Optimal Vibration Control of Guyed Masts

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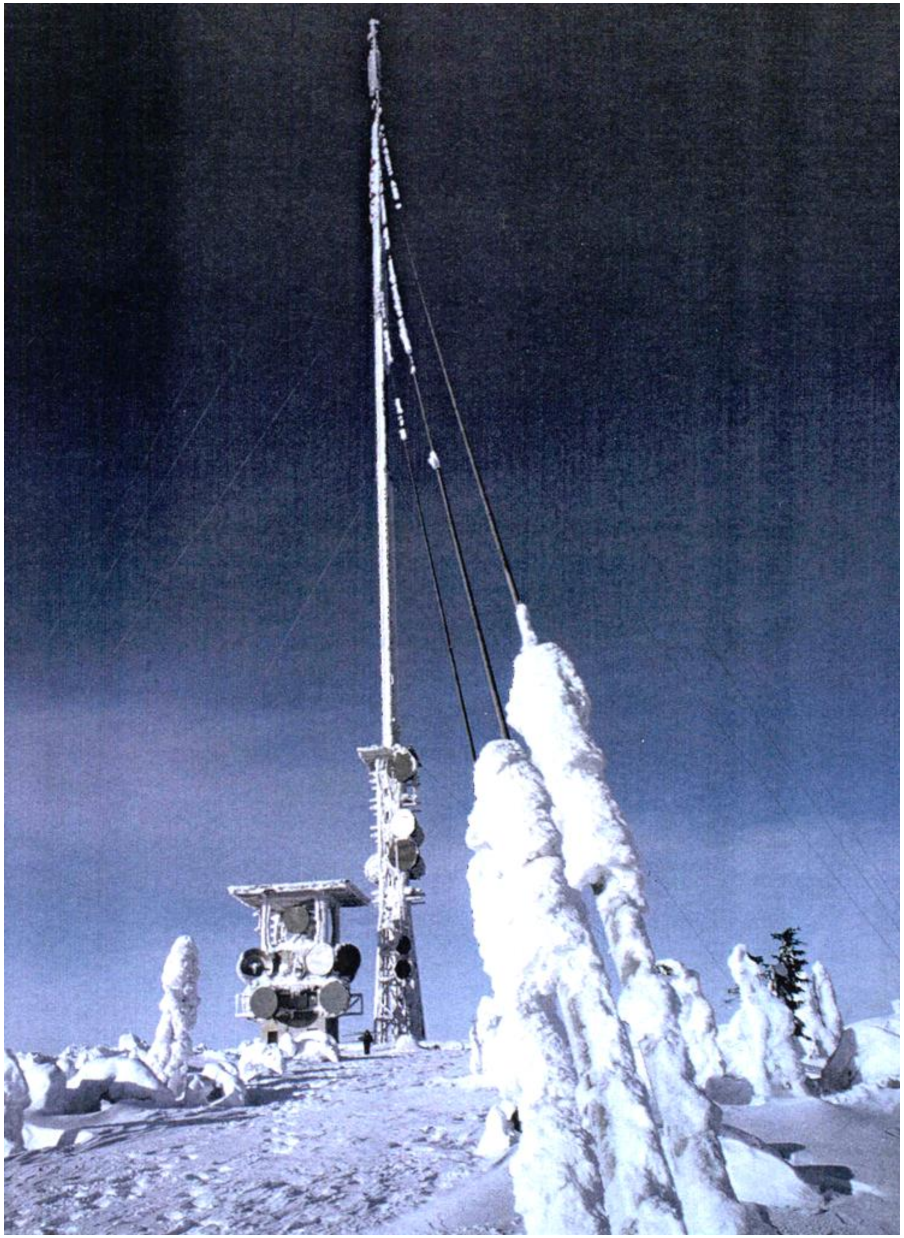
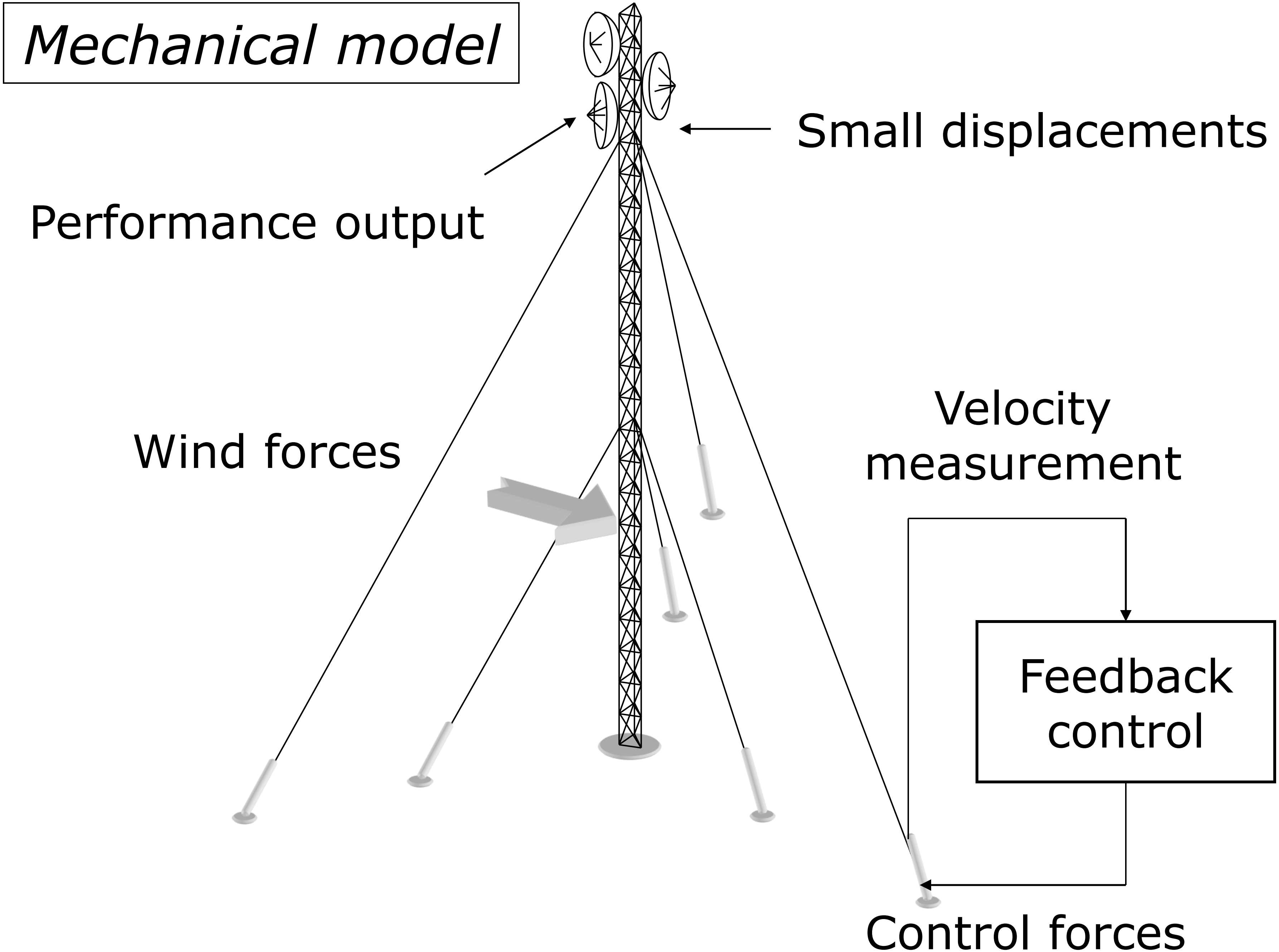
Motivations

