is confirmed in the FLD (**Fig. 3a**) where the points corresponding to the critical area lie above the forming limit curve.

Finite element simulation of the forming process has been carried using a five-zone model of the tailor welded blank [7] discretised with BST shell elements with different material properties taken for the parent material, heat affected zone and weld bead. The stress-strain curves given

PhD, Jerzy Rojek, mechanical engineer; *Professor, Michal Kleiber,* civil engineer, *Professor, Antoni Piela*, mechanical engineer, Department of Metal Forming, Silesian University of Technology, Katowice/Poland, *PhD, Rafal Stocki,* civil engineer, Department of Computational Science, Institute of Fundamental Technological Research, Warsaw/Poland, *PhD, Jaroslaw Knabel,* civil engineer, Department of Computational Science, Institute of Fundamental Technological Research, Warsaw/Poland

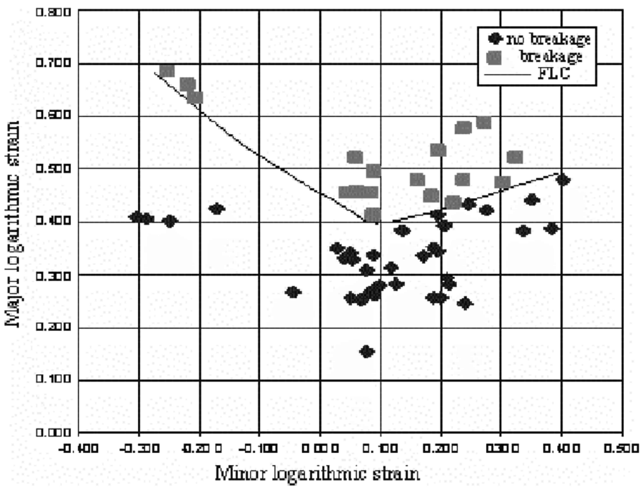

Fig. 1: Forming limit diagram and determination of the forming limit curve

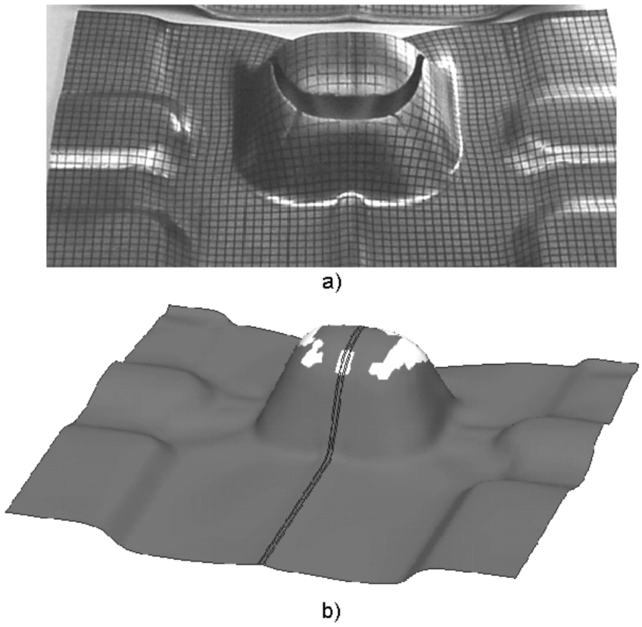

Fig. 2: Final shape with fracture: a) experiment, b) numerical simulation

by Eq. (1) have been defined by the following parameters: $K = 598$ MPa, $a = 0.038$ and $n = 0.224$ for the parent material, $K = 787$ MPa, $a = 0.043$ and $n = 0.302$ for the heat affected zone, and $K = 818$ MPa, $a = 0.045$ and $n = 0.308$ for the weld bead. Transverse anisotropy with the average Lankford parameter $r = 1.56$ was considered using the Hill'48 model.

Numerically obtained deformed shape is shown in Fig. 2b with marked zones of probable failure. Predicted zones of breakage are in agreement with the experimental results. Places of probable fracture are predicted using the forming limit diagram which is shown in Fig. 3b.

