Verification of assumptions in dynamics of lattice structures

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Outline of presentation

1. Motivation
2. Examples of lattice structures
3. Joints in lattice structures
4. „Pin joint” assumption
5. Real structural behavior in dynamics
6. Examples
7. Conclusions

1. Motivation

Methods in Structural Health Monitoring

1.1. Global methods

Vibration-based methods (Operational Modal Analysis)

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{D_0}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{w}$$

$$\boldsymbol{y} = \boldsymbol{C_q}\boldsymbol{q} + \boldsymbol{C_v}\dot{\boldsymbol{q}} + \boldsymbol{C_a}\ddot{\boldsymbol{q}}$$

Transformation to state space $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix}$

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{w} \\ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{w} + \boldsymbol{v} \end{cases}$$

Identified mass, damping and stiffness matrices (→ $\boldsymbol{A}$)

Measured quantities (→ $\boldsymbol{y}$)

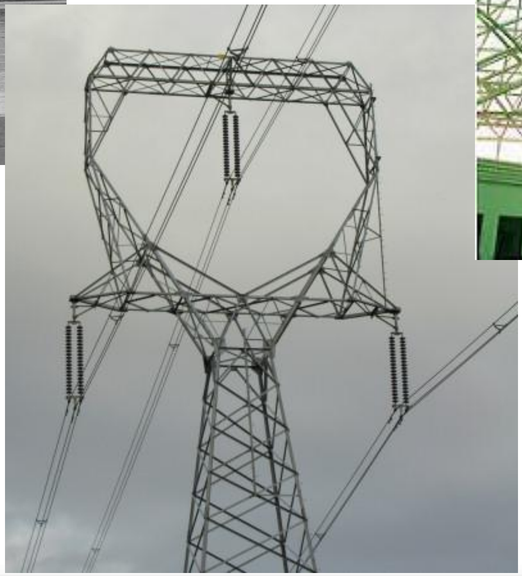
1.2. Local methods

Ultrasonic waves propagation, Vibrothermography

2. Examples of lattice structures

A large number of structures are build of system of rods (beams) commonly called trusses, frames and lattice structures.

Truss bridges



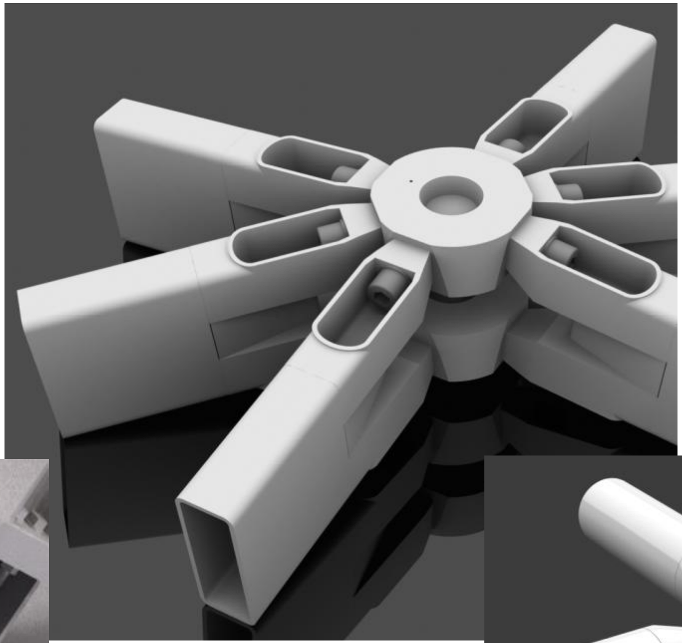
Transmission towers

Double layer grids

3. Joints in lattice structures

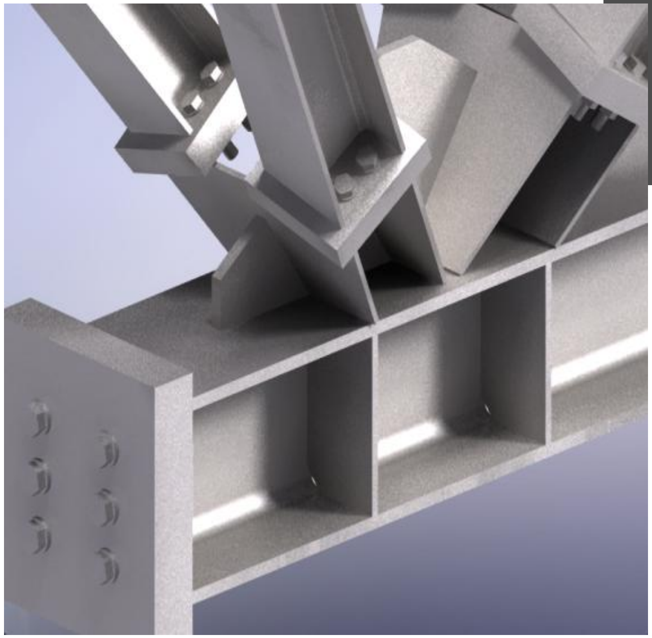
In most cases in such structures elements are rigidly connected.

FF-System
Free form

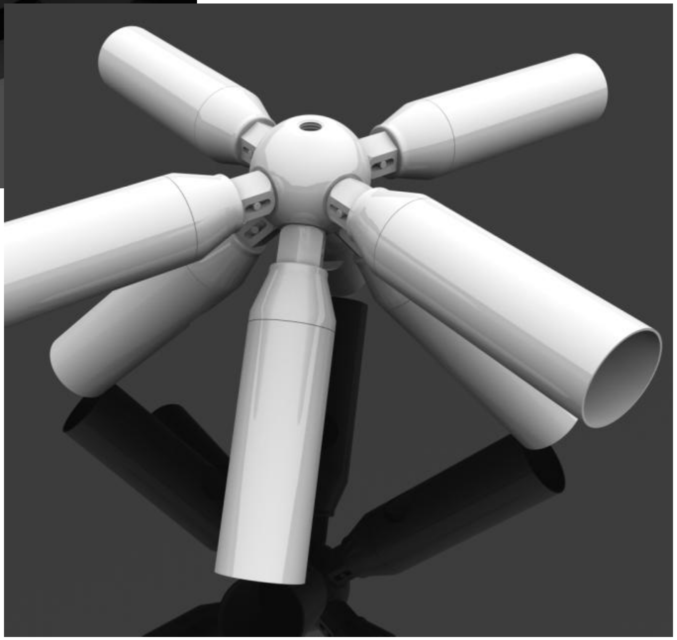


Novum System Products

Classical welded joint

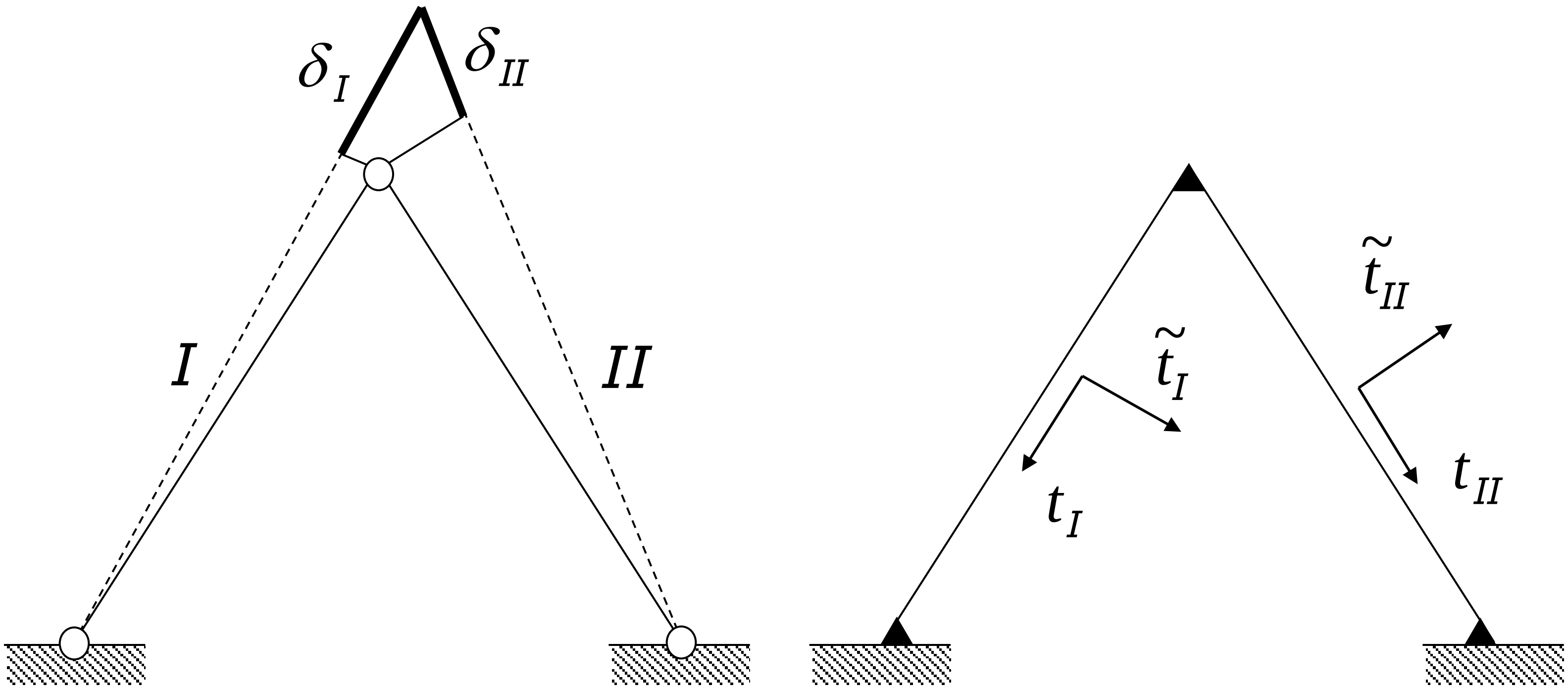


KK-System
Kugel knoten



4. „Pin joint” assumption

The origin of the „pin joint” assumption



Equilibrium equations

- for pin joint

$$\begin{cases} C_I t_{I_1} \delta_I + C_{II} t_{II_1} \delta_{II} = P_1 \\ C_I t_{I_2} \delta_I + C_{II} t_{II_2} \delta_{II} = P_2 \end{cases}$$

$$C_I = \frac{EA_I}{l_I}$$

- for rigid connection

$$\begin{cases} -\tilde{K}_I \tilde{t}_{I_1} \tilde{\kappa}_I + C_I t_{I_1} \delta_I - \tilde{K}_{II} \tilde{t}_{II_1} \tilde{\kappa}_{II} + C_{II} t_{II_1} \delta_{II} = P_1 \\ -\tilde{K}_I \tilde{t}_{I_2} \tilde{\kappa}_I + C_I t_{I_2} \delta_I - \tilde{K}_{II} \tilde{t}_{II_2} \tilde{\kappa}_{II} + C_{II} t_{II_2} \delta_{II} = P_2 \\ \tilde{K}_I \vartheta_I + \tilde{K}_{II} \vartheta_{II} = 0 \end{cases}$$

$$\tilde{K}_I = \frac{EI_I}{l_I^3}$$

$$\frac{C}{\tilde{K}} = \frac{EA}{EI} l^2 = \left(\frac{l}{i}\right)^2 \rightarrow 10^4$$

