Rolling contact wave problem – numerical investigation

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Streszczenie

W pracy przedstawiono numeryczne aspekty modelowania zjawiska kontaktu tocznego. Uwzględniono sprężysto-lepkie zawieszenie koła toczącego się po sztywnym podłożu. Wykazano, że podczas toczenia pojawia się oscylacyjny ruch materiału wokół osi obrotu. Istotne zjawiska wiążą się ze strefą kontaktu oraz tarciem. Numeryczna symulacja zjawiska falowego napotyka na trudności związane z dyskretyzacją obwodu koła oraz nieciągłością prędkości w chwili kontaktu. Zaproponowano sposób modelowania, pozwalający uzyskać zadawalające wyniki. Podano przykłady obliczeń z wykorzystaniem różnych podejść.

1 Introduction

The road transportation in many countries is overcrowded and for therefore the attention is focused now on the railay systems. In the railway transportation both the load carrying capacity and speed of trains increased consederably in recent years. It involves new problems of exploitation: faster wear of rail surfaces and wheel tires. Circular geometry of wheels and plane surface of rail heads lose their perfect shape. Both on the rail head and the wheel ring wave-shaped deformations can be observed. They are called corrugations. Several papers published by the authors [1, 2, 3] of this paper deal with the wear, especially generated by the phenomena in the contact zone. In cities noise generated by tramways or even underground trains effects the environment. In long distance trains it can be tiring for passengers. From the technological point of view spurious effects of mechanical phenomena decrease the life time of rail and wheels.

[1]Supported by KBN 8 T12C 046 21

Another case where similar phenomenon occurs are vehicle breaks and clutches. High frequency oscillations generated between break shoes and friction disks or disks of the clutch considerably reduce the life time of elements.

In publications dealing with numerical modelling of rolling contact problem the wave analysis is not popular. The aim of this work is the presentation of the numerical questions met in discrete analysis. It is the discontinuity of the boundary conditions in the contact, rotation of the geometry and the stress field in the time integration process, discretization of the boundary by a polygon line and the influence of the boundary nodes on the dynamic response. Further problems are the following: self excitation in higher (300 km/h) velocity range, influence of non-linear material properties (visco-plasticity), non-linear friction, torsional vibration of wheel/axle system, influence of plate bending state for cone-shaped wheel, approach to optimization of resulting parameters. We can see that the subject is complex and that is why research centers in the world work intensively in the field.

Different hypotheses were assumed as a base of investigation. Some of them can be easily rejected, others are intensively investigated. In the literature the following cases are pointed as a source of corrugations:

- imperfections in rail joints,
- cone form of wheels which results in different linear speed of left and right wheel; it causes snaking of trains and generally, disturbs steady motion,
- periodical structure of rails (sleepers); instability of motion on the periodically placed supports [4],
- contact problems between wheel and rail; stick and slip zones which vary with high frequency (horizontally) and generate waves which deform elastically, then plastically both contact surfaces [5, 6, 7],
- residual stress caused by manufacturing and service of rails and wheels [8],
- non-linear friction law in the stick zone [6],
- influence of material hardening [9],
- deformation of elements of wheel/axle as a result of the impact during rolling motion,
- instability of wheel-sets motion [10, 11],
- strong hits of a perfectly round wheel rolling on the waved rail [12].

The important contribution to the problem was published in [11, 13, 14]. The physical continuous model is treated analytically. The wheel tire is modeled by the elastic curved Rayleigh's beam joined with the axle by means of the continuous elastic Winkler type foundation. The elastic foundation constituting the wheel disk carries out the load in three directions: circumferential, radial and vertical to the plane of the wheel. Curved beam theory ensure the real shape of the cross-section. Visco-elastic properties of the wheel material are described by the Kelvin-Voigt model. Frequency response functions for forced vibrations of the railway wheel (Fig. 1) proves the significant amplitude increase for the frequency 100 Hz with the velocity 200 km/h. There is no doubt that wave propagation analysis is essential.