Ice and wind pressure are the most common reasons of mast failures.




Mechanical model
Small displacements
Performance output
Wind forces
Velocity measurement
Feedback control
Control forces

Guyed mast dynamics

Finite element model

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{D}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{B}_f f(t) + \mathbf{B}_u \mathbf{u}(t)$$

Modal truncation

$$\mathbf{q}(t) = \boldsymbol{\varphi}_c \boldsymbol{\eta}_c(t) + \boldsymbol{\varphi}_r \boldsymbol{\eta}_r(t) \approx \boldsymbol{\varphi}_c \boldsymbol{\eta}_c(t)$$

Discrete time state-space model

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}_1 f_k + \mathbf{B}_2 \mathbf{u}_k \qquad \mathbf{x}_k = \begin{bmatrix} \boldsymbol{\eta}_c(t_k) \\ \dot{\boldsymbol{\eta}}_c(t_k) \end{bmatrix}$$

$$\mathbf{z}_k = \mathbf{C}_1 \mathbf{x}_k \qquad \text{(Performance output)}$$

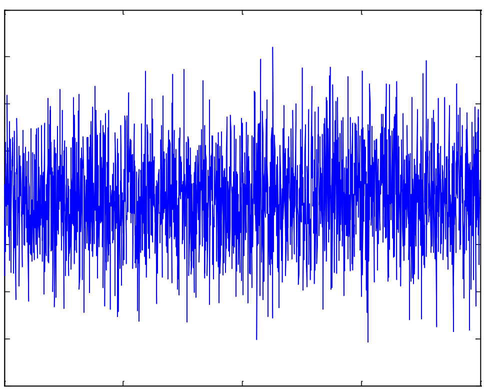
$$\mathbf{y}_k = \mathbf{C}_2 \mathbf{x}_k + \mathbf{v}_k \qquad \text{(Measurement)}$$