Where

$$t_{I_1}\delta_I \quad \text{and} \quad \tilde{t}_{I_1}\tilde{\vartheta}_I$$

are of the same order then the product

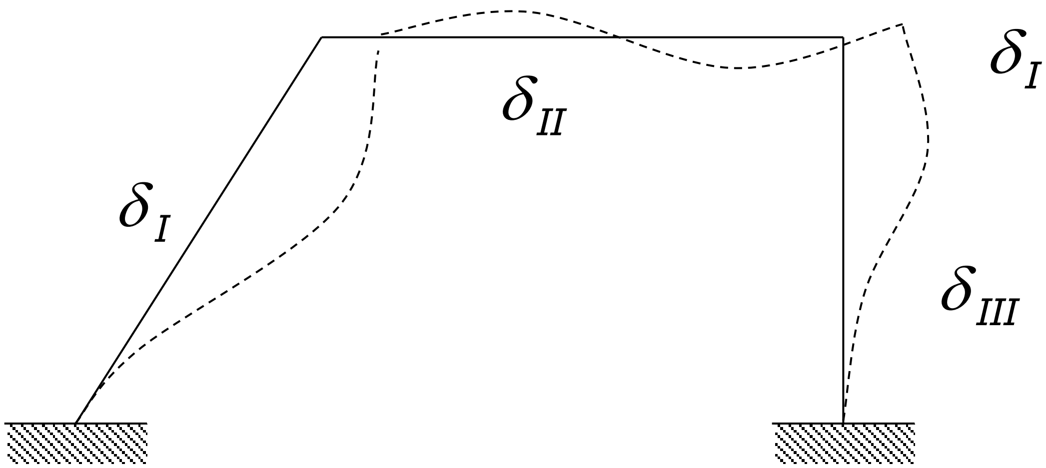
$$\tilde{K}_I\tilde{t}_{I_1}\tilde{\vartheta}_I$$

can be neglected comparing with $C_I t_{I_1}\delta_I$

$$\tilde{K}_I\tilde{t}_{I_1}\tilde{\vartheta}_I <<< C_I t_{I_1}\delta_I$$

Neglecting bending term is equivalent to the assumption that two elements are „pin joint”.

For structures when displacements are caused by bending only, „pin joint” assumption can not be applied.

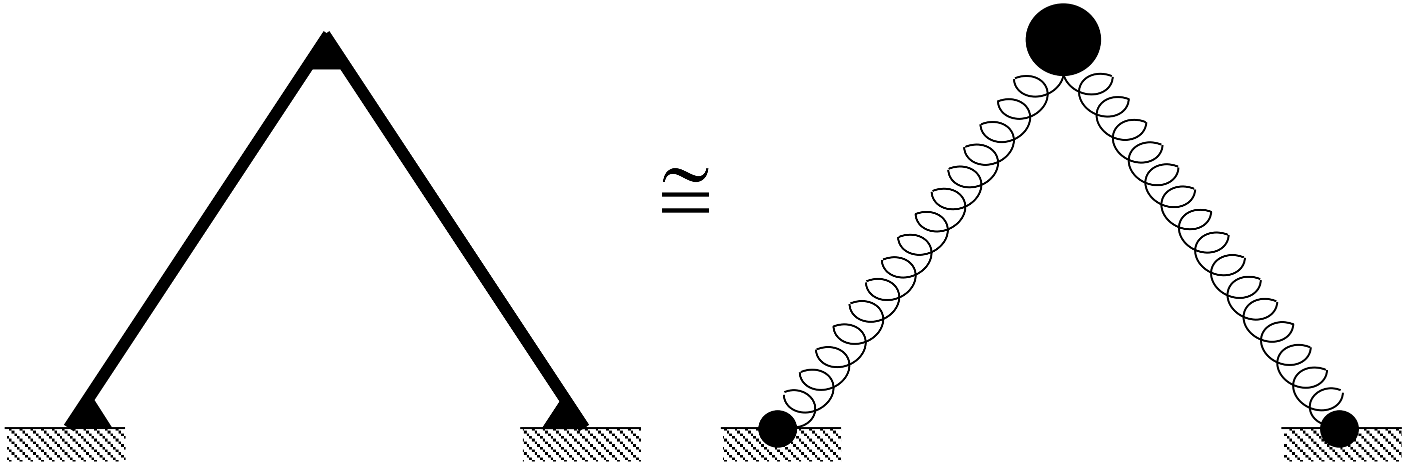


$$\delta_I = \delta_{II} = \delta_{III} = 0$$

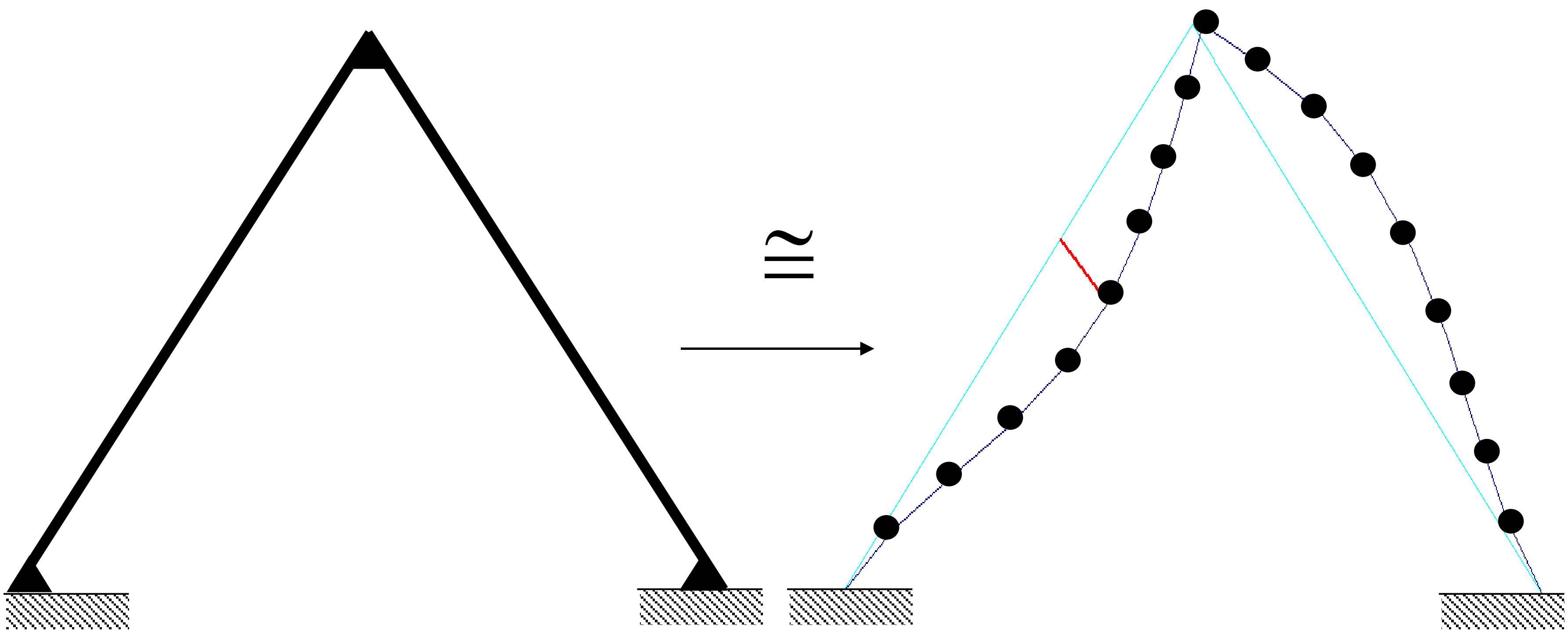
5. Real structural behavior in dynamics

The static „pin joint" assumption has been incorporated in truss dynamics.

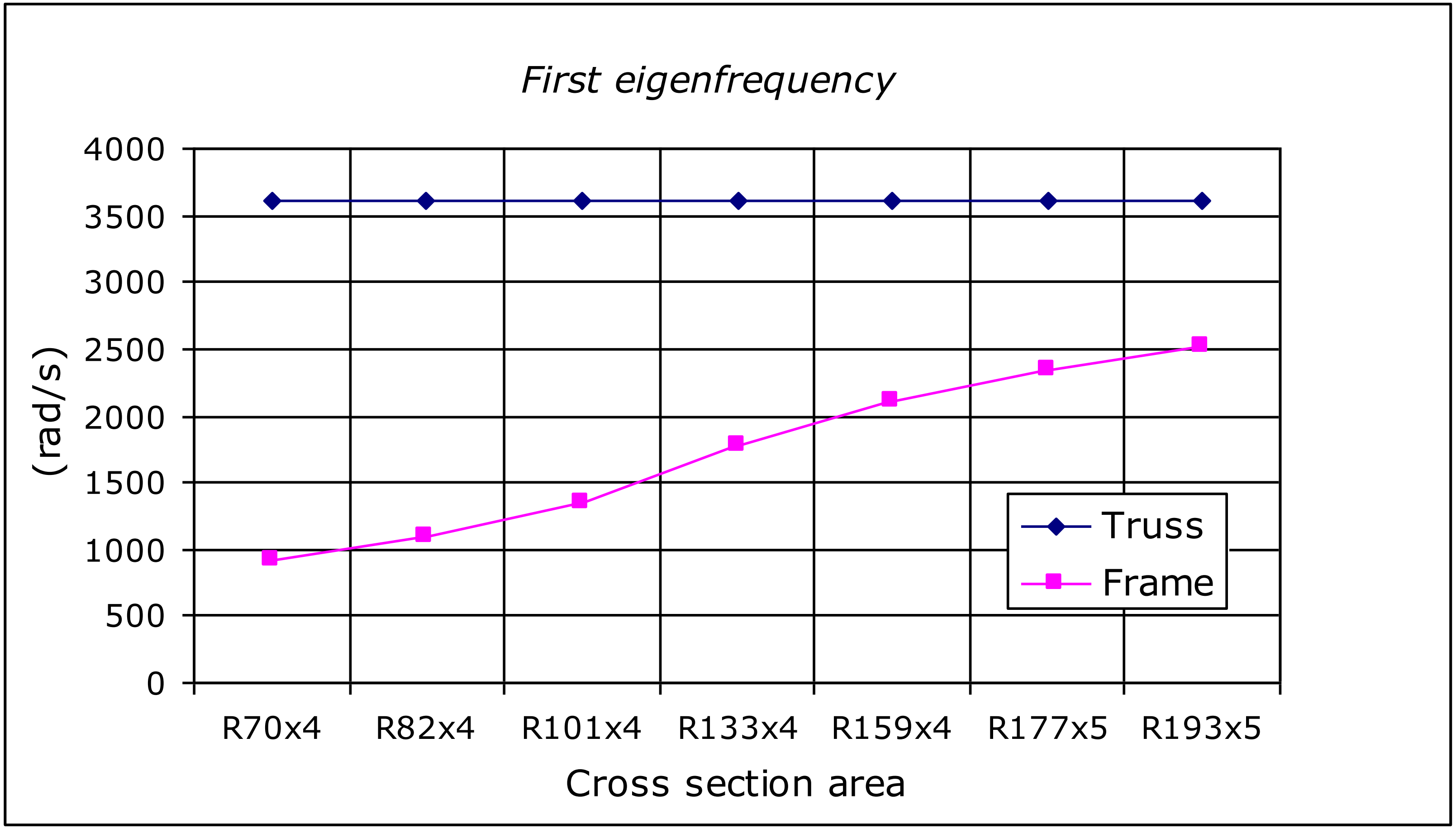
Equivalent masses of beams are attached to pin joints. Problem is reduced to vibration of concentrated masses connected with massless springs.