We try to prove that

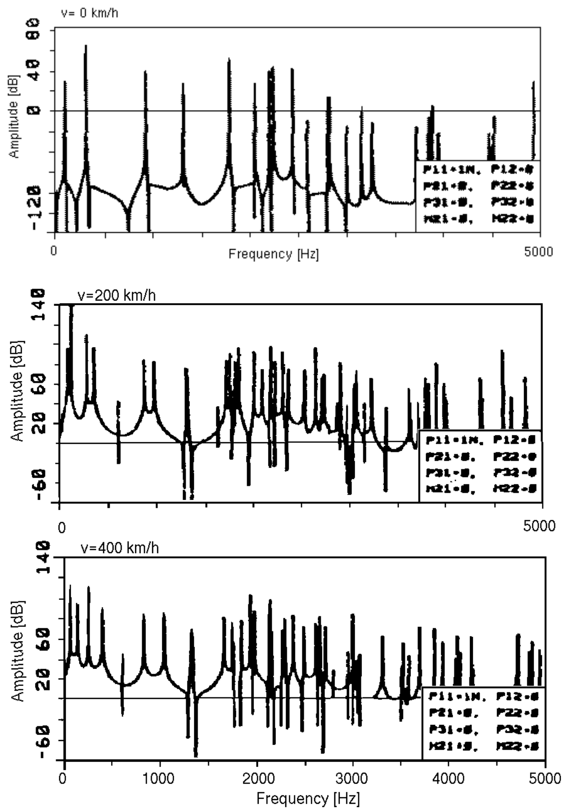

Figure 1: Frequency response functions for rotating railway wheel.

- in the steady rolling the waves generate periodic stress concentration on the surfane of the wheel,
- the contact force is periodic with relatively high frequency,
- circumferential oscillations with moderate frequency occur and in such a case the friction law can flow on the final results.

2 Numerical approach

The task is divided into stages, which enable us to investigate the phenomenon selectively. We consider the wheel suspended rigidly, elastically with damping or rolling over the waved surface (Fig. 2). In the early approach the coordinate system was bound with the rotating

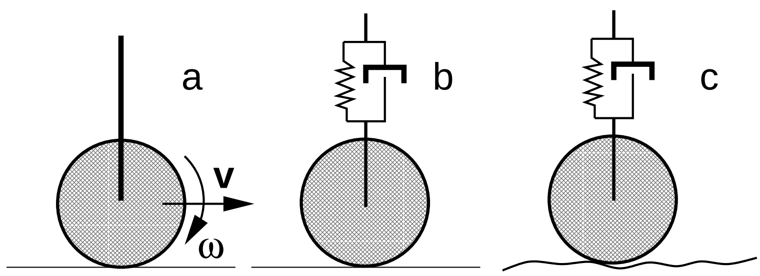

Figure 2: Problems investigated numerically.

wheel. In such a case the mesh geometry was practically fixed while the contact boundary conditions were subject to rotation (Fig. 2a). The elastic suspension disabled the same approach without relatively complicated new formulation. That is why in further tests (Fig. 2b) we assumed the classical approach, with the fixed coordinate system.

2.1 Velocity in the contact

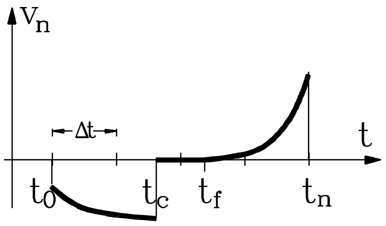

Figure 3: Relative normal velocity in the time interval $[t_0, t_1]$.

There are many ways in which boundary conditions can be taken into account. We used two of them. In the first one the contact conditions are imposed on displacements. The time integration scheme should have displacements as the main unknown vector to be solved. Thus we preserve the continuity of displacement field. The second method [15] uses the velocity formulation. In this case the boundary conditions imposed to the velocity result in penetration, unless more complex algorithm is assumed.