Variations of sheet stamping processes

Sheet stamping is a process influenced by many parameters, which can be sources of scatter. The most important factors influencing the forming results are identified by Col [8] as follows:

- **Material variability.** Although a very great progress has been made in sheet metal production techniques, material has some scatter in its mechanical properties. Material structure is never absolutely homogenous. The thickness has also some variation.
- **Variability of tooling and presses.** The shape and roughness of the tool material is of great importance. Tool shape changes due to wear are located in places with high contact pressure like die radii and drawbeads.
- **Process variables.** Variation of blankholder pressure is an important source of scatter. Modification of the stamping velocity can influence the result of stamping. Increase of the velocity can lead to a significant rise of the tool temperature.

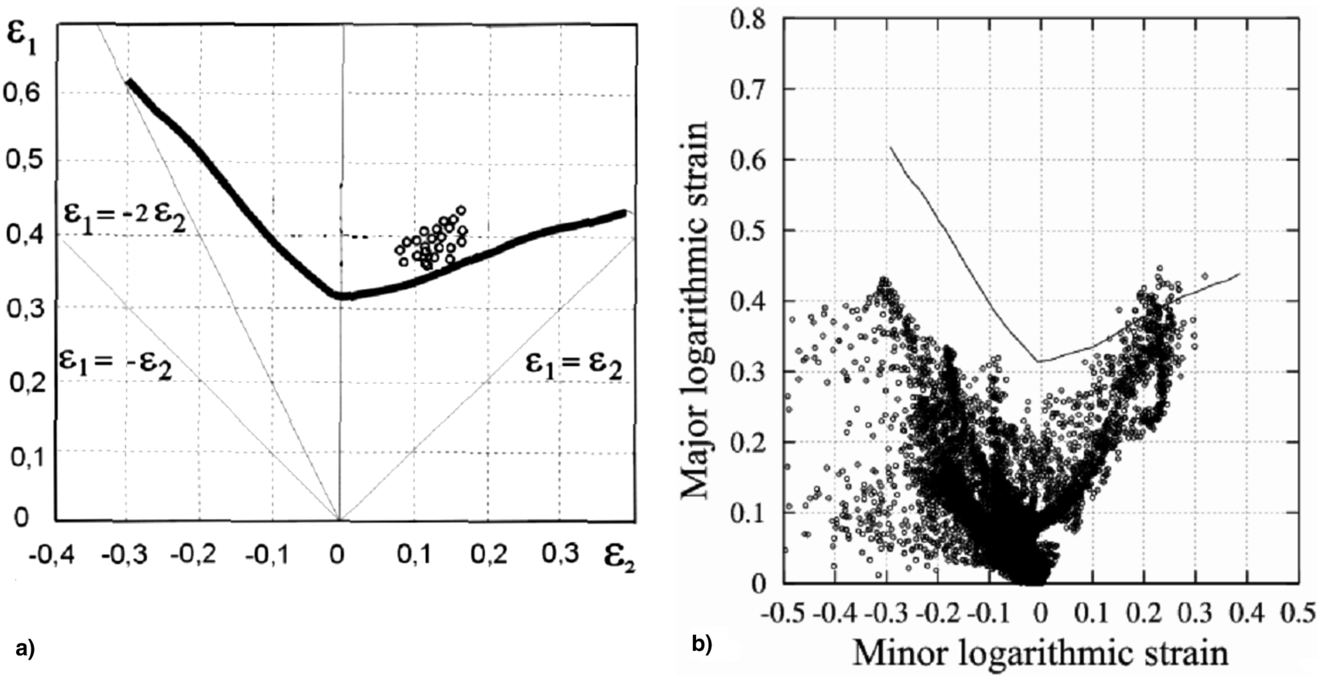

Fig. 3: Forming Limit Diagram: a) experiment, b) numerical simulation

- **Lubrication.** Lubrication is a very important parameter and also very difficult to control. Amount of oil can change locally. The tribological conditions can also be changed by the tool temperature.
- **Unpredictable factors.** Bad positioning of the tool, parameters of mechanical parts (springs, gas springs).