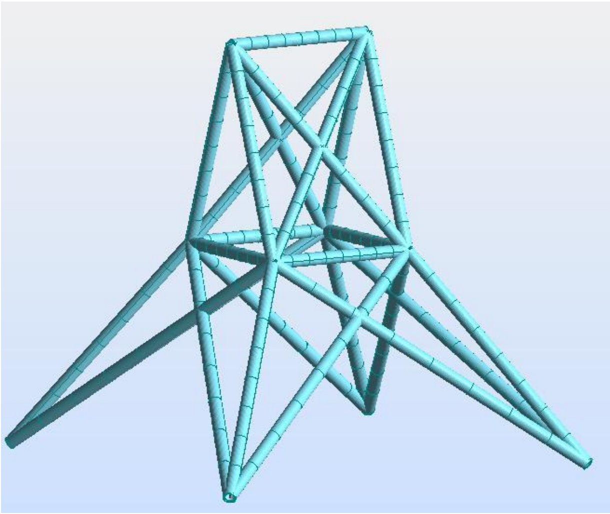
In real, rigidly connected structure, beam mass is distributed along its length and transversal motion assumed.

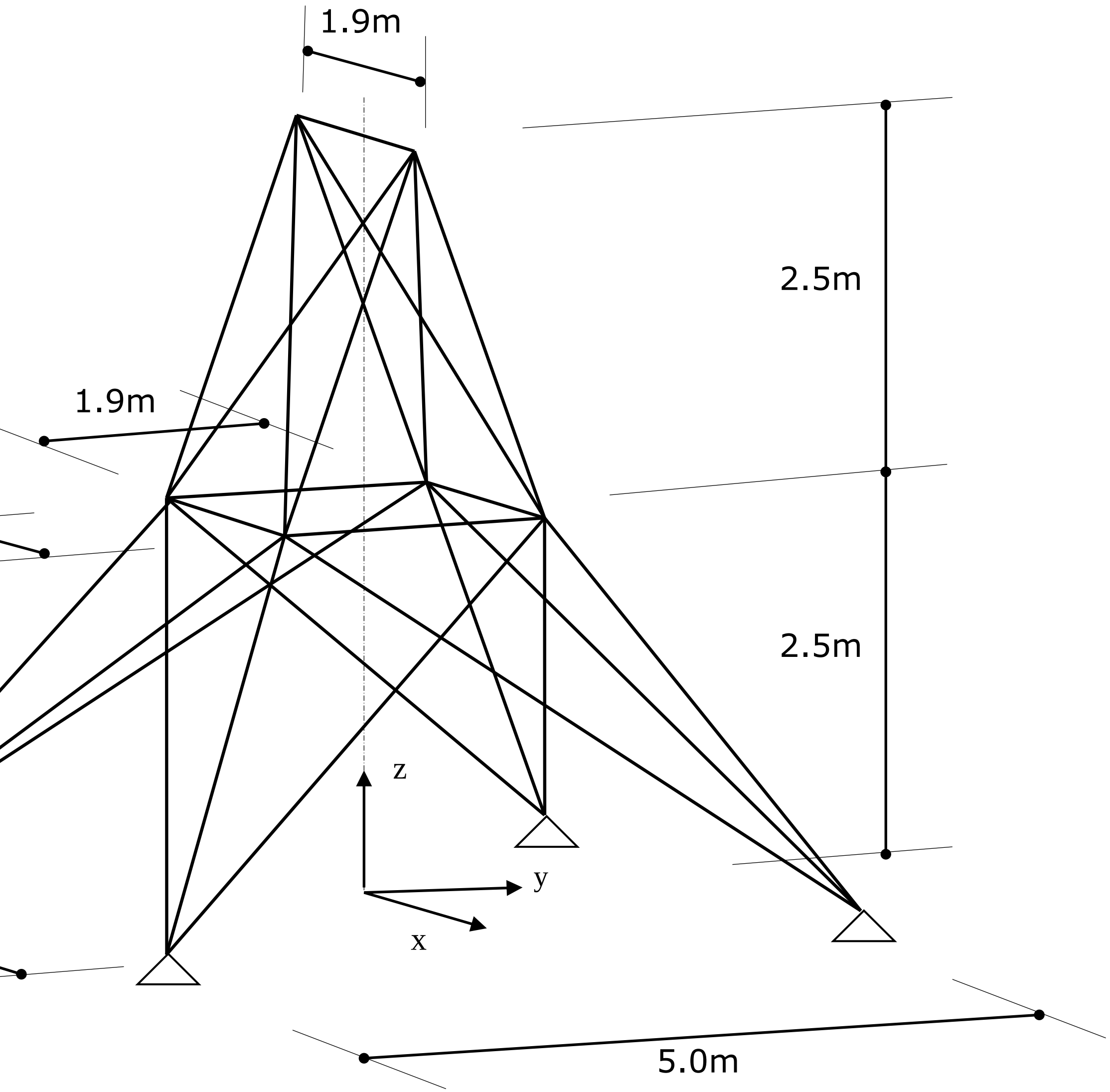


First eigenfrequency for both cases



25 bar structure

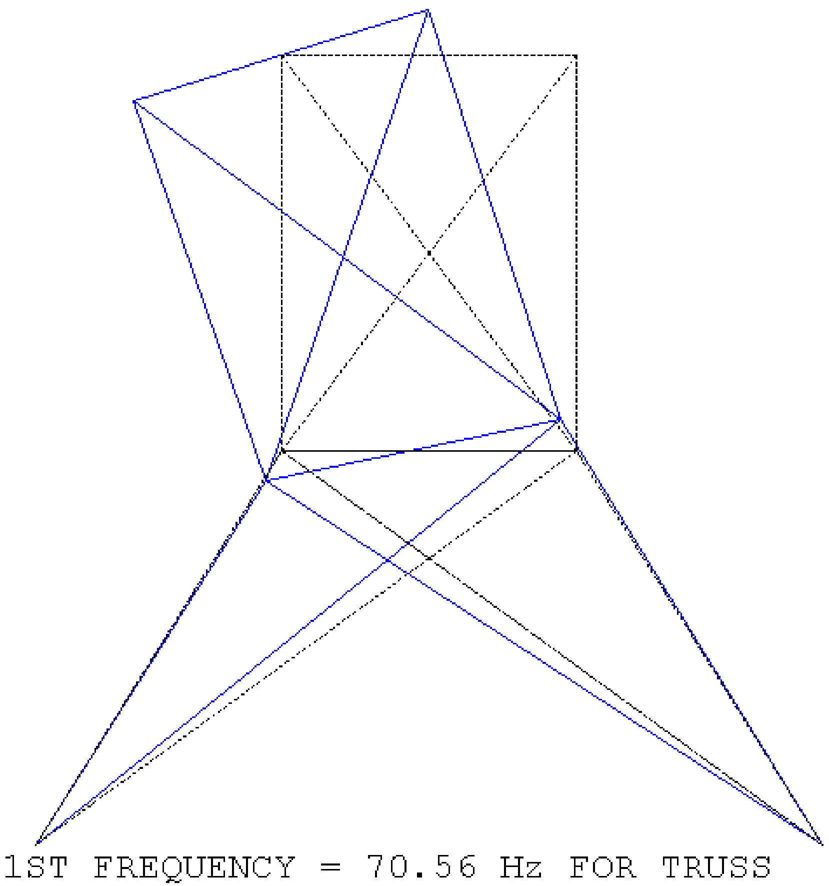


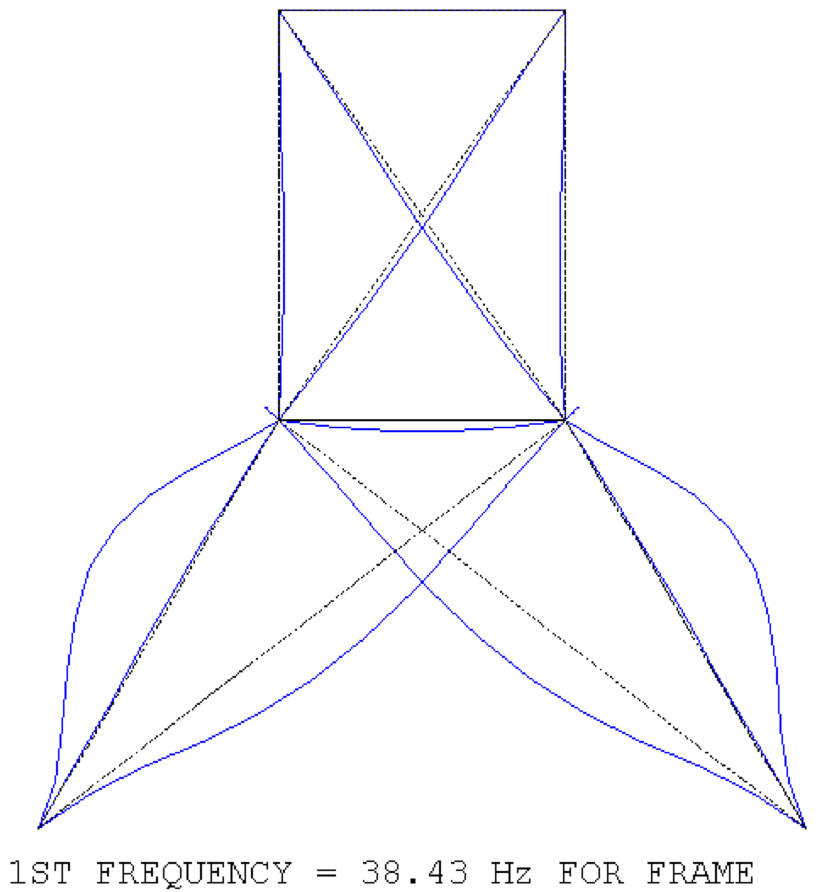


Cross section areas:

Circular hollow section
Ø159x8

First mode shape of 25 bar structure





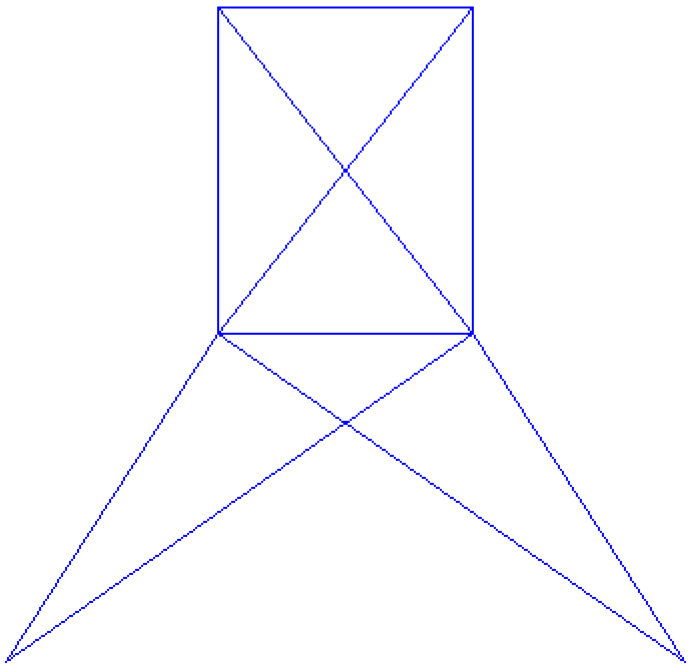
I – pin-joint model

f=70.56 Hz

II – rigid-joint model

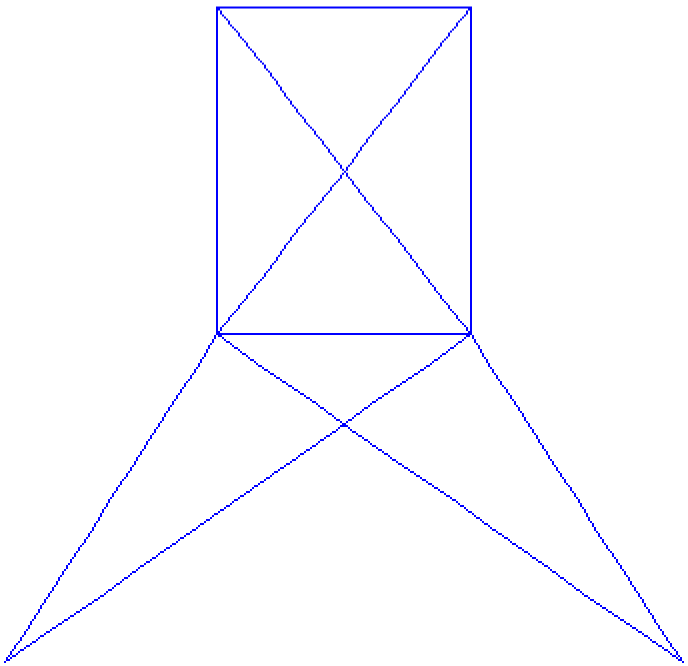
f=38.43 Hz

First mode shape animation



I – pin-joint model

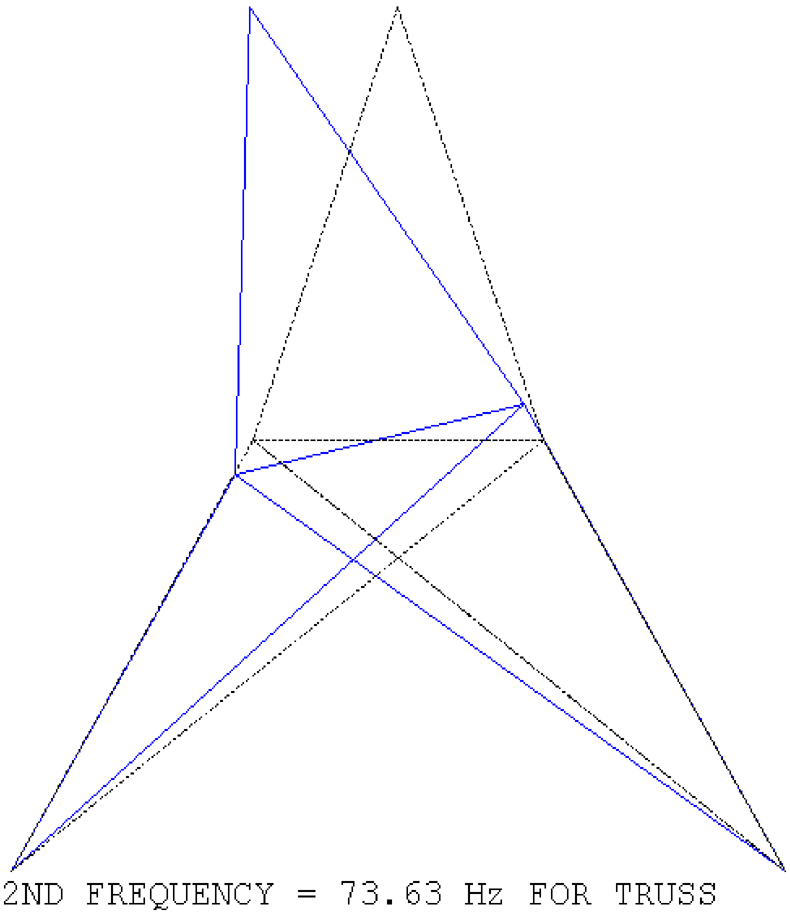
f=70.56 Hz



II – rigid-joint model

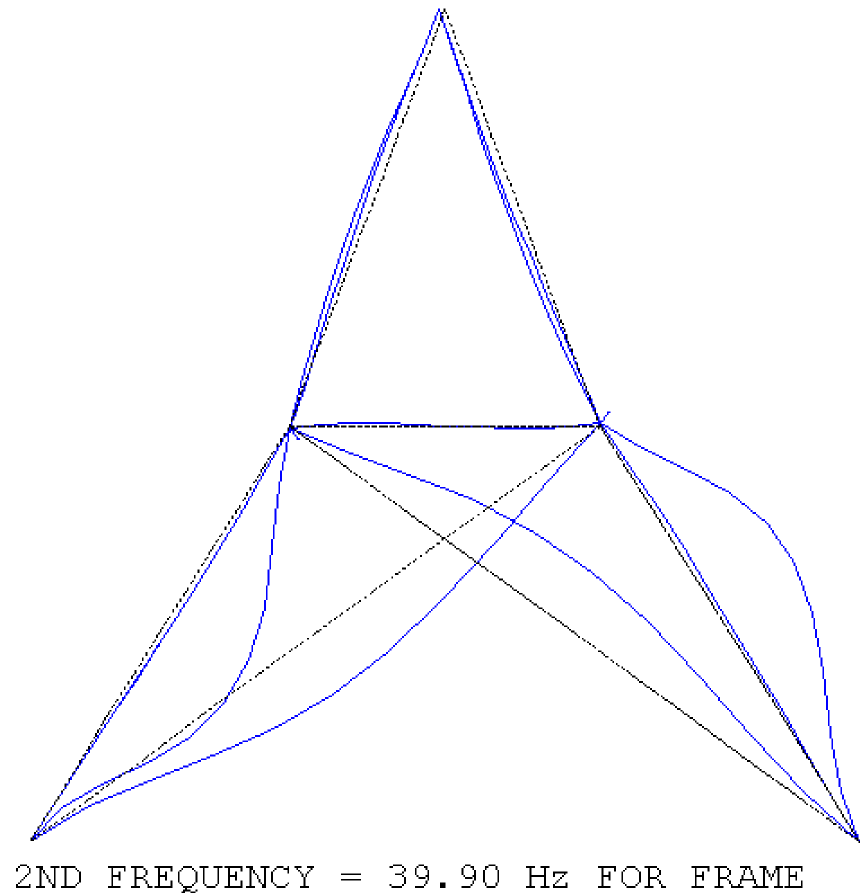
f=38.43 Hz

Second mode shape of 25 bar structure



I – pin-joint model

f=73.63 Hz



II – rigid-joint model

f=39.90 Hz

Truss tower

Cross section areas:

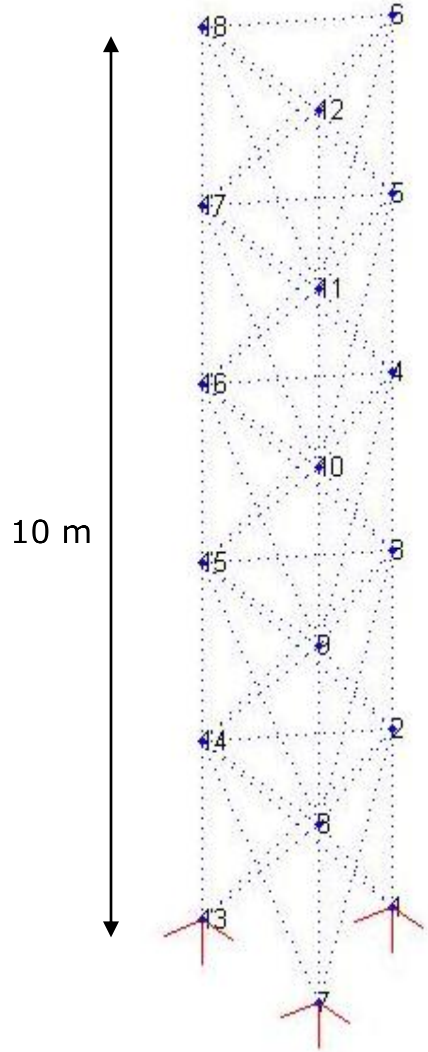
CHS Ø159/8

Material:

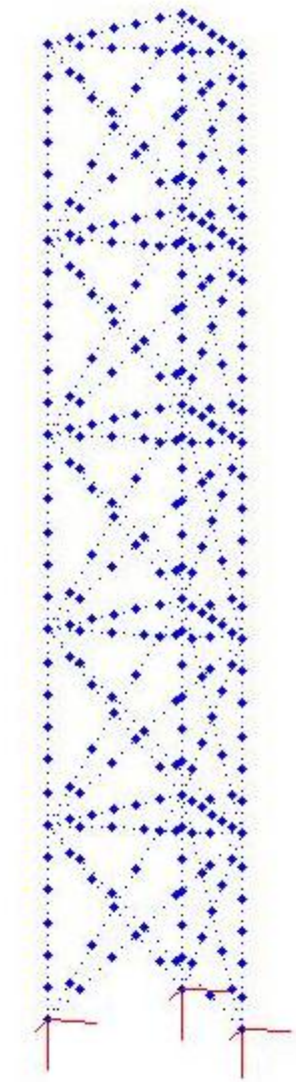
Steel

Ex=205 10^6 kPa

ρ = 7850 kg/m^3

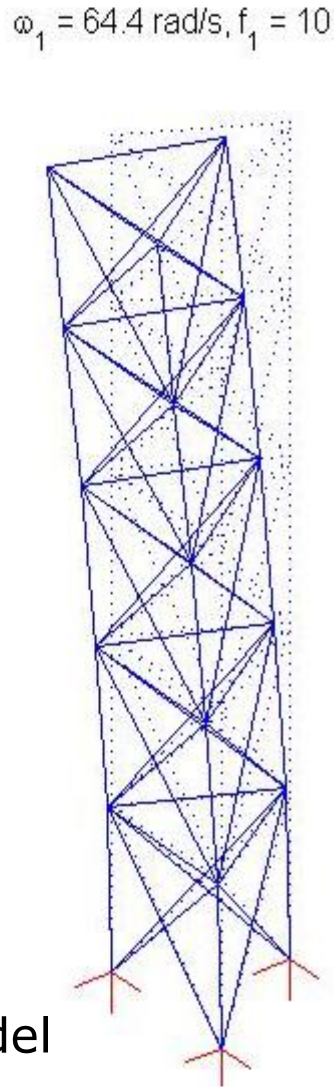


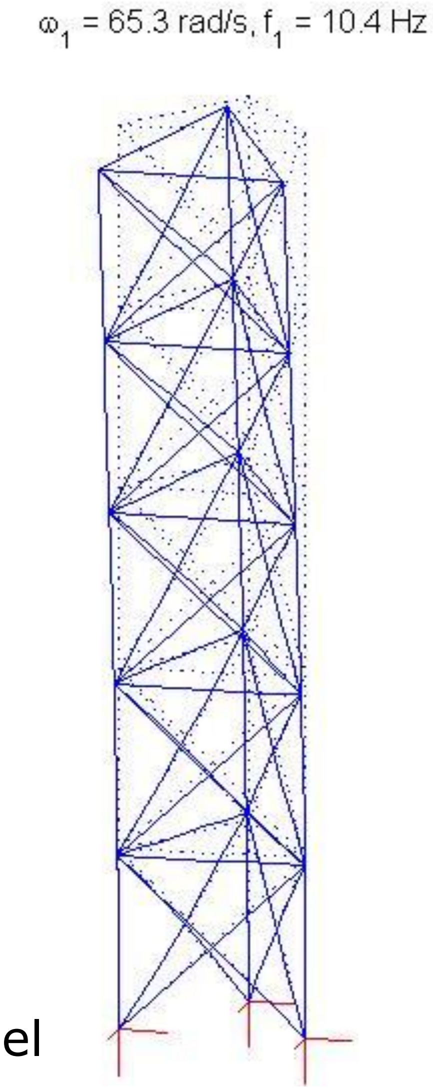
I pin-joint model



II rigid-joint model

First mode shape of tower truss





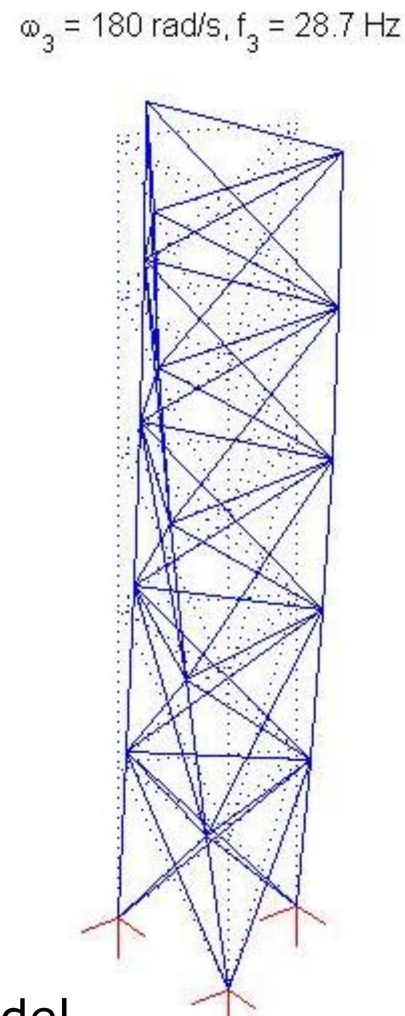
Pin-joint model

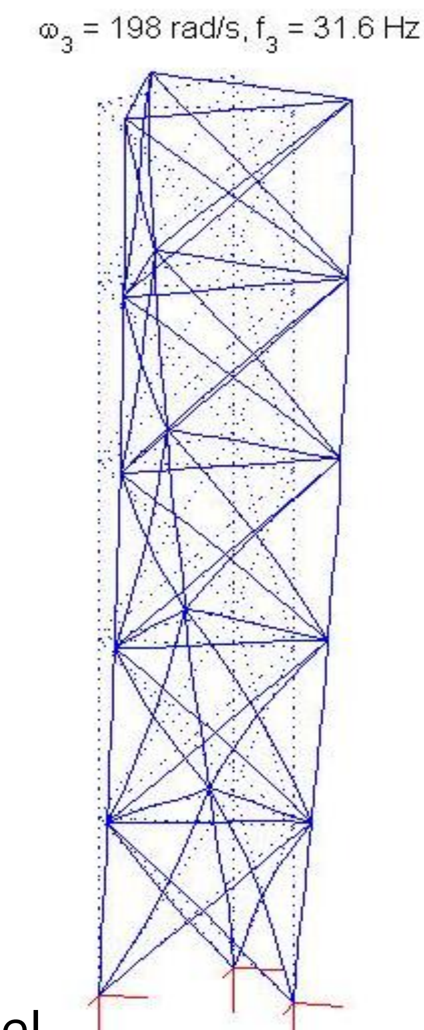
1st bending mode

Rigid-joint model

1st bending mode

Third mode shape of truss tower





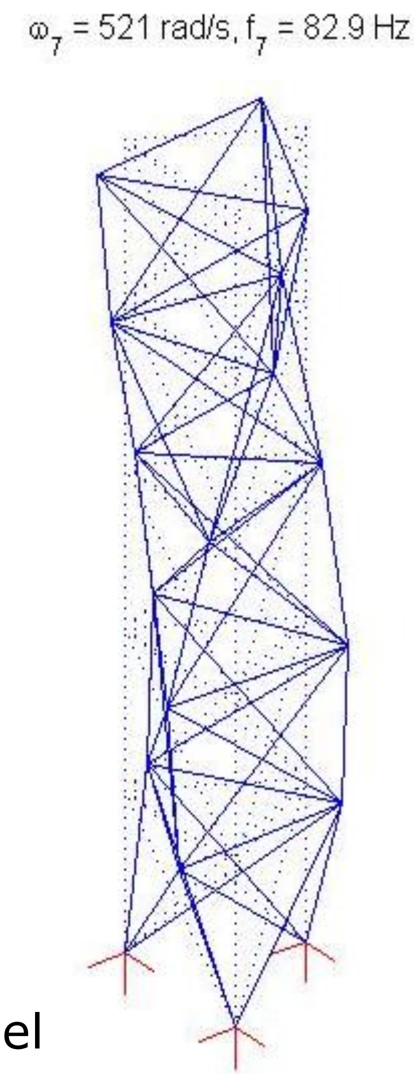
Pin-joint model

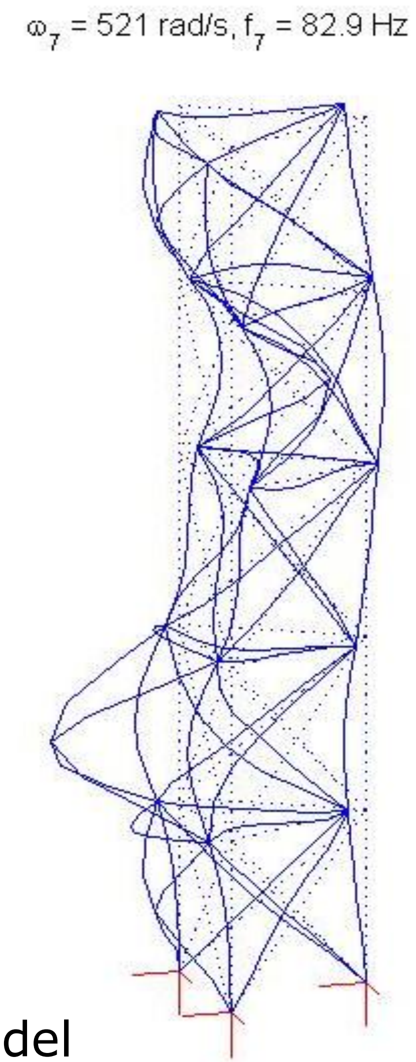
1st torsional mode

Rigid-joint model

1st torsional mode

7th mode shape of truss tower





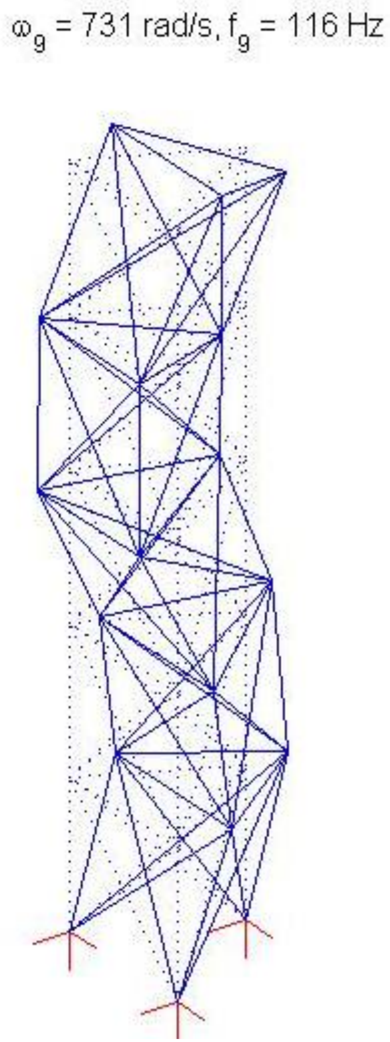
Pin-joint model

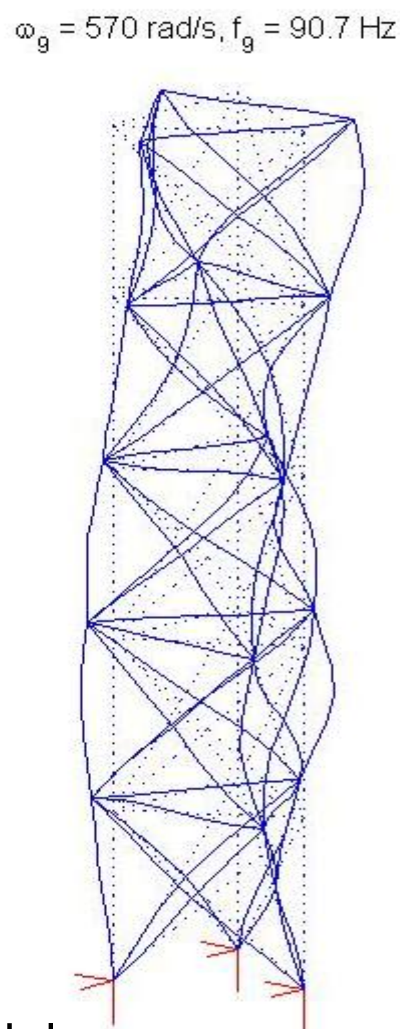
2nd torsional mode

Rigid-joint model

Local bending mode of bracings

9th mode shape of truss tower





Pin-joint model

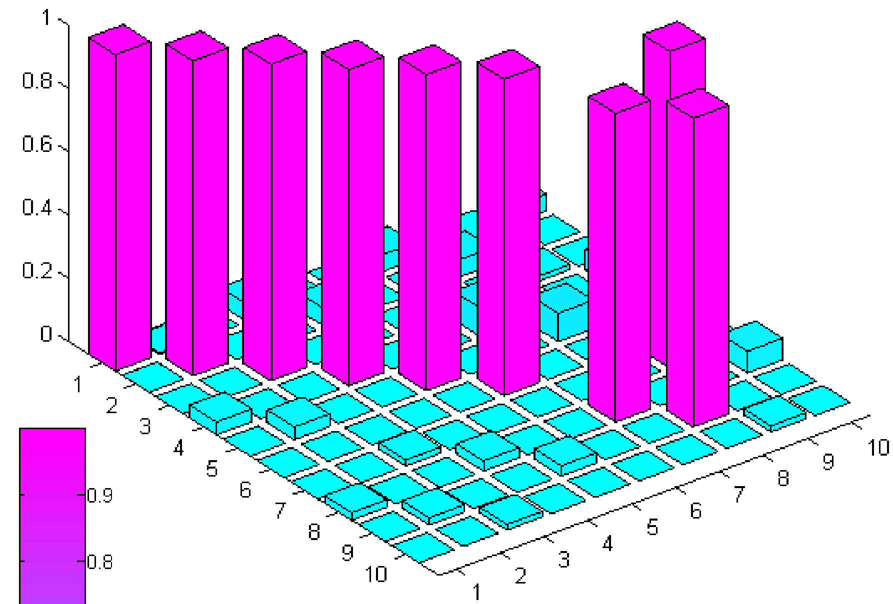
2^{nd} bending mode

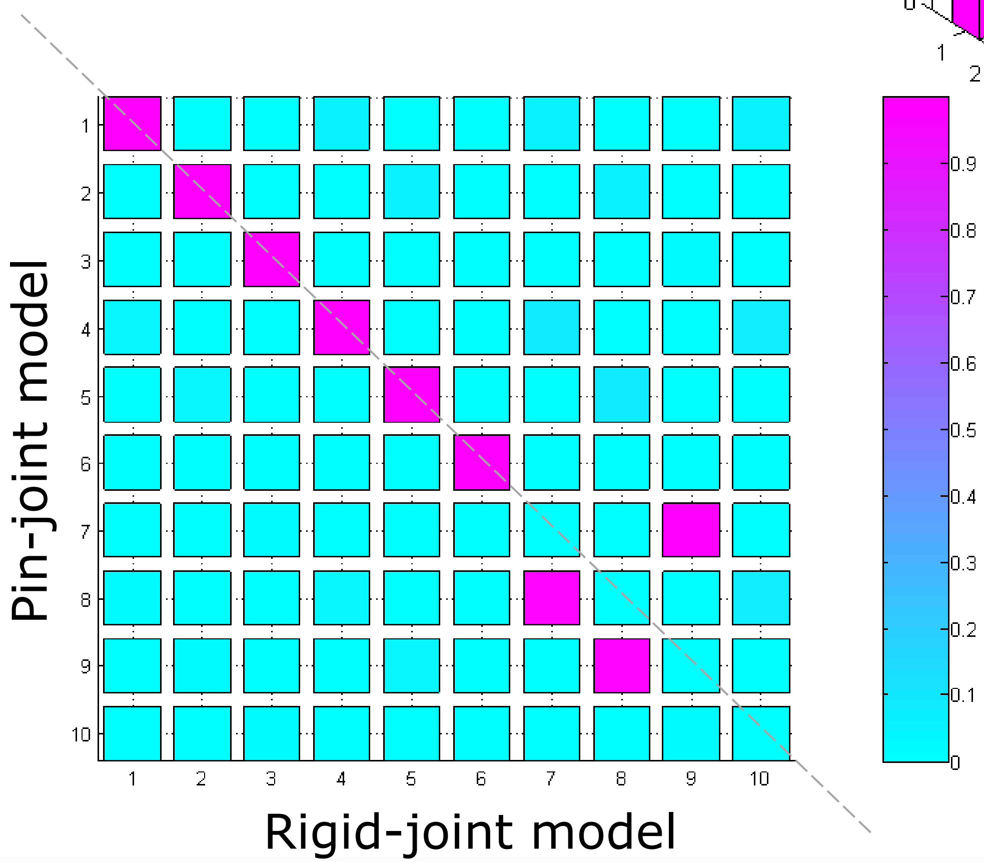
Rigid-joint model

2^{nd} torsional mode