Now let us discuss the discontinuity of the velocity in contact. This discontinuity shown in Fig. 3 can be removed. It is necessary to impose the relations derived from the contact conditions to the motion equation in the time step proceeding the detection of the body penetration, i.e. in time t_0 if the penetration was detected in t_1. The imposed restrictions have to reduce the velocity of the point to such a value v_0, for which in the next time step $v_1 = 0$ and $x_1 = x_{cont}$ (Fig. 4). In the simplest case shown in Fig. 4 the velocity at t_0 must get the value

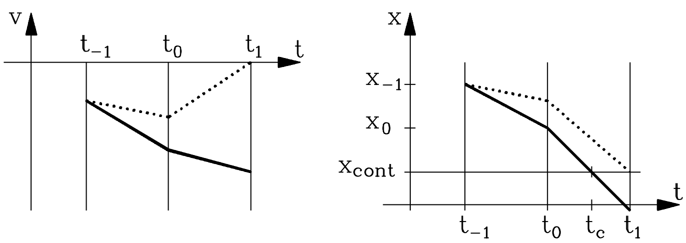

Figure 4: Reduction of the velocity near the contact: continuous line – free motion, dashed line – motion with constraints.

$$v_0 = \frac{x_{cont} - x_{-1}}{h} - (1-\beta)\, v_{-1} \quad . \tag{1}$$

The parameter β is used in the difference relation which determins the displacement

$$x_1 = x_0 + [(1-\beta)v_0 + \beta v_1]\, \Delta t \tag{2}$$

The resulting position of the point is

$$x_0 = (1-\beta)x_{-1} + \beta\, x_{cont} + (1-\beta)^2 h v_{-1}. \tag{3}$$

At $t = t_1$ the velocity v_1 equals to 0 and x_1 equals to x_{cont}.

It should be emphasized that the reduction of the velocity before the contact results in decreasing the energy of the system. It can be implemented in two ways: by imposing

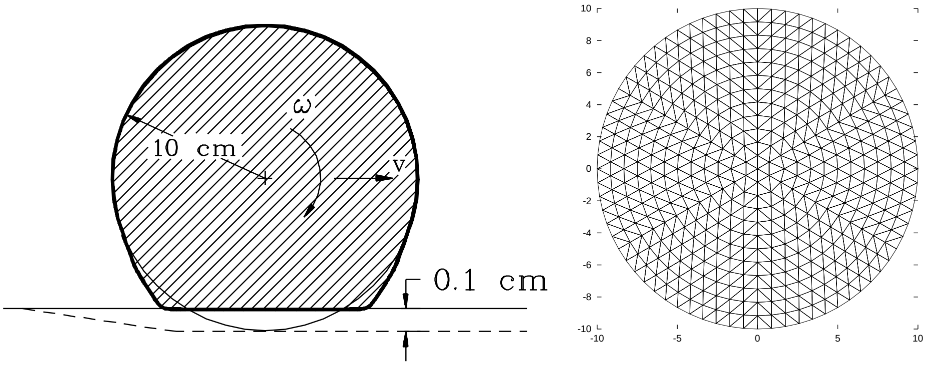

Figure 5: The scheme of the rolling wheel and the spatial mesh assumed in calculation.

the constraint on the velocity of a specified joint or by imposing a predefined breaking force. Both approaches are equivalent since the value of the force is equal to the reaction of contact force in the case of the condition imposed on the velocity of the point. The breaking force acts during the step Δt. As the effect the node hits an obstacle with the lower velocity and the impact returns less of the momentum after the reflection. The momentum taken off must be given back to the system in the first step in which the point moves freely after the contact.

2.2 The space-time modeling of contact problem

Dynamic contact problems are characteristic of fast varying contact domains. In some problems the precise definition of the contact zone is of fundamental importance. Contact phenomena with friction that involve vibration of the stick and slip type require both the small time step of the integration of the differential equation of motion and refined mesh in that region. The finite element method gained its popularity since it is relatively simple and universal in applications. However, in certain problems the F.E.M. is difficult since its discrete form does not allow to investigate the problem with the required precision. For example, the varying contact zone, extended between two nodes in spatial mesh requires subintegration of resulting matrices to evaluate more precisely friction contribution. Much more natural approach is to modify the spatial mesh and subintegrate the differential equation in time, in required regions only.

The spatial adaptation of the mesh in structural dynamics can rarely be found in the literature (for example [16, 17, 18]). However, the simplest interpolation of displacement, velocity and acceleration vectors were discussed there with particular reference to additional joint. Such a discontinuous path to the refined/coarsened mesh changes the problem under consideration: local and global stiffness and temporary distribution of acceleration and velocity, compared with the problem solved with the constant mesh. The adaptation procedure may incorporate greater error than the simple classical computation. It is well visible if higher modes are not damped. Although smoothing by physical or numerical damping enhances the quality of the solution, we can not accept such a technique without restrictions.

