# Simple human-structure interaction model of walking on a flexible surface



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### Human induced vibrations

There exists a problem of excessive oscillations caused by humans. An example of such a problematic event occurred at the Millennium Bridge in London, which on the opening day swayed and twisted with the amplitude of up to 70 mm.



A better understanding of the phenomenon of human induced vibrations could prevent such problems in future.

## Objectives

We present a new humanstructure interaction (HSI) model of walking on a flexible surface. A human is considered as a mass point, located at the body's Center of Mass (COM). The mass moves along a predefined trajectory, which deforms together with the surface on which the human walks.

# Problem

The equations of motion of the system are time-dependent and discontinuous. They can be written in the form of the second order differential equations of structural dynamics, but with the right-hand forcing dependent on the deformation of the surface.

 $\ddot{\boldsymbol{\eta}} + 2\mathbf{Z}\boldsymbol{\Omega}\dot{\boldsymbol{\eta}} + \boldsymbol{\Omega}^2\boldsymbol{\eta} = \mathbf{B}_i\mathbf{f}(\boldsymbol{\eta},\dot{\boldsymbol{\eta}},\ddot{\boldsymbol{\eta}},t)$ 

- $\mathbf{Z} = \operatorname{diag}(\zeta_1, \zeta_2, ..., \zeta_{n_m})$
- $\mathbf{\Omega} = \operatorname{diag}(\omega_1, \omega_2, ..., \omega_{n_m})$
- $\mathbf{B}_i$  is the influence matrix

Normalized ground reaction force



The forces of motion, equal to the sum of inertial and gravitational forces acting on the mass, are transferred to the surface at prescribed foot positions.

 $\mathbf{f}(\mathbf{x} = \mathbf{r}_i, t \in [t_i, t_{i+1})) = -m\ddot{\mathbf{x}}_m(t) + m\mathbf{g}$ 

 $\mathbf{x}_m(t) = \mathbf{x}_r(t) + \mathbf{u}(\mathbf{x}_s, t)$ 



#### Results

We present a numerical example of a human walking on a long beam structure. The motion of the beam is described by three mode shapes, representing its vertical, lateral, and torsional deflections.



The following data have been used for the bridge:

 $L = 50 \,\mathrm{m}$  (length),  $W = 4 \,\mathrm{m}$  (width)  $\rho = 200 \,\mathrm{kg/m^2}$  (mass per unit area)

The values of  $\eta_1(t)$  for the passage of the walker over the bridge, for the three COM's trajectories



## Conclusions

The proposed HSI model is simplified but advantageous. It captures the basic features of motion, is computationally inexpensive, and amenable to improvements and extensions.

### References

P. Dallard et al, *The London Millennium Footbridge*, The Structural Engineer, Volume 79 No.22 (2001)