Correlation of modes

$$\mathrm{MAC}_{ij} = \frac{\left(\boldsymbol{\varphi}_i^{T} \tilde{\boldsymbol{\varphi}}_j\right)^2}{\left(\boldsymbol{\varphi}_i^{T} \boldsymbol{\varphi}_i\right)\left(\tilde{\boldsymbol{\varphi}}_j^{T} \tilde{\boldsymbol{\varphi}}_j\right)}$$

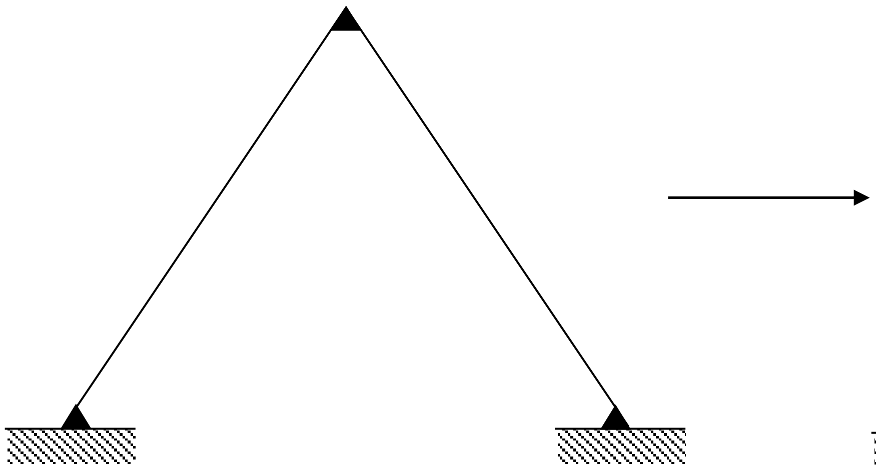


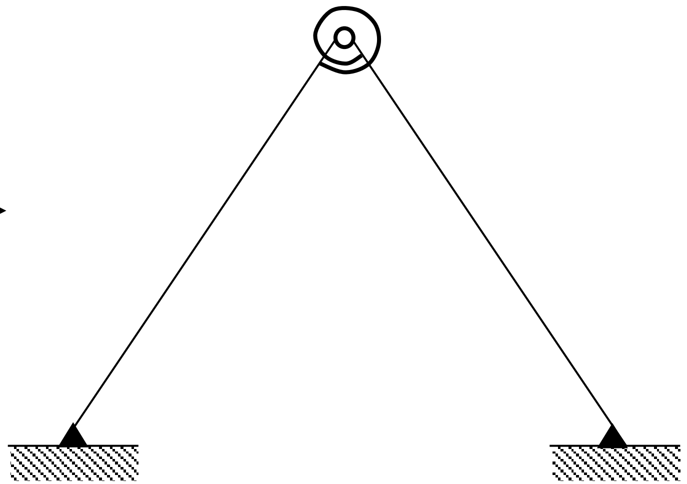


$\boldsymbol{\varphi}_i$ mode shapes of pin-joint model

$\tilde{\boldsymbol{\varphi}}_i$ mode shapes of rigid-joint model

Damaged joint modeling





$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{0}$$

$$\left(\boldsymbol{K} - \lambda_i^2 \boldsymbol{M}\right)\boldsymbol{\varphi}_i = \boldsymbol{0}$$

$$i = 1, 2, \ldots, N_m$$

$$\boldsymbol{M}\ddot{\boldsymbol{r}} + \left(\boldsymbol{K} + \Delta\boldsymbol{K}\right)\boldsymbol{r} = \boldsymbol{0}$$

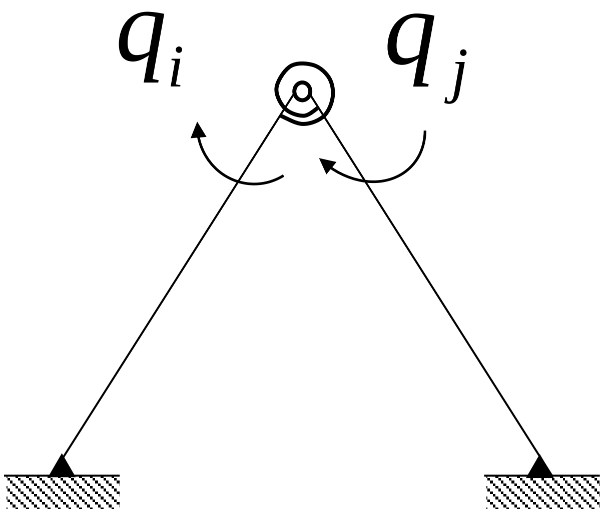
$$\left(\boldsymbol{K} + \Delta\boldsymbol{K} - \omega_l^2 \boldsymbol{M}\right)\boldsymbol{\psi}_l = \boldsymbol{0}$$