Wind fluctuations

Wind forces and spectrum $\mathbf{B}_f = \begin{bmatrix} \rho_a A_1 C_d \bar{v} \cos\alpha \\ \rho_a A_1 C_d \bar{v} \sin\alpha \\ 0 \\ \rho_a A_2 C_d \bar{v} \cos\alpha \\ \rho_a A_2 C_d \bar{v} \sin\alpha \\ 0 \\ \vdots \end{bmatrix}$

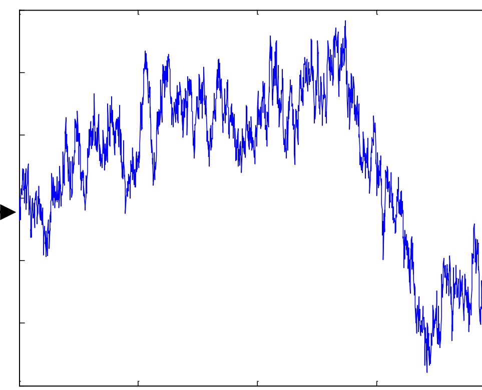
$$S_f(\varpi) = 4800 \bar{v} \kappa \frac{\beta \varpi}{(1 + \beta^2 \varpi^2)^{3/2}}$$

where ρ_a – air density, C_d – drag coefficient



$$H(j\varpi) \approx \sqrt{S_f(\varpi)}$$

Davenport filter



Controllability

Classical criterion of controllability

$$\text{rank}\begin{pmatrix}\mathbf{B}_2 & \mathbf{A}\mathbf{B}_2 & \mathbf{A}^2\mathbf{B}_2 & \dots & \mathbf{A}^{2N-1}\mathbf{B}_2\end{pmatrix} = 2N$$

where N – number of modes

Modal transformation

$$\ddot{\boldsymbol{\eta}}_c(t) + 2\boldsymbol{\Xi}\boldsymbol{\Omega}\,\dot{\boldsymbol{\eta}}_c(t) + \boldsymbol{\Omega}^2\,\boldsymbol{\eta}_c(t) = \boldsymbol{\varphi}^T\mathbf{B}_u\mathbf{u}(t)$$

where $\boldsymbol{\eta}_c(t)$ – modal coordinates

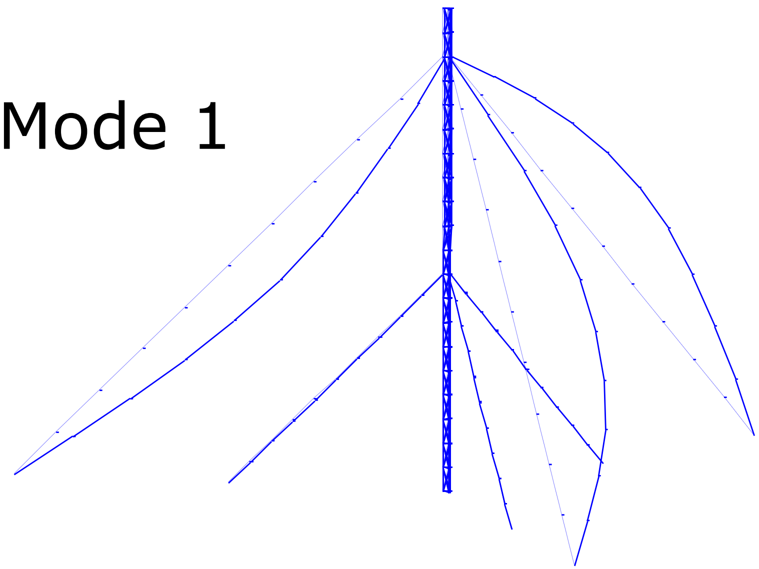
Modal controllability index

$$\mu_j = \boldsymbol{\varphi}_j^T\mathbf{B}_u\mathbf{B}_u^T\boldsymbol{\varphi}_j \qquad j = 1,2,\dots,N$$