The basics of the space-time finite element method was described in [5, 19, 20, 21]. First the displacement formulation was developed. Then the same idea was extended to derive velocity formulas [22, 15, 23, 24].

2.3 Example and results

In the numerical analysis of the rolling contact problem we shall limit the investigation to the range where the contact occurs. Other factors such as friction, plastic deformation, hardening, can simply be added following the classical scheme. As an example we take the wheel with the radius R=10 cm, thickness 1 cm, made of steel (E=2.05·10^7 N/m^2, ν=0.3, ρ=7.83 g/cm^3). It rolls on the rigid base with an angular speed ω. The linear velocities taken into account were of the range 90–180 km/h. The elastic material in plane stress was assumed. The domain was discretized with 864 triangles and 469 nodes (Fig. 5). The uniform mesh density was applied for the reason of wave nature of the process and stress concentration passing throughout the domain.

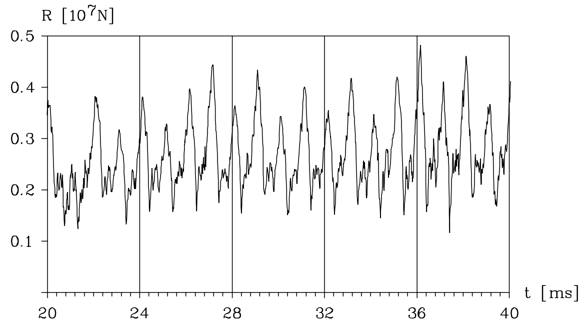

Figure 6: Contact force in successive turns in the case of ω=0.3·10^{-3} rad/s

To avoid multiple rotations of matrices effected by the rotation of the structure and in the same time the accumulation of round–off errors the rotation of the rigid base over the fixed wheel was assumed. All the forces arising from the circular motion were introduced. In the first stage the wheel, which turns is settled slowly on the rigid base (in numerical simulation the base which turns presses slightly the fixed wheel). The depth of penetration (flattening) reaches finally d=0.1 cm (Fig. 5). In order to avoid the influence of the initial conditions and to reduce the effect of wave reflections and interference the comparatively large numerical damping was assumed. The value of the parameter γ [15] was equal to 0.2 and it corresponded to the logarithmic decrement of damping $\Lambda = 0.03$. In practice it allowed to damp vibration according to the first eigenform and the period $T \approx 80$ μs in 95% during the first 1/4 turn of the wheel.

The elastic-plastic material with hardening was assumed in computation. The second invariant of stresses J_2 was integrated in successive phases of the full turn. It enables us to show the distribution of stresses in the material [9]. Final form of the diagram depends on problem parameters. In the presented example corrugations are successfully flattened. However, in the case of other material coefficients concentrations of stresses under the wheel surface increases.

Computation shows that the contact force vary, even when the motion is steady and well damped. Selected part of a wheel turn with the speed ω=0.3·10^{-3} rad/s is presented in Fig. 6. The stresses in the rotating disk in two speeds are depicted in Fig. 7. The analysis exhibits the periodical distribution of the wear on the wheel surface which can occur during exploitation. The number of contact force oscillations decreases along with the increase of the speed. It was observed for example in [25, 26] for a rubber wheel. However, in those publications the authors treat the problem as an eigenvalue problem. They do not solve the initial boundary problem. The estimated diagram of the relation between the number of oscillations in one full turn and the velocity ω is shown in Fig. 8. The value of the contact force increases with the increase of the velocity ω. The investigation was performed for a full turn of the wheel. If the number of waves due to a turn is not an integer (*i.e.* the phase shift occurs after each turn), then the diagram is