An increasing importance of product quality and the objective of zero-defect production have increased the reliability of sheet forming processes [9]. Steel suppliers have reduced significantly variation of thickness and mechanical properties which helped to reach high technical level of fabrication of sheet metal parts. Nevertheless it can be found out, cf. [9], that despite using materials that have very uniform thickness and mechanical properties, variation of the stamping process is observed due to the influence of other factors listed above.

Reliability problem formulation

Influence of the random parameters on the failure of sheet forming processes can be analysed using the theory of reliability. Parameters describing the sheet metal forming process posses of nondeterministic nature are treated as random variables, say X_1, X_2, $\ldots$, X_n. They are called the basic variables and constitute a random vector **X** whose samples $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^T$ belong to the Euclidian space. The probability measure is defined by the joint probability density function $f_{\boldsymbol{X}}(\boldsymbol{x})$ of the random vector **X**. The failure criterion, due to material fracture etc., is usually expressed by the equation $g(\boldsymbol{x}) = 0$, called the limit state surface. It divides the Euclidian space into two regions: the failure domain $\Omega_f = \{\boldsymbol{x} : g(\boldsymbol{x}) \le 0\}$ and the safe domain $\Omega_s = \{\boldsymbol{x} : g(\boldsymbol{x}) > 0\}$. Hence, the failure probability of the structural system is determined by the following integral:

$$P_f = \mathrm{P}\left[\boldsymbol{X} \in \Omega_f\right] = \mathrm{P}\left[g(\boldsymbol{x}) \le 0\right] = \int_{\Omega_f} f_{\boldsymbol{X}}(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \tag{2}$$

where $\mathrm{P}[A]$ means the probability of the random event A. In practice the direct integration appears to be impractical. Therefore, some approximate methods have been developed to assess the value of failure probability.

In the commonly used approach the problem of the reliability calculation is appropriately transformed, $\boldsymbol{U} = \boldsymbol{T}(\boldsymbol{X})$ (see e.g.[10, 11]), into the standard normal space where probability density function $f_{\boldsymbol{U}}(\boldsymbol{u}) = \prod_{i=1}^{n} \varphi(u_i)$ becomes the product of the n one-dimensional standard normal probability density functions of random variables $U_i = T_i(\boldsymbol{X})$. The idea of the reliability analysis is shown in Fig. 4. The limit state condition is also transformed $g(\boldsymbol{x}) = 0 \rightarrow g\left[\boldsymbol{T}^{-1}(\boldsymbol{u})\right] = 0$. Then, the axial symmetry of the probability density function $f_{\boldsymbol{U}}(\boldsymbol{u})$ assures for any linear function $l_{\boldsymbol{U}}(\boldsymbol{u}) = \beta - \boldsymbol{\alpha}^T \boldsymbol{u} = 0$, the following equality to be true

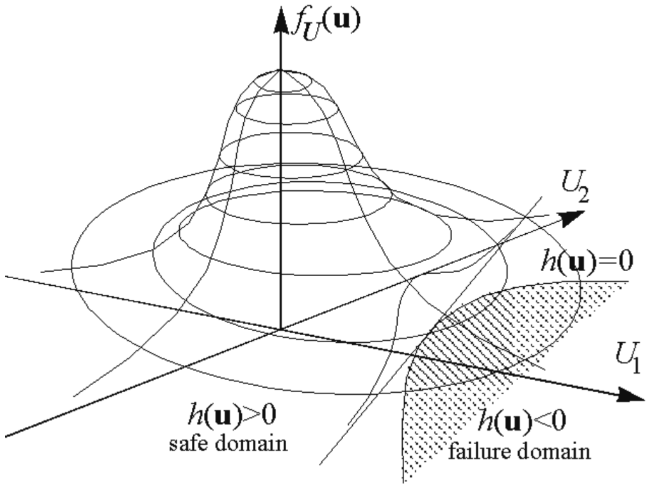

Fig. 4: Idea of reliability analysis

$$P_f = \mathrm{P}\left[l(\boldsymbol{U}) \le 0\right] = \int_{\{\boldsymbol{u}: l(\boldsymbol{u}) \le 0\}} f_{\boldsymbol{U}}(\boldsymbol{u}) \mathrm{d}\boldsymbol{u} = \Phi(-\beta) \tag{3}$$