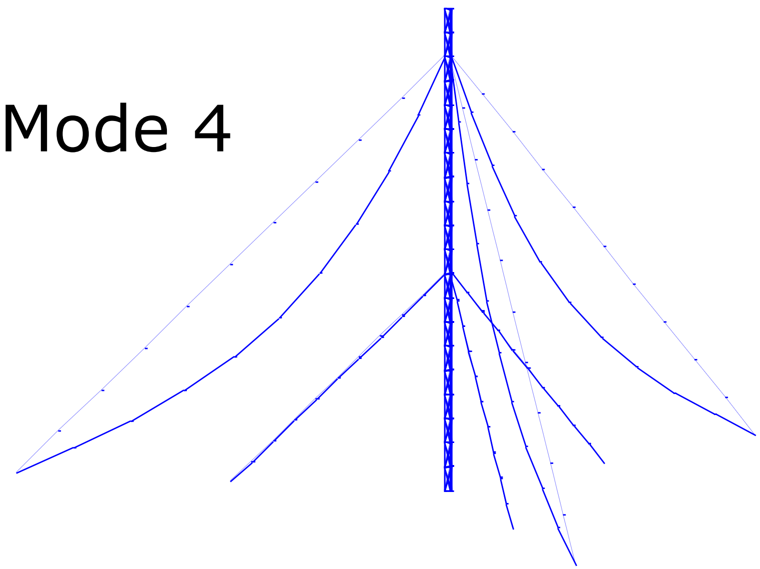
Modal analysis

shown displacements are out of scale

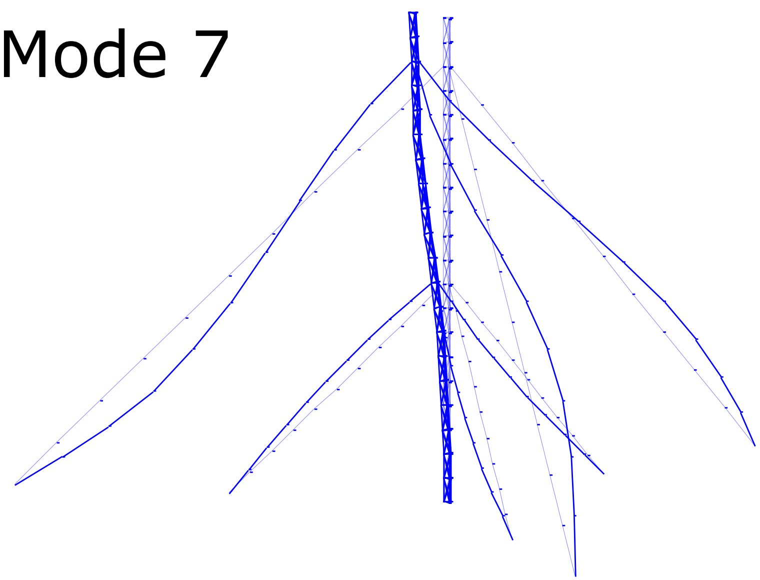
Mode 1



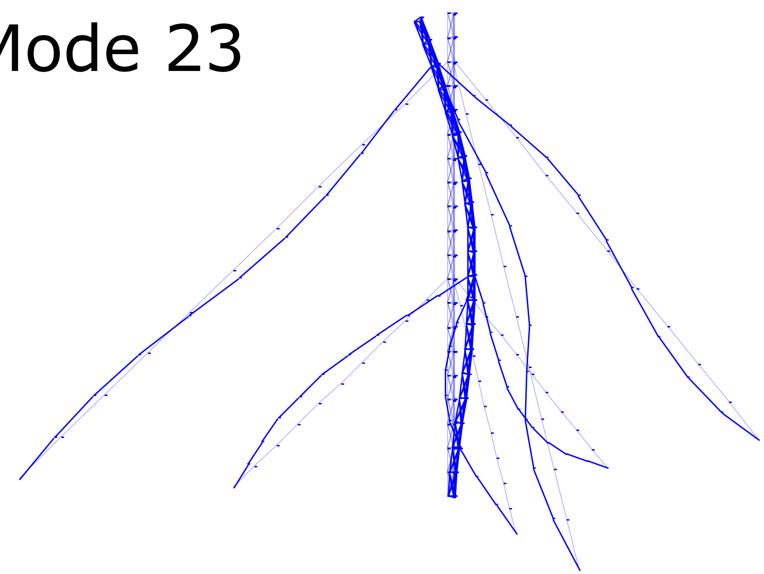
Mode 4



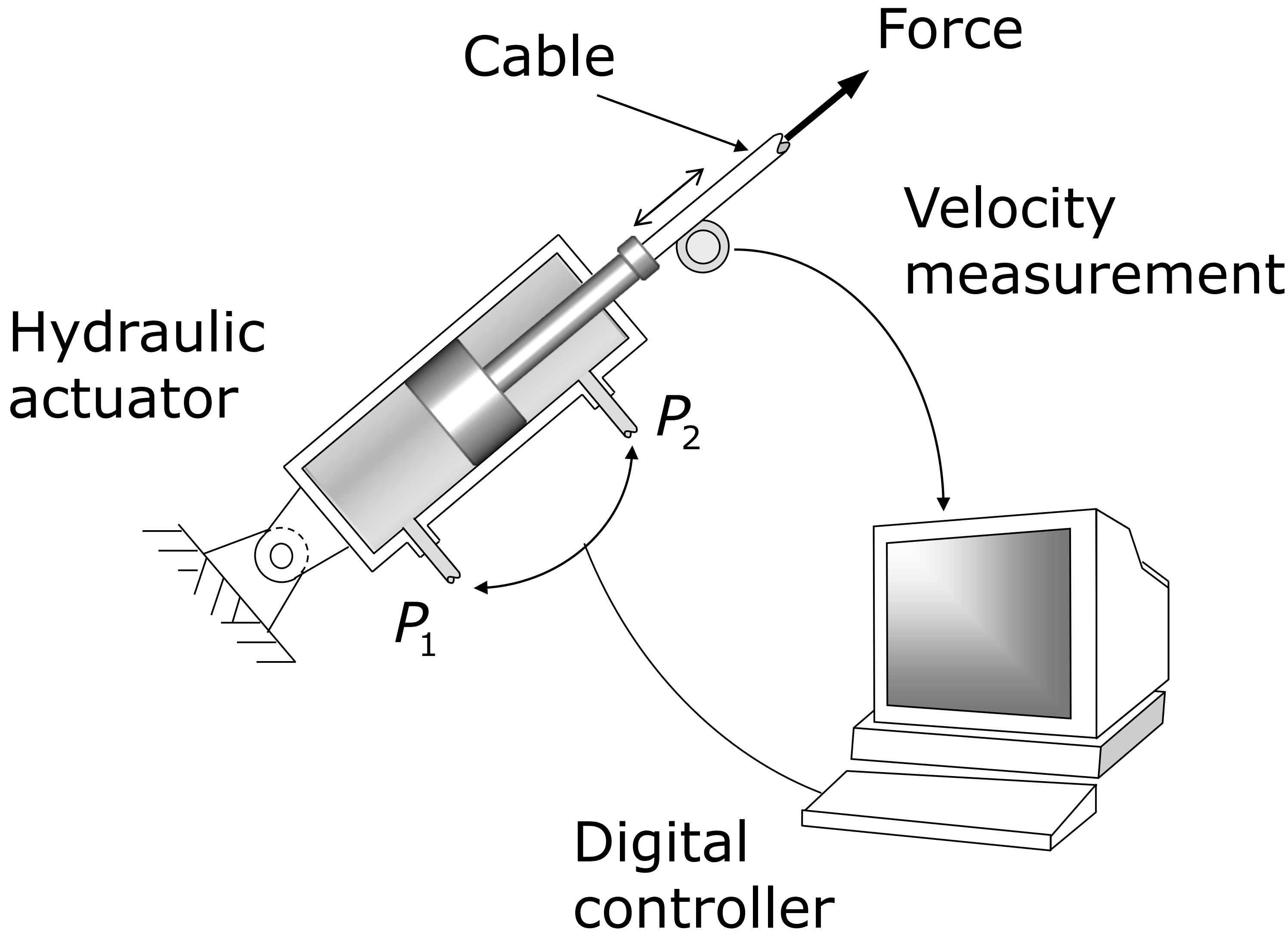
Mode 7



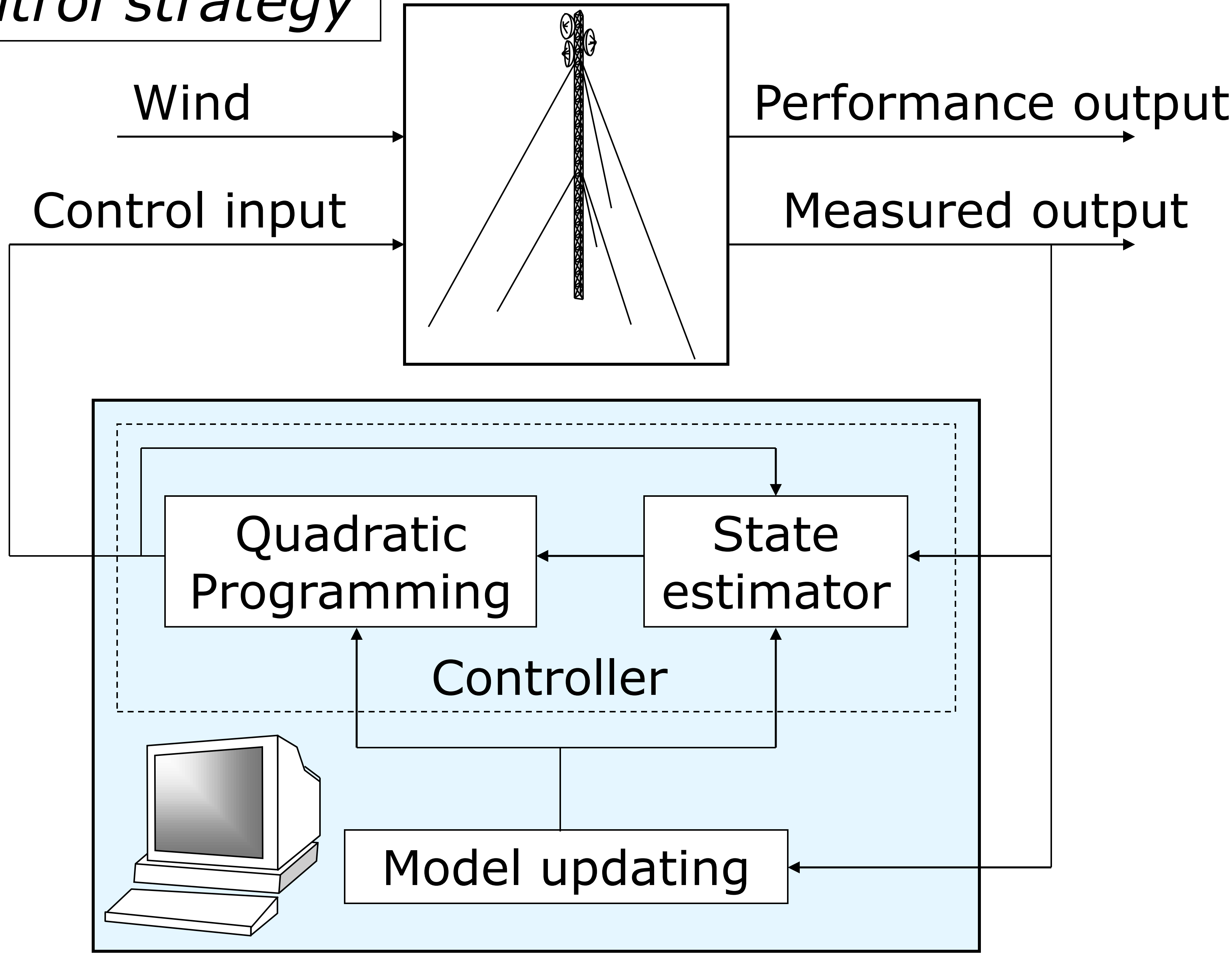
Mode 23



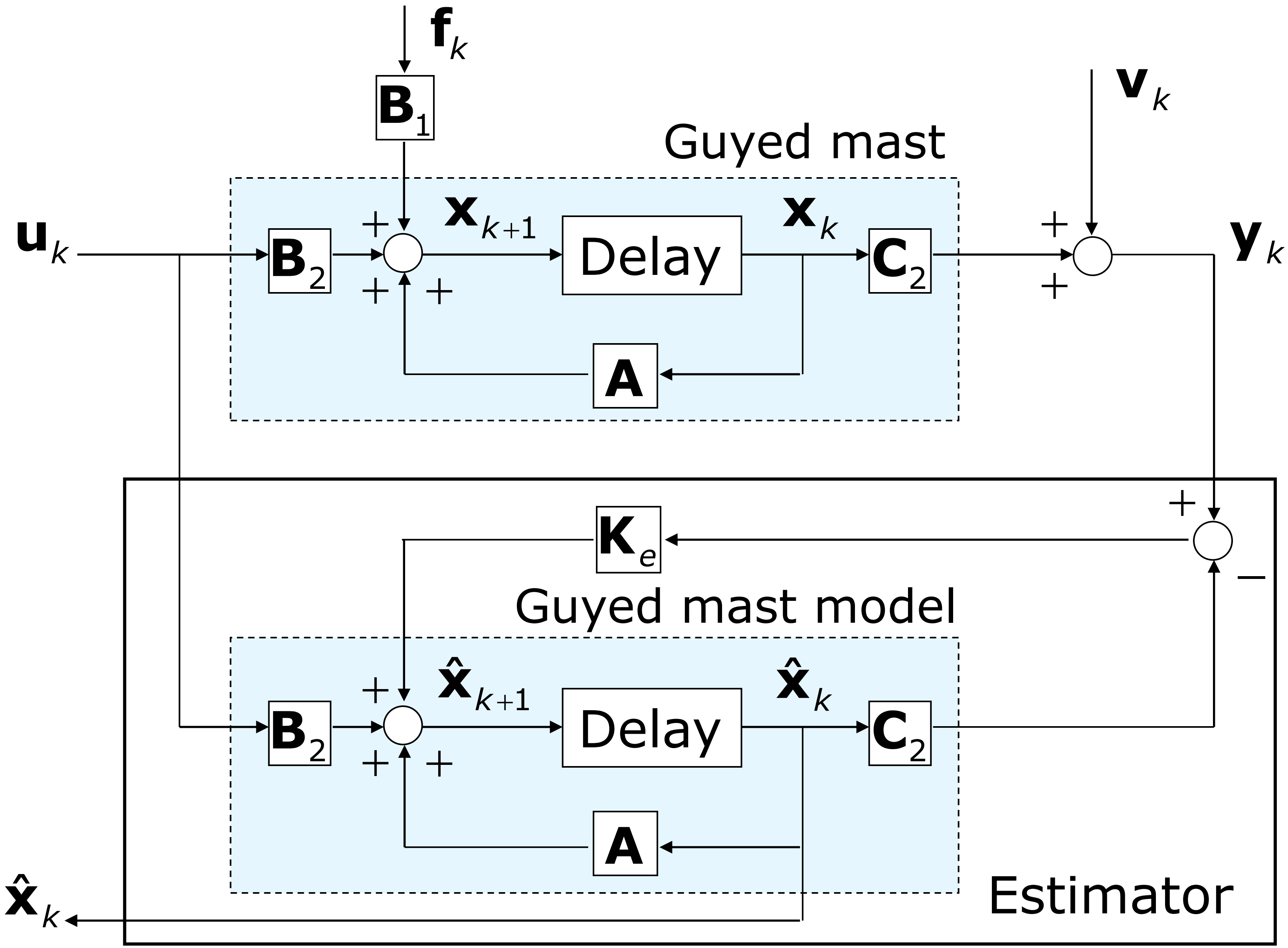
Control system scheme