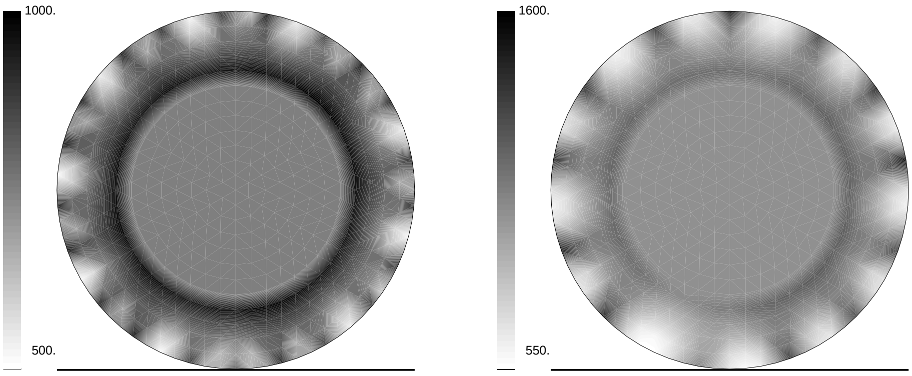

Figure 7: Stresses I_2 in the wheel.

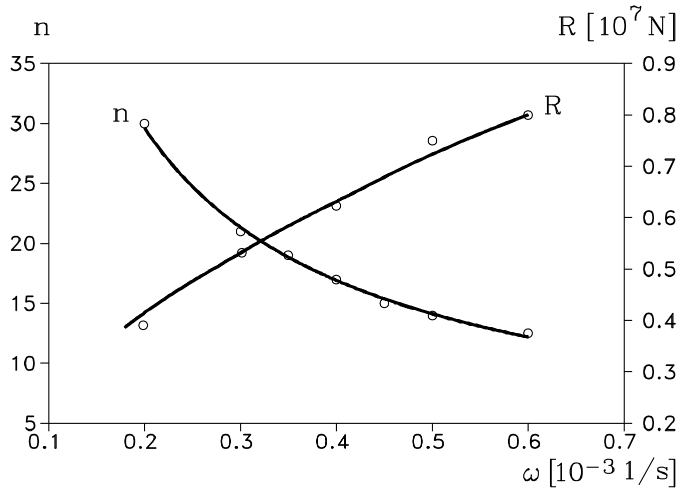

Figure 8: Number of cycles of the reaction and its maximal value in relation of the angular velocity ω.

disturbed in the vicinity of the lower point of the wheel, from which the solution starts and on which is finished.

3 Elastic suspension

The problem depicted in Fig. 2b was solved with the use of unstructured mesh (Fig. 9). The radius of the wheel $R = 50$ cm. The depth of the penetration was 0.4 cm. The number of joints on the circumderence exceeded 200. High flattening of the wheel in the contact area did not eliminated the influence of boundary joints on the reaction force (Fig. 11). Selected eigenfrequencies are related to the well known forms with increasing number of wave on the surface (Fig. 10). Vertical reaction in the centre of the wheel, in which the elastic suspension is placed is depicted in Fig. 12. We can notice peaks that exceed the average value of reaction. Since the numerical and physical damping imposed to the system was significant, we expect the physical origin of these peaks. Reliable answer would be done if the full animation of displacement and velocity field was performed. Horizontal displacement of the contact point related to the angular way is depiceted in

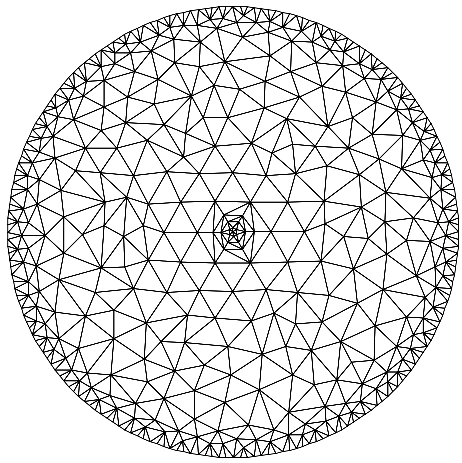

Figure 9: Irregular finite element mesh.