$$l = 1, 2, \ldots, N_f$$

$$\Delta\boldsymbol{K} = \sum_{j=1}^{N_k} k_j \boldsymbol{B}_j$$

Stiffness correction

Damaged joint modeling continued

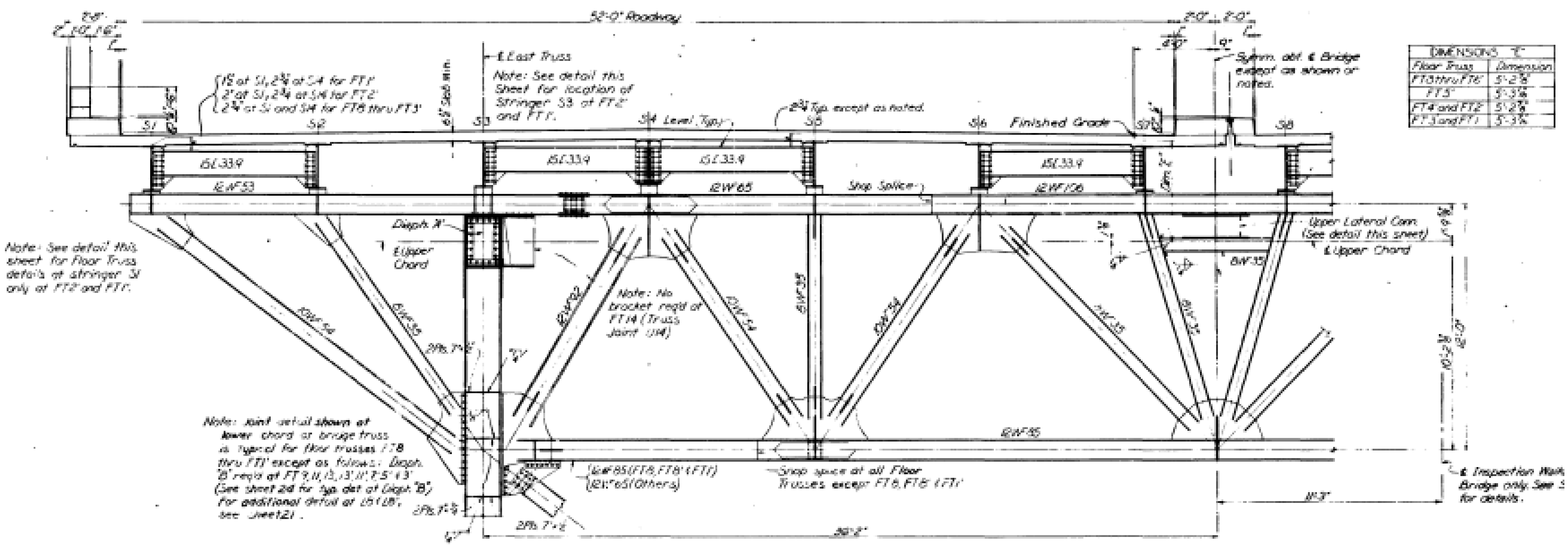


$$
\begin{array}{ccccccc} 1 & 2 & \ldots & i & \ldots & j & \ldots & n_{DOFs} \end{array}
$$

$$
\boldsymbol{B_j} = \begin{bmatrix} 0 & 0 & & & \cdots & & & 0 & 0 \\ 0 & \ddots & & & & & & \iddots & 0 \\ & & & & & & & & \\ & & & 1 & 0 & -1 & & & \\ \vdots & & & 0 & 0 & 0 & & & \vdots \\ & & & -1 & 0 & 1 & & & \\ 0 & \iddots & & & & & & \ddots & 0 \\ 0 & 0 & & & \cdots & & & 0 & 0 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ \vdots \\ i \\ \vdots \\ j \\ \vdots \\ n_{DOFs} \end{array}
$$

I-35W truss bridge

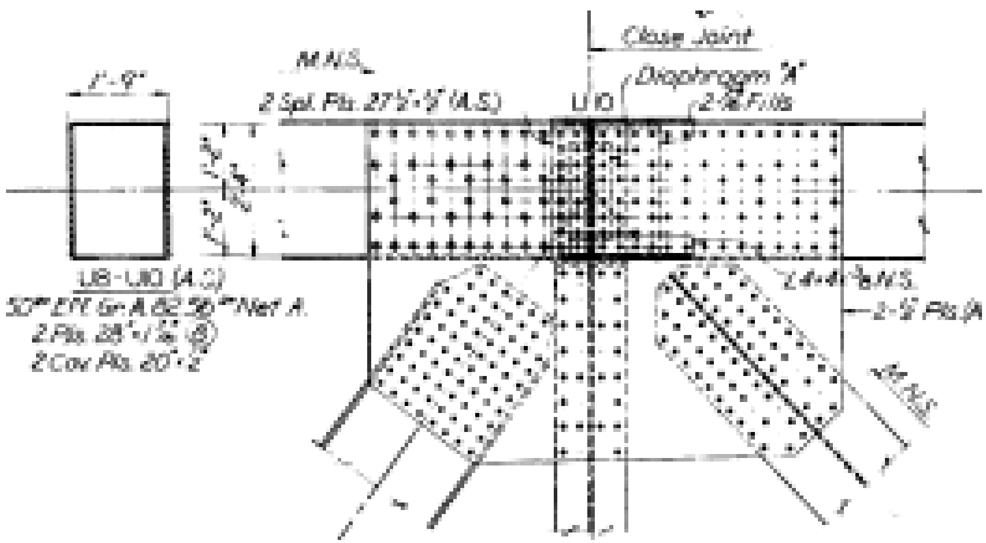
Floor truss according to construction plans



*) Astaneh-Asl, A., "*Progressive Collapse of Steel Truss Bridges, the Case of I-35W Collapse*", 7th International Conference on Steel Bridges, Guimarães, Portugal, 4-6 June, 2008.

Collapse

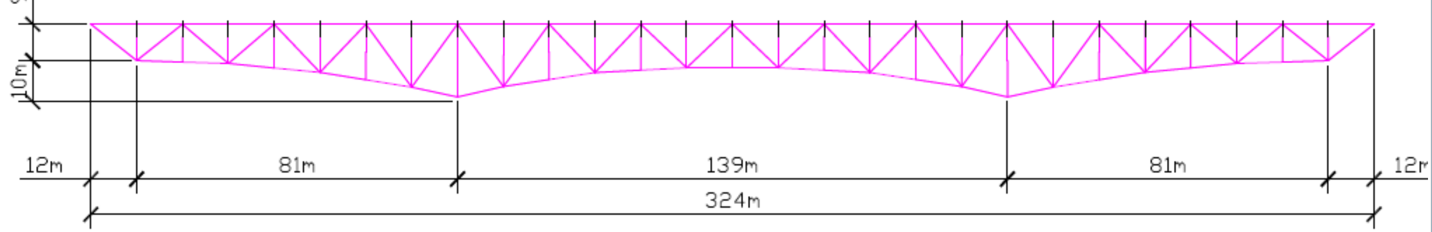
On August 1, 2007, the 40 years old I-35W steel deck truss bridge in Minneapolis, suddenly and without almost any noticeable warning collapsed.



