Light scattering by «soft» particles of arbitrary shape and size

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Abstract

In this paper is proposed the simple method of analysis of the process of light scattering by soft particles of arbitrary shape, based on investigation in the three-dimensional spectral space. As a results the approximation of Rayleigh-Gans is generalized to the particles of arbitrary size and shape. Comparison with the accurate according to the Mie theory for the sphere showed that a error in the method is ~20%. For illustration of the possibilities of method the indicatrices and integral scattering cross-sections were calculated for the sphere, the spheroids, the parallelepipeds and the cylinders.

1. Introduchtion

The methods of the analysis of the light scattering process by the nonspherical particles are complicated and the obtained results do not allow clear physical interpretation [1]. In this paper a new approximation method is represented of the analysis of the light scattering process of by the soft particles of arbitrary shape and size. The basics of the method are: 1. Interaction of light with the particle is divided into two parts [2]: scattering in the narrow sense, i.e., secondary radiation of light by the substance of particle; and diffraction (in the narrow sense) of incident wave on the aperture in screen, which have the shape of the particle shadow. 2. The analysis of scattering (in the narrow sense) is made in the three-dimensional spectral space with the aid of the three-dimensional Fourier transform of the scattering particle. The use of Kotelnikov-Shannon decomposition makes it possible to extend the Rayleigh-Gans approximation to the particles of arbitrary size and shape [3]. 3. The summing up of the scattered and diffracted fields is made taking into account the fact that the power of the diffracted field must be equal to the power of scattered field.

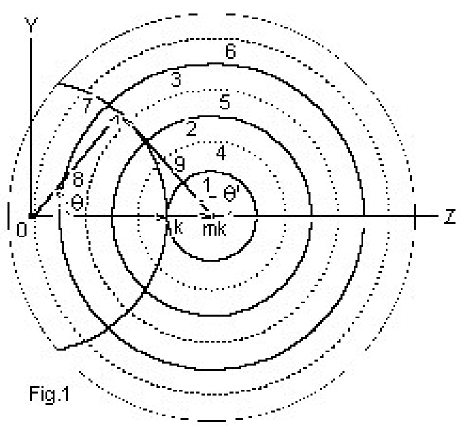
2. Scattering (in the narrow sense) light by the particle

Let us examine the particle, whose shape is described by the arbitrary function F(**r**) , F(**r**) ≡ 1 inside the particle and F(**r**) = 0 outside it. Refractive index of the substance of particle and environment are n_1 and n_2, accordingly; relative refractive index $m = n_1/n_2$, for the soft particles $m \approx 1$. The plane monochromatic electromagnetic wave is incident on particle. The wave is propagated along the axis Z, it is polarized along the axis X: $E = E_0 e^{-i\omega t + ik_0 z}$, $k_0 = \omega/c$.

For the case m ≈ 1 it is possible to use the WKB-approximation essentially close to the approximation of anomalous diffraction [2]. Accordingly, the wave in the particle will be plane, but with a different wave vector $k_1 = mk_0$: $E_i = E_{i0} e^{-ik_1 z}$. Under the action of the exciting light in each element of volume of the particle appears a vector of electrical polarization with the amplitude $p(r) = E_i \Delta\varepsilon$, where $\Delta\varepsilon = \varepsilon_1 - \varepsilon_2 = 3(m^2 - 1)/(m^2 + 2)$. Single-component (in this case) $p(r)_x$ is the source of secondary wave $E_s(r)$, i.e., the wave is scattered (re-emitted) by the particle. It is convenient to investigate the scattering field in the three-dimensional spectral space [3]. The three-dimensional space spectrum of the perturbation of dielectric constant (Fourier decomposition in terms of the wave vector $\overline{\rho} = \overline{q}a$) will be $\Delta\varepsilon(\overline{\rho}) = \Delta\varepsilon V f(\overline{\rho})$, where the normalized spectrum of particle $f(\overline{\rho}) = F(\overline{\rho})/V$, $F(\overline{\rho}) = a^3 \iiint F(\overline{r}')e^{-i\overline{\rho}\overline{r}'} dr'^3$; $V = \chi a^3$ is the volume of particle, χ is the coefficient, depending on the shape of particle. For the particle of a limited volume *f(ρ)* is continuous with the maximum at $\rho = 0$. Since the distribution of *p(r)* is a product $\Delta\varepsilon$ and E_i, the spectrum of *p(q)* is a convolution of their spectra. The spectrum of plane wave is a δ-function. Respectively, the convolution of spectra is to transfer of zero spectra $\Delta\varepsilon$ into the extreme points of the wave vector k_1 (Fig.1). Further with the aid of the expansion of Kotelnikov-Shannon [3] it shows that the contribution to the scattering give only those spectral components of *p(q)*, which intersect by Ewald's sphere of the radius k (Fig.1), i.e., satisfy Bragg's conditions. The angular distribution of scattering field is determined by the expression, close to the Rayleigh-Gans formula:

$$E_s(\vartheta,\varphi) = \frac{3(m^2-1)}{4\pi(m^2+2)\sqrt{2}}\frac{\chi}{\sqrt{\chi_1}}\rho^2 f(q_B,\vartheta_B,\varphi)\sqrt{2Z_0P_0}\sqrt{1-\sin^2\vartheta\cos^2\varphi} \qquad (1)$$

where $\chi_1 = S/a^2$, S is the area of shadow; $q_B = 2\rho\sqrt{m\sin^2(\vartheta/2)+(m-1)^2}$, $\vartheta_B =$ $=-\mathrm{asin}(\sin\vartheta/\sqrt{m\sin^2\vartheta+(m-1)^2}$ and φ are coordinates of the spectral component, intersected by Ewald's sphere; $Z_0 = \sqrt{\mu/\varepsilon}$ is wave impedance of space; P_0 is power of the incident wave to S. Main difference from Rayleigh-Gans approximation is shift of the spectrum center of *p(q)* from Ewald's sphere (Fig.1), in other words the replacement $q = 2k\sin\vartheta/2$ to the value $\overline{q}_B$, which satisfies Bragg's conditions. The displacement confine increase E_s and makes it possible to use the formula (1) for particles of arbitrary size and to calculate the scattered (in the narrow sense) field of the "soft" particle of any shape.



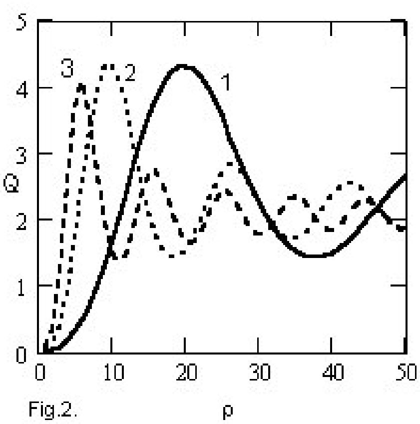


Fig.1. Construction in the spectral space. 1 – 3 are maxima and 4 – 6 are minima of spectral density (based on the sample of a sphere); 7 is Ewald's sphere; 8 is $\overline{k}_s$; 9 is $\overline{q}$.

Fig.2. Calculated Q for the sphere with m = 1.1 (1), m = 1.2 (2) and m = 1.33 (3).

3. Diffraction (in the narrow sense)

The second part of the complete field is determined by diffraction. The scattered energy is drawn from the incident wave, in which is formed the hole with the shape of the particle shadow. This process can be interpreted as the emission of the aperture the wave in antiphase. Field distribution of this wave in the far zone is possible in the scalar approximation easy to find by the method of Huygens-Kirchhoff-Fresnel [4]. Since the diffraction wave is intended for the compensation in the incident wave of the fields scattered, the energy transferred by it must correspond precisely to the energy of the scattered field, but they will not be determined by the energy, which falls to the area S.

4. Complete field scattered by particle

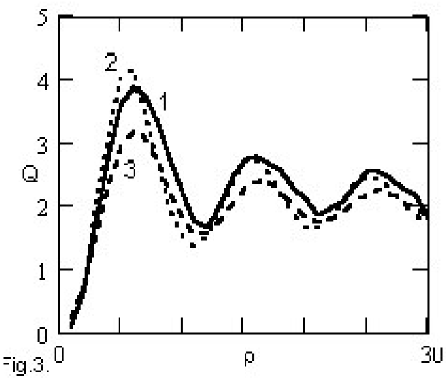
The distribution of complete field makes it possible to determine the indicatrix of scattering and efficiency factor (or integral scattering-cross section) $Q_s = P_s/P_0$. The finding of complete field is conducted three stages: 1. Calculation of the scattered field; 2. Calculation of the diffracted field taking into account the standardization of its energy; 3. Summing up of these fields.

5. Scattering of light by sphere

For the illustration of method and estimation of its error let us examine scattering by the sphere of radius *a*, whose three-dimensional space spectrum is described by the known formula:

$$f(\rho) = 3[\frac{\sin\rho}{\rho} - \cos\rho]/(\rho)^2 \qquad (2)$$

In Fig.2 are shown the calculated graphs Q(ρ) and comparison with the Mie theory (Fig.3,4). It is evident, that an error of the method is of the order ~20%. In Fig.5 are shown the indicatrices. Construction in the spectral space (Fig.1) makes it possible to interpret the obtained results. It is evident that the maxima and the minima Q(ρ) correspond to falling on the Ewald's sphere of maxima and minima of the spectrum f(ρ). It is evident that in the case of sphere with not very small ρ the main lobe is defined, in essence, by diffraction, its intensity grows as ρ^2, while side lobes are determined, in essence, by scattering; moreover, with the growth ρ their envelope does not change, changes only the number and the direction of lobes.