where the coefficients, $-\alpha_i$, $i = 1, 2, \ldots, n$, are the components of the normalized gradient of the hyperplane $l_{\boldsymbol{U}}(\boldsymbol{u}) = 0$, i.e. $\boldsymbol{\alpha}^T \boldsymbol{\alpha} = 1$, $\beta = \mathrm{sign}[l(\boldsymbol{0})]\delta$ is the signed distance δ between the hyperplane and the origin, and $\Phi(\cdot)$ is the standard normal distribution. Thus, the linear approximation of the transformed limit state surface $h(\boldsymbol{u}) = 0$ in the point closest to the origin (so-called design point $\boldsymbol{u}^*$) provides a simple estimate of the failure probability of structural system

$$P_f = \mathrm{P}\left[h(\boldsymbol{U}) \le 0\right] \approx \mathrm{P}\left[l(\boldsymbol{U}) \le 0\right] = \Phi(-\beta) \tag{4}$$

where β is called the reliability index. This approach is called the first order reliability method (FORM). The reliability index $\beta = \mathrm{sign}[h(\boldsymbol{0})]\delta^*$ is determined as a solution of the following optimization problem:

$$\delta^* \equiv \|\boldsymbol{u}^*\| = min\|\boldsymbol{u}\| \quad \text{subject to: } h(\boldsymbol{U}) \le 0 \tag{5}$$

Various optimization techniques can be employed. Gradient-based optimization techniques can be used but only if the limit state function is differentiable.

Insensitive to this requirement are simulation (Monte Carlo) methods (see [12] for the review of simulation methods). Fundamental one is the crude Monte Carlo approach, where the samples **x** of the random vector **X** are being generated from the joint probability density function $f_{\boldsymbol{X}}(\boldsymbol{x})$. To compute the probability of failure the following estimator of the mean value is employed

$$\hat{\mathrm{E}}\left[\chi_{\Omega_f}(\boldsymbol{X})\right] = \frac{1}{K}\sum_{k=1}^{K} \chi_{\Omega_f}(\boldsymbol{X}_k) = \hat{P}_f \tag{6}$$

where $\chi_{\Omega_f}(\boldsymbol{X})$ is the indicator function of failure domain and K is the number of samples. In real life problems where the expected $P_f = (10^{-7} \div 10^{-3})$, to get the accurate result, with, say coefficient of variation $v_{\hat{P}_f} = 5\%$, it is required to perform $K = 4 \cdot 10^5 \div 4 \cdot 10^9$ simulations. This computational cost is certainly not acceptable.

The method that allows to significantly reduce the number of required simulations is the so-called adaptive Monte Carlo (AMC) method. This alternative approach consists in seeking the design point during sampling by `moving' the sampling density based on the information from the previous samples.

Another method insensitive to the limit state function differentiability requirement approach is response surface method (RSM). RSM estimates limit state function (LSF) in vicinity of the design point, and then standard, gradient reliability optimization method (FORM) is used to search *exact* design point on LSF approximation.

Reliability analysis of sheet forming operations

From the discussion in Section 4 it can be seen that there are different uncertainties in the sheet metal forming, from variations of material properties to many changing process factors, that lead to uncertainties in the results of practical realization and numerical analysis for a given process. For the same reasons a forming limit curve can be regarded as bounding the safe zone with some probability only. The safe zone is considered as the one where failure is highly improbable, while the failure zone is regarded as the one defining strain states with a high probability of failure. Usually between the two zones, safe and failure, a critical zone (marginal zone) is introduced (**Fig. 5**), with the probability of failure high enough so that the strain state cannot be considered safe. The present work is aimed to quantify these qualitative notions.

We take advantage of the forming limit diagrams and define the limit state function as the signed minimal distance from the FLC of the point corresponding to principal strains in the given finite element (Fig. 5). In the adopted sign convention the minus sign is for the points above the curve. Depending on the realization of the vector of random variables the different points in the sheet metal may be located close to the FLC. Considering also the shape of the FLC (piecewise linear with vertices) and some `numerical noise' introduced by using the explicit dynamic approach in the finite element analysis the reliability analysis was based on the methods insensitive to the limit state function differentiability requirement, first of all most efficient response surface method and adaptive Monte Carlo method.