Control strategy



State estimator concept



Optimal estimator

Initial condition, disturbances

$$E(\mathbf{x}_0\mathbf{x}_0^T) = \mathbf{M}_0 \qquad E(\mathbf{f}_k\mathbf{f}_l^T) = \mathbf{Q}\delta_{kl}$$

and measurement errors

$$E(\mathbf{v}_k\mathbf{v}_l^T) = \mathbf{R}\delta_{kl}$$

Maximum likelihood estimate of state $\mathbf{x}_m$

$$\hat{\mathbf{x}}_{k+1} = \mathbf{A}\hat{\mathbf{x}}_k + \mathbf{B}_2\mathbf{u}_k + \mathbf{K}_e(\mathbf{y}_k - \mathbf{C}_2\hat{\mathbf{x}}_k) \qquad \hat{\mathbf{x}}_0 \text{ given}$$

$$\mathbf{K}_e = \mathbf{A}\mathbf{P}_k\mathbf{C}_2^T\mathbf{R}^{-1} \qquad k = 0,1,\dots,m$$

$$\mathbf{P}_k = \mathbf{M}_k - \mathbf{M}_k\mathbf{C}_2^T(\mathbf{C}_2\mathbf{M}_k\mathbf{C}_2^T + \mathbf{R})^{-1}\mathbf{C}_2\mathbf{M}_k$$

$$\mathbf{M}_{k+1} = \mathbf{A}\mathbf{P}_k\mathbf{A}^T + \mathbf{B}_1\mathbf{Q}\mathbf{B}_1^T$$

Optimal control

Performance index

$$\mathrm{J} = \mathbf{z}_n^T \mathbf{\Phi}_n \mathbf{z}_n + \sum_{k=0}^{n-1} \mathbf{z}_k^T \mathbf{\Phi} \mathbf{z}_k + \mathbf{u}_k^T \mathbf{\Psi} \mathbf{u}_k$$