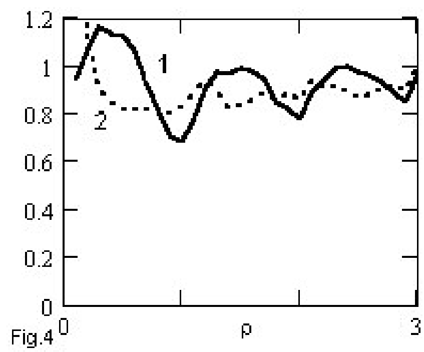
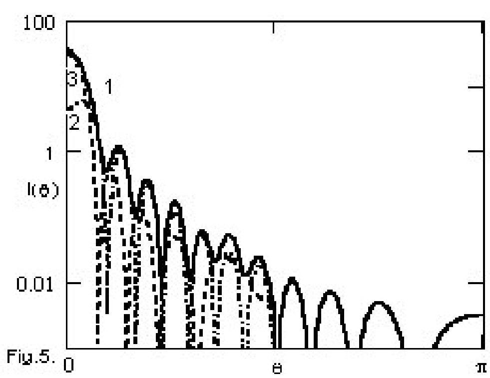


Fig.3. Calculated Q (1), Q with data of Mie (2) and data of van de Hulst (3) for sphere m =1.33.
Fig.4. Deviation of calculated Q (1) and data of van de Hulst (2) from data of Mie.



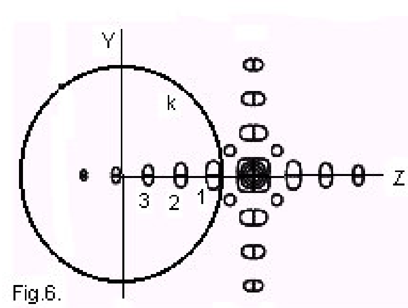


Fig.5. The sphere indicatrix with m = 1.33 , ρ =16, (1), its components of "scattering" (2) and "diffraction" (3).
Fig.6. Construction in the spectral space for the cube. 1 – 3 are maxima, k is Ewald's sphere.

6. Scattering of light by a parallelepiped

The three-dimensional spectrum of parallelepiped takes the form:

$$f(\rho,\vartheta,\varphi) = \frac{\sin(\rho\sin(\vartheta)\sin(\varphi))}{\rho\sin(\vartheta)\sin(\varphi)}\frac{\sin(\rho\sin(\vartheta)\cos(\varphi))}{\rho\sin(\vartheta)\cos(\varphi)}\frac{\sin(\Gamma\rho\cos(\vartheta))}{\Gamma\rho\cos(\vartheta)} \qquad (3)$$

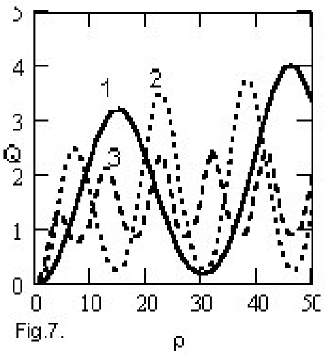


Fig.7.

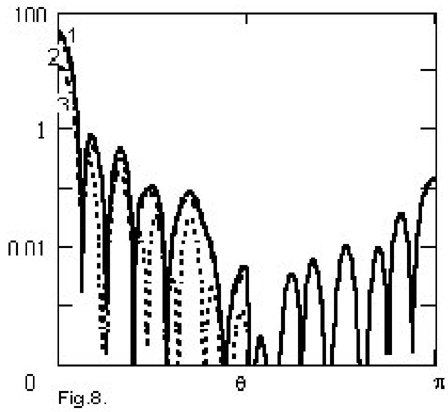


Fig.8.

Fig.7. Calculated Q for the cube for m = 1.1 (1), m = 1.2 (2) and m = 1.33 (3).

Fig.8. The cube indicatrix with m = 1.33 , ρ =16, (1), its components of "scattering" (2) and "diffraction" (3).

The spectrum of parallelepiped principally differs from the spectrum of sphere [5]. Its main maxima are arranged along the axes XYZ, their amplitude linearly decreases with their number (Fig.6). They form in the space is three-dimensional cross, between which is arranged the three-dimensional net of side-line maxima with much smaller amplitude. The salient of spectrum determine a difference in the characteristics of the light scattering by parallelepiped from by sphere. Since the main contribution to the scattering introduce the maxima of spectrum on the axis Z, upon the falling to the Ewald's sphere of zero spectra (when thickness it is multiple to the integer of waves) scattering in practice does not occur, $Q \to 0$ (Fig.7).The contributions of scattering and diffraction to main lobe of indicatrix are close; however, the side lobes scattering practically does not give contribution as a result of the fact that the main maxima from the axis Y do not fall on Ewald's sphere (Fig.7.,8).

7. Conclusion

Besides the sphere and the parallelepiped, also the characteristics of scattering by spheroids (prolate and oblate) and cylinders of finite length are investigated. All results obtain natural explanation from the point of view of space spectra. An error in the results of calculations is approximately the same (order 20%) as in most of the approximate methods, for example, of the van de Hulst's method [2], but the time, required for the calculation of the scattering characteristics usually does not exceed 1 hour, in the majority of the cases being ~10 minutes.

Acknowledgement

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