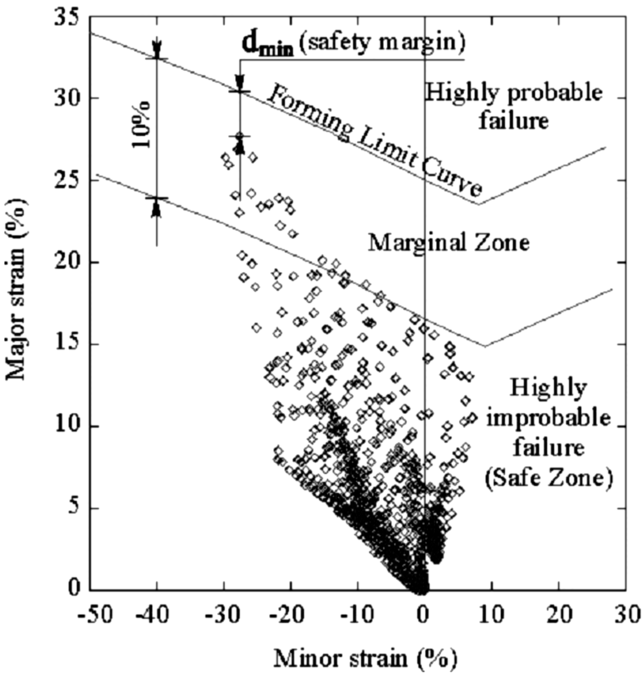

Fig. 5: Typical forming limit diagram and definition of the limit state function.

Numerical illustration

Deep drawing of a square cup, the benchmark problem at the Numisheet'93 conference [13] has been analysed. Geometry definition can be found in [13]. The material properties are taken as follows: aluminium, thickness 0.81 mm, Young's modulus $E = 71$ GPa, Poisson's ratio $\nu = 0.33$, uniaxial true stress-true strain curve $\sigma = 576.8(0.01658 + \varepsilon^p)^{0.3593}$ MPa, friction coefficient $\mu = 0.16$. The blankholding force is 19.6 kN.

Prior to a stochastic analysis the deterministic analysis of deep drawing of a square cup has been carried out. **Figure 6** presents deformed shape with contours of logarithmic thickness strains for the punch travel of 20 mm. The major and minor strains are plotted in the forming limit diagram in Fig. 7. Here the strains are very close to the FLC. The failure conditions are first met for the punch travel of 20 mm. This indicates the danger of failure. This is in agreement with experimental results - in [14] the failure in laboratory tests at punch stroke of 19 mm is reported.

The stochastic description of the system involves 3 random variables: the initial thickness of the sheet metal *t*, the hardening exponent *n* and the Coulomb friction coefficient μ - between sheet metal and punch, die and blankholder, respectively. Full correlation among friction coefficients describing these three states is assumed which appears close enough to reality. The random variables are assumed to be lognormally distributed with mean values and standard deviations shown in **Tab. 1**.

The reliability analysis employing RSM was performed. The AMC method and, in one case, crude Monte Carlo