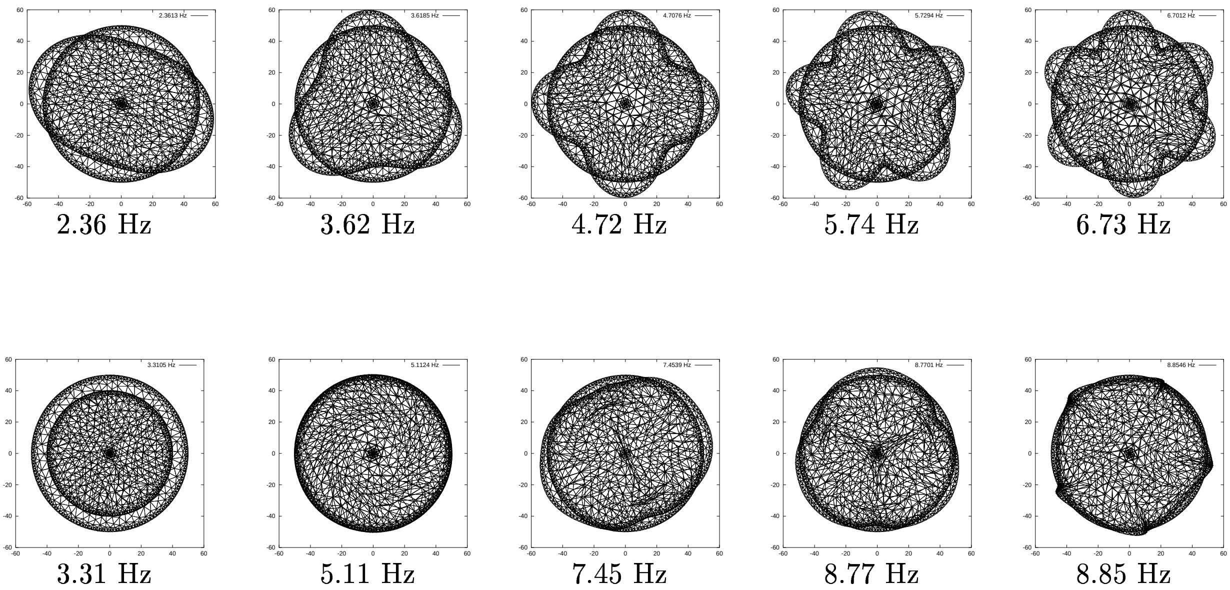

Figure 10: Selected eigenforms of a disk: polygon form (upper row, circumferential oscillations (lower row).

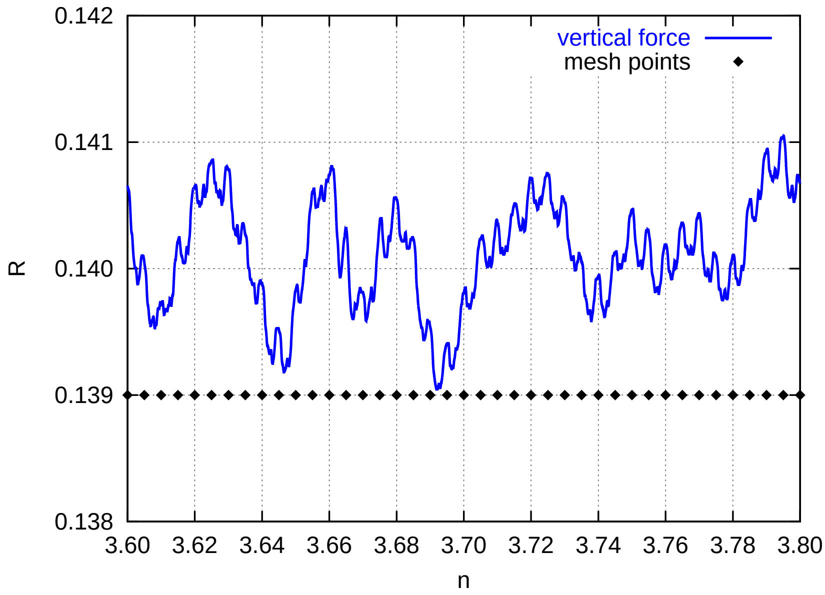

Figure 11: Vertical force slightly influenced by the mesh.

Fig. 13. Jumps of curves coincide with peaks of reaction in Fig. 12.