where $\mathbf{z}_k$ – performance output

Quadratic programming with constraints

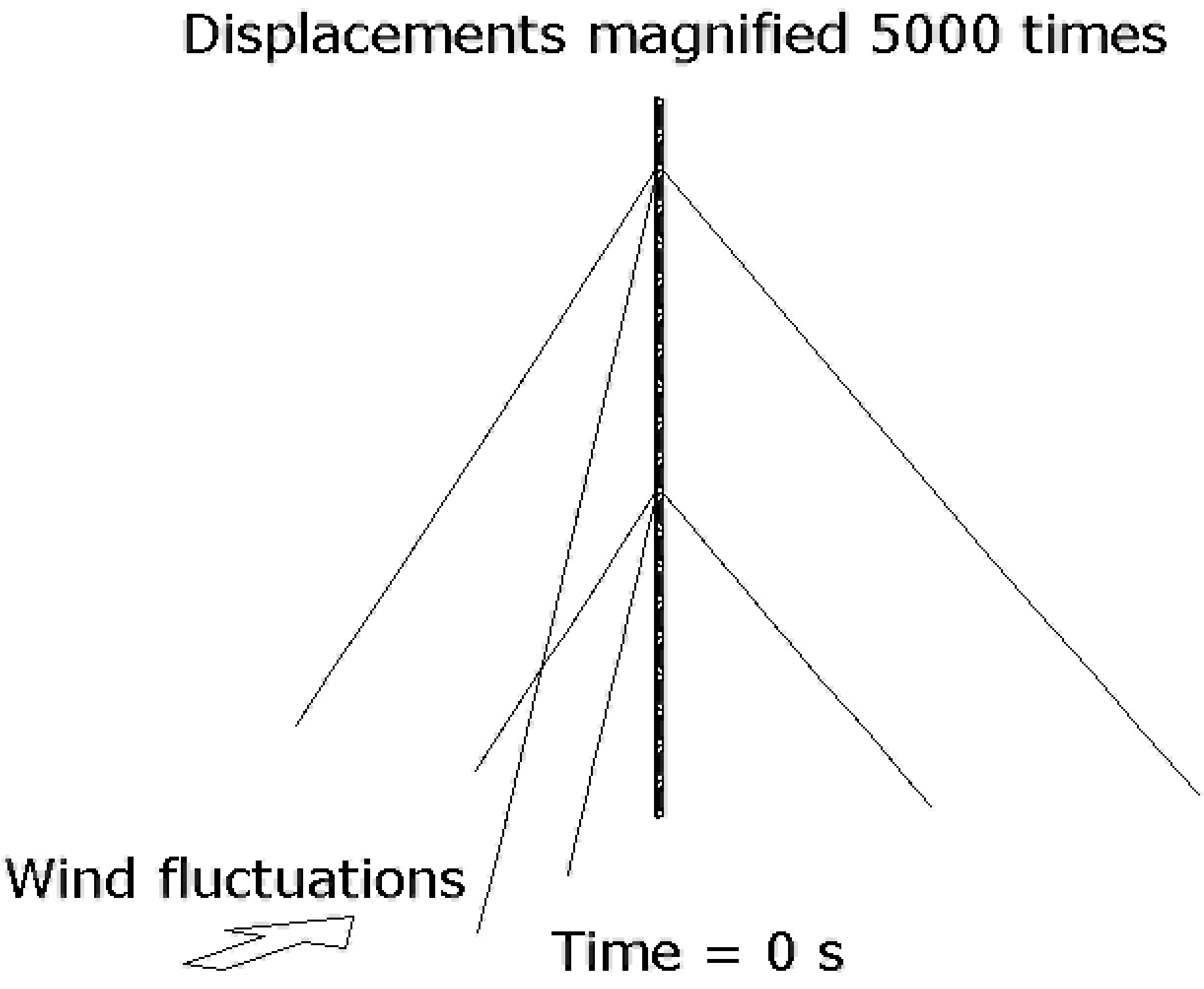
$$\mathrm{J} = \mathrm{J}_0 + \mathbf{U}^T \mathbf{W} \mathbf{U} + 2\mathbf{V}\mathbf{U}$$

$$\mathbf{U}_{\min} \le \mathbf{U} \le \mathbf{U}_{\max}$$

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_{n-1} \end{bmatrix}$$

$$\mathbf{U}^{OPT} = \arg \min_{\mathbf{LU} \le \mathbf{b}} \mathbf{U}^T \mathbf{W} \mathbf{U} + 2\mathbf{V}\mathbf{U}$$

Numerical simulations



Conclusions

- A 3D FEM model of guyed mast under control forces was created
- External loading was included in the form of a stochastic wind model of Davenport spectrum
- Controllability of individual mode shapes of the mast was determined
- A model based state estimator for the mast was constructed
- Numerical simulation of a control process was presented, displaying significant reduction in vibration amplitudes of the top of the mast

Acknowledgements

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