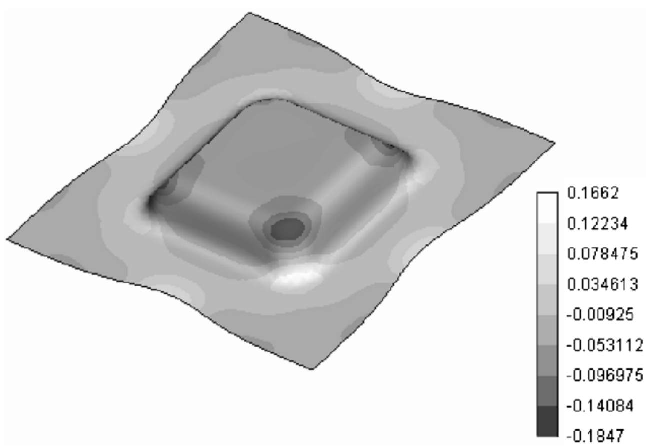

Fig. 6: Distribution of logarithmic thickness strains at 20 mm depth of drawing

Table 1: Random variables

Variable	Mean value	Std. dev.
t	0.81 mm	0.04 mm
μ	0.16	0.015
n	0.3593	0.020

techniques were used to check the accuracy of the RSM. The objective of the reliability analysis was to study a change of probability of failure in terms of the safety margin. This allows us to verify the need of the marginal zone with a width of 10% which is used in practice. Change of the safety margin in the example studied has been obtained by the change of the depth of drawing. The results are presented in **Tab. 2**. Punch strokes between 16 and 20 mm were analysed, which corresponds to safety margin ($d_{\min}$) variation from 7.44% to 0.77%, the values being obtained in the deterministic analysis based on the mean values of the random variables. The corresponding change of probability of failure P_f ranges from 0.0001 to 0.373 (the reliability index β evaluated by the RSM (FORM) varies from 3.718 to 0.324). Table 2 compares results of the linear approximation RSM analysis with the results obtained by the quadratic approximation RSM (SORM). The values of P_f estimated by these two methods are almost equal. The linear approximation RSM analysis of one case required 23 to 59 LSF calls, while the quadratic approximation RSM analysis needed 38 to 74 LSF calls. The calculations performed with the more accurate RSM analysis based on the quadratic approximation prove that LSF relationship is almost linear in that region. Thus, the more effective RSM analysis would be sufficient in this case to get accurate results. Relatively small numbers of LSF calls for the RSM analysis are in contrast with large number (about 1000) of simulations necessary for the AMC method.

From the results in Tab. 2 it can be seen the probability of failure decreases fast with the increase of safety margin. The safety margins used in practice ensure sufficient reliability of stamping processes.

Conclusions

Methodology developed for reliability calculations of structures is applied to estimate reliability of sheet metal forming operations. Forming Limit Diagrams (FLD) used in the industrial practice as a criterion of material breakage in the manufacturing process are treated as the limit state function for reliability analysis. Computationally efficient response surface method was chosen for reliability assessment.

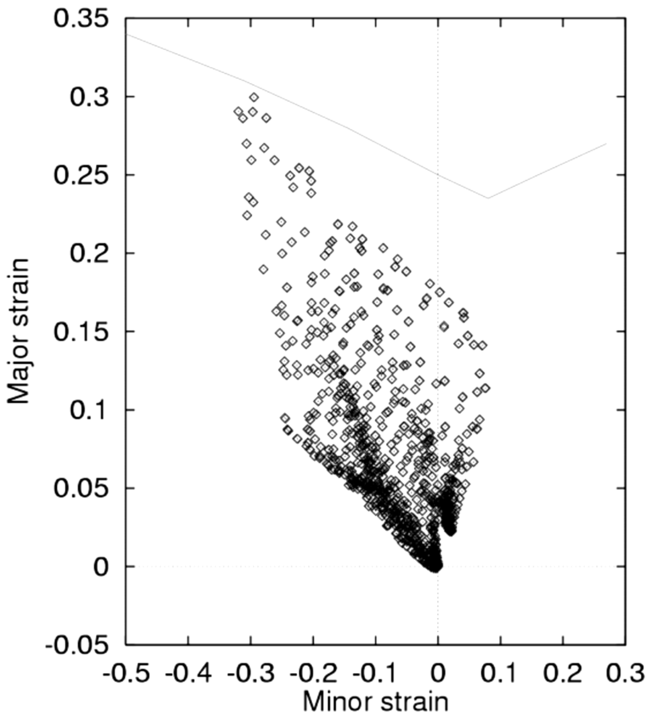

Fig. 7: Forming limit diagram for 20 mm depth

Table 2: Results of metal forming reliability analysis

Punch stroke mm	d_{min} %	RSM (FORM)			RSM (SORM)			AMC		
		P_f	β	N	P_f	β	N	β	v_{Pf} (%)	N
16	7.44	0.0001	3.718	39	0.000099	3.723	54	3.721	7.10	10^3
18	3.77	0.0485	1.660	29	0.0476	1.669	44	1.670	5.32	10^3
20	0.77	0.3730	0.324	23	0.395	0.267	38	0.342*	5.89	500

*Crude Monte Carlo method

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