4 Conclusions

The efficient method for analysis of dynamic contact problem is presented. The soft way method [15] with modified contact condition described by velocities provides for a convenient treatment of the dynamic contact problem, even in the case of large time steps. The presented method is successfully applied to the problem of corrugations. Even in the simplest case of the material property one can notice the oscillation of the contact force. The resulting stress distribution is stationary if the observation is carried out in the rotating coordinate systems and for the particular value of the angular velocity. If the plastic material was used, the deformation would polygonize the wheel surface permanently. Then successive passages of the wheel over the rail increase the wear by the dynamic feedback [27]. The friction introduced to the contact region can changes quantitative relations. It is shown that neither imperfections of rail junctions nor periodic placement of sleepers generate corrugations. Simple stationary motion is disturbed by the propagation of waves from the contact point. In our case the load is introduced kinematically. In the real problem, despite of different type of loading, the situation can be similar due to considerable inertia of the wheel-set.

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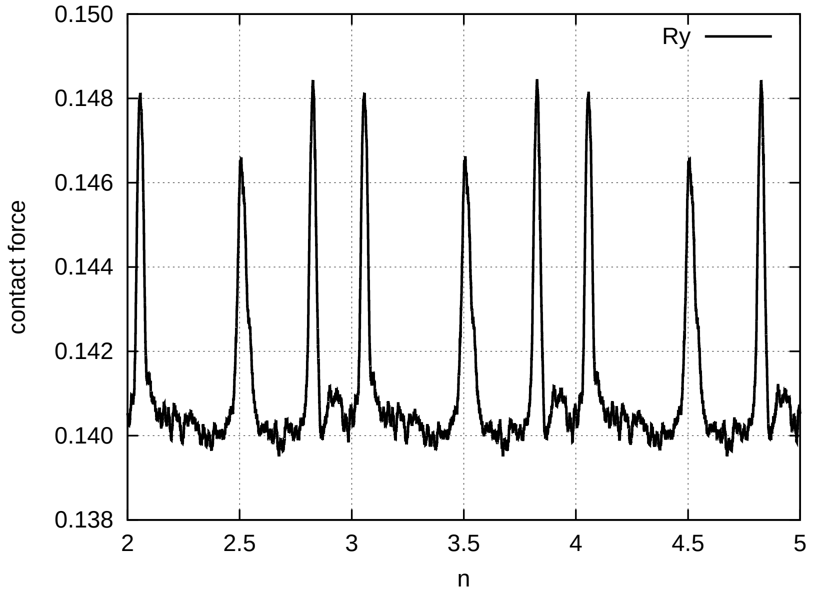

Figure 12: Vertical reaction in the suspension point.

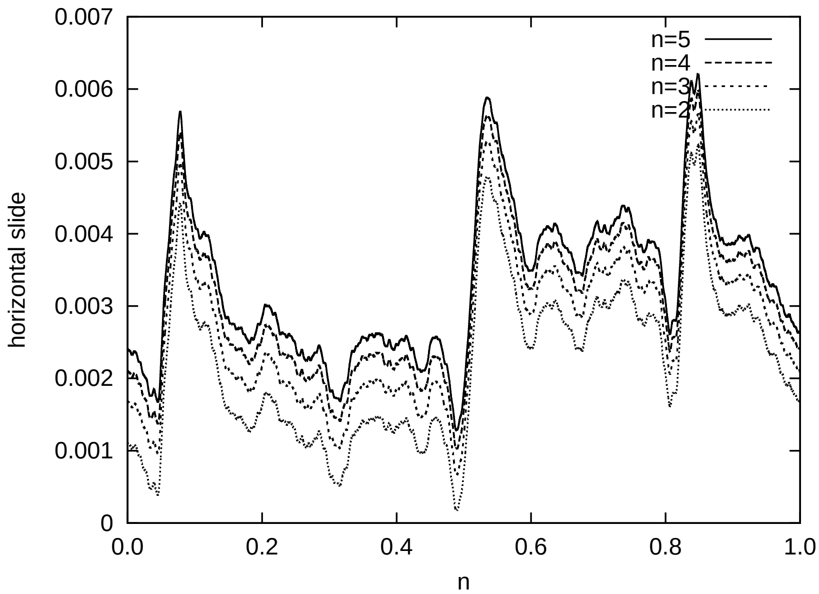

Figure 13: Horizontal displacement of the central contact point.

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