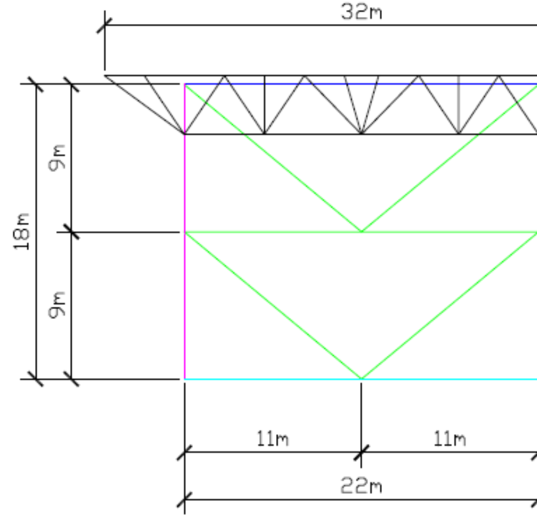
Joint detail

Computational model

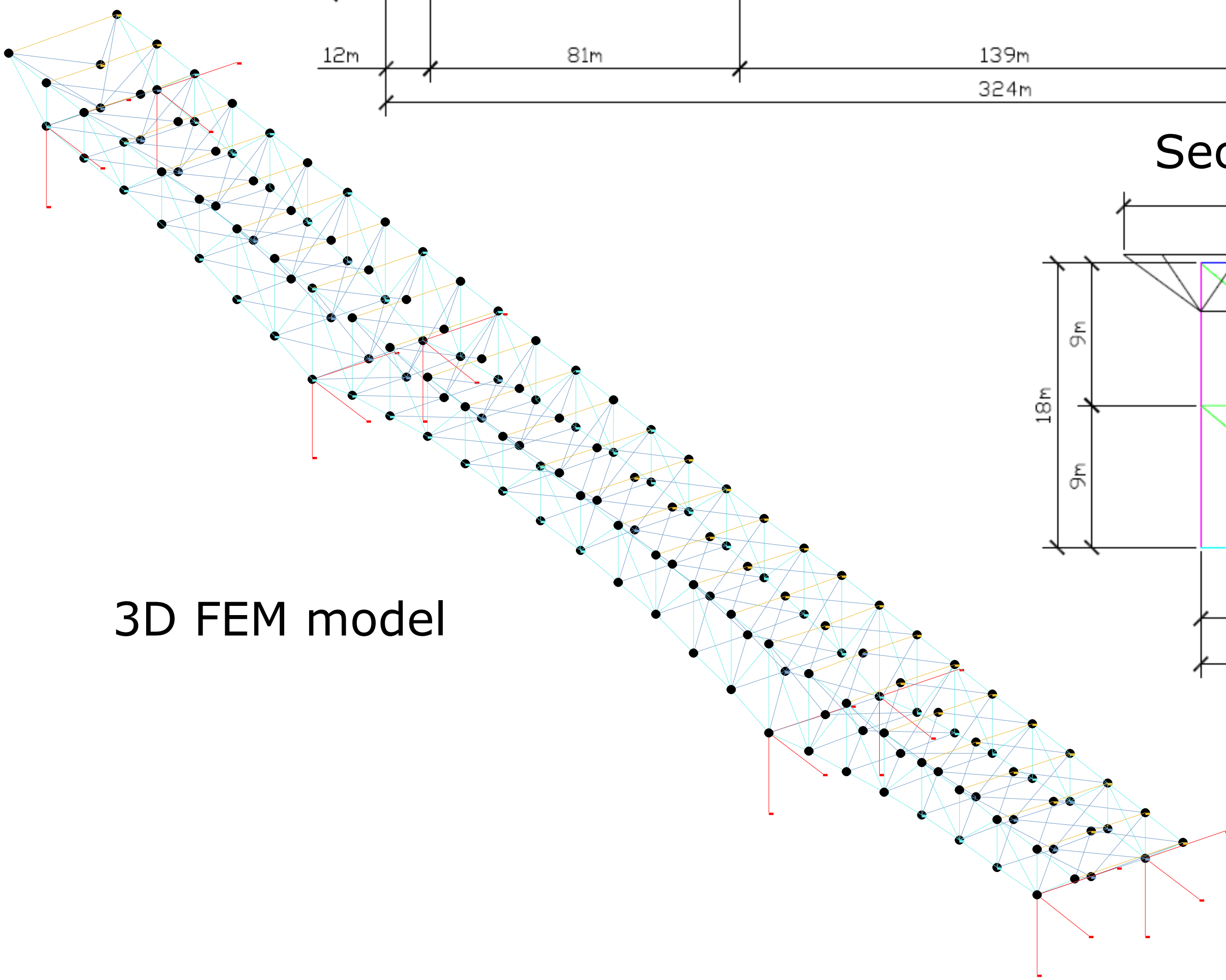
Elevation of truss



Section near pier 7



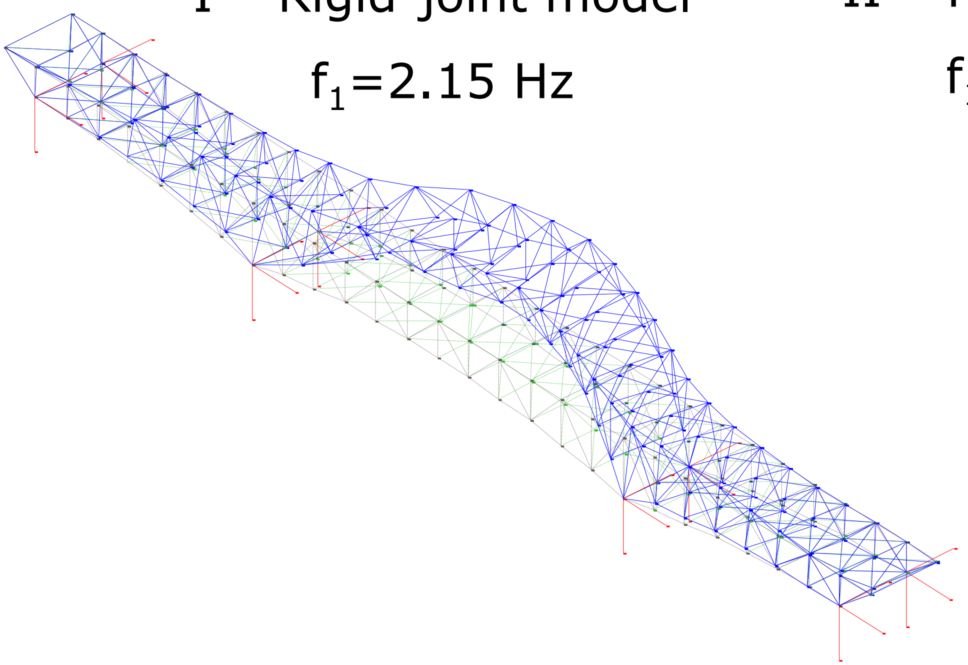
3D FEM model



1st mode – out-of-plane bending

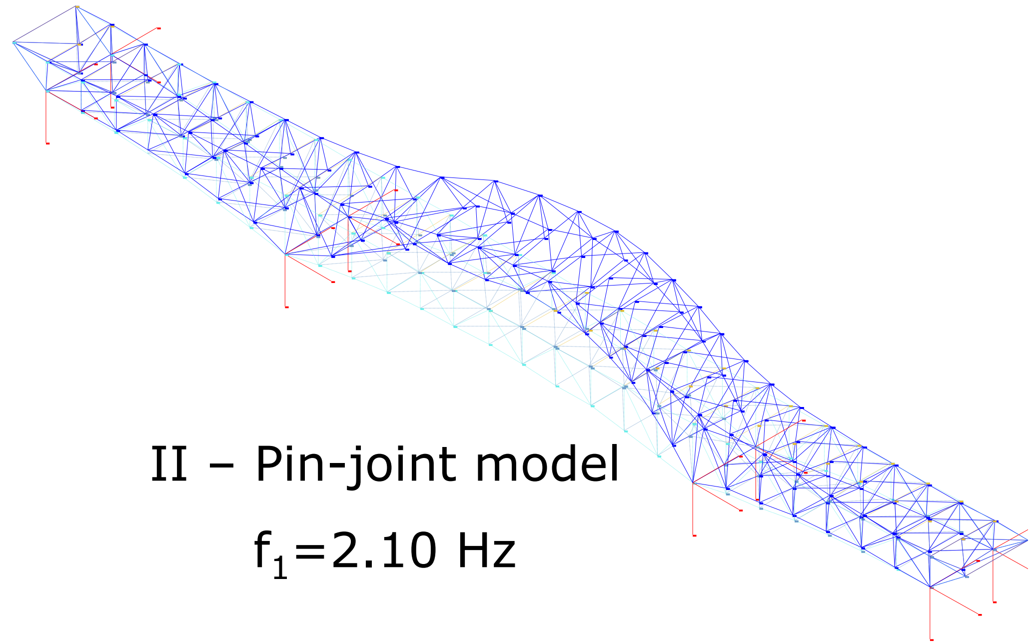
I – Rigid-joint model

f_1=2.15 Hz

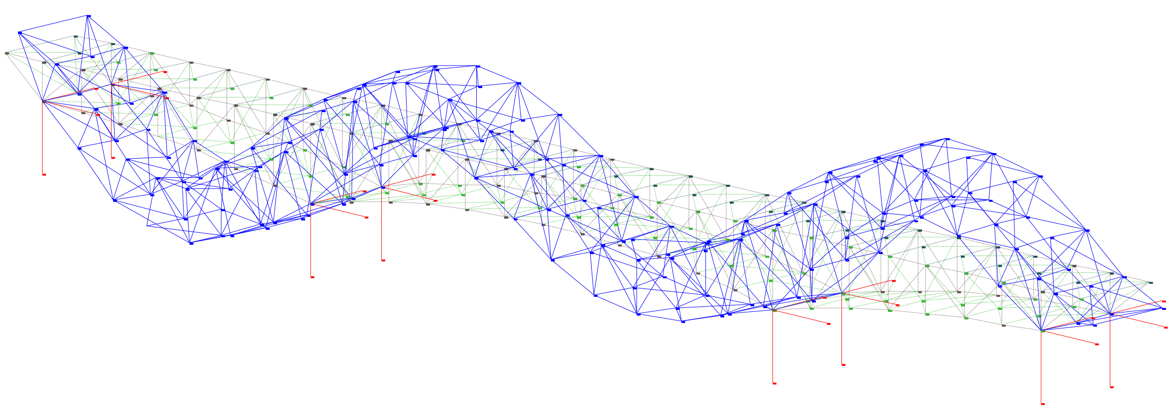


II – Pin-joint model

f_1=2.10 Hz



6th mode shape



I – Rigid-joint model

f_6=4.34 Hz

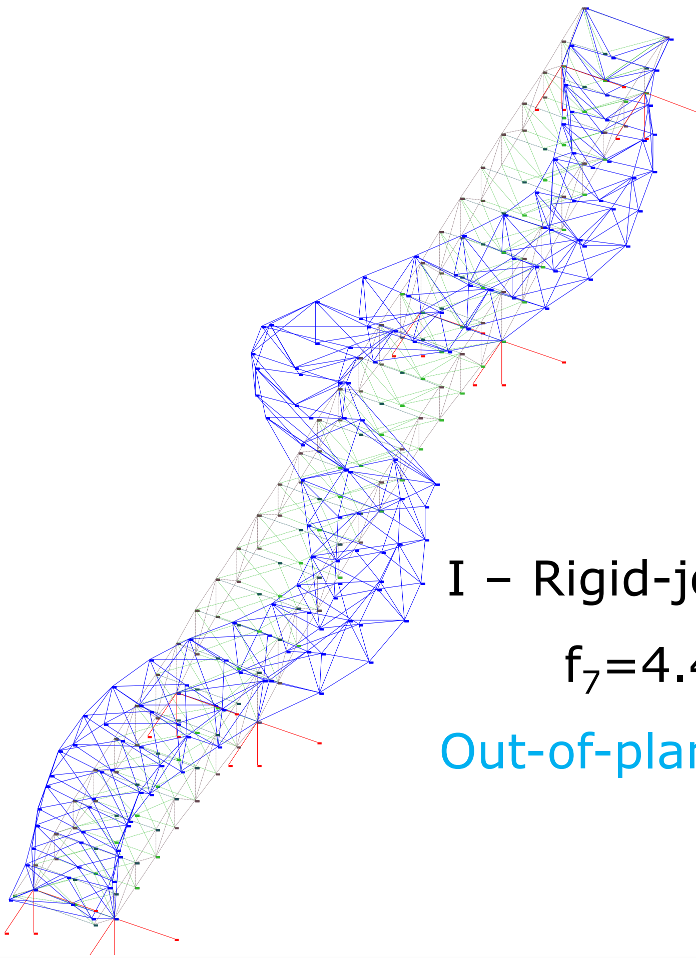
In-plane bending

II – Pin-joint model

f_6=4.44 Hz

Out-of-plane bending

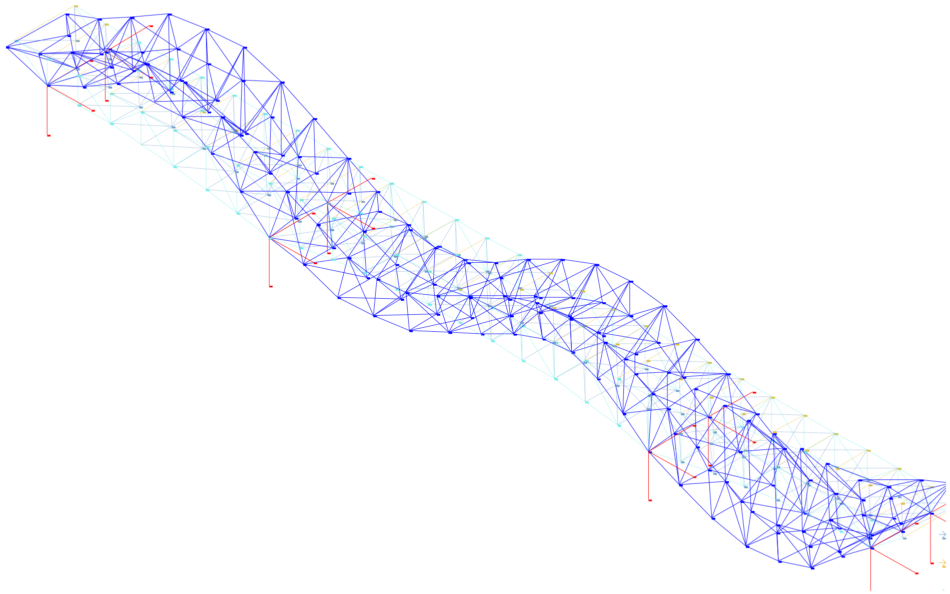
7th mode shape



I – Rigid-joint model

f_7=4.48 Hz

Out-of-plane bending



II – Pin-joint model

f_7=4.49 Hz

In-plane bending

7. Conclusions

- Theoretical background, together with presented examples show, that the pin-joint assumption in truss dynamics can lead to considerable errors in finding eigenfrequencies and eigenmodes.

- The pin-joint assumption doesn’t allow to monitor, real structural joints, which are more often subjected to damages than prismatic rolled structural elements.

Acknowledgments

The authors would like to thank:

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Thank you for your attention.