Combined on/off-line load ID

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Determining location of force

Equations of motion in time domain

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{D}_d\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}(t)$$

Fourier transformation

$$\mathbf{Q}(j\varpi) = \int_{-\infty}^{\infty} \mathbf{q}(t) e^{-j\varpi t} dt$$

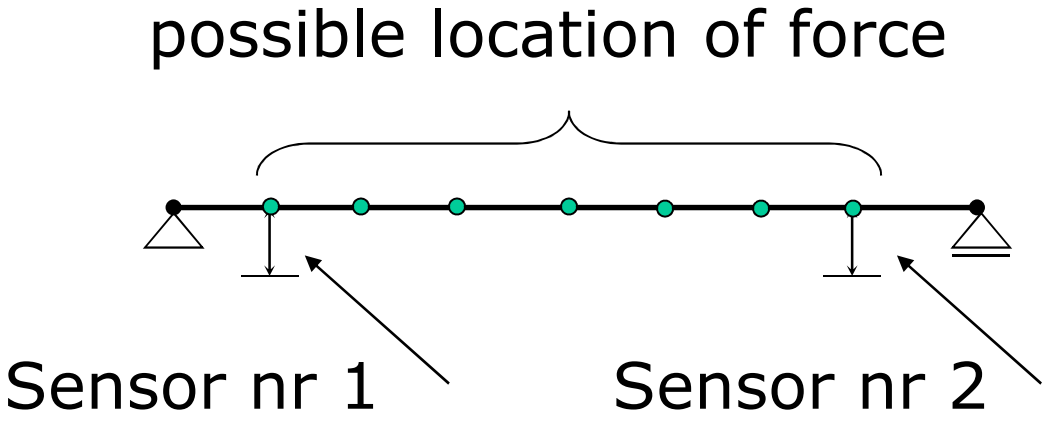
Equations of motion in frequency domain

$$\left(\mathbf{K} + j\varpi\mathbf{D}_d - \varpi^2\mathbf{M}\right)\mathbf{Q}(j\varpi) = \mathbf{F}(j\varpi)$$

Applying unit load $\mathbf{F}(j\varpi) \Rightarrow \mathbf{H}(j\varpi)$ freq. response fun.

$$F_i(j\varpi) = \frac{Q_k(j\varpi)}{H_{ik}(j\varpi)}$$

Determining location of force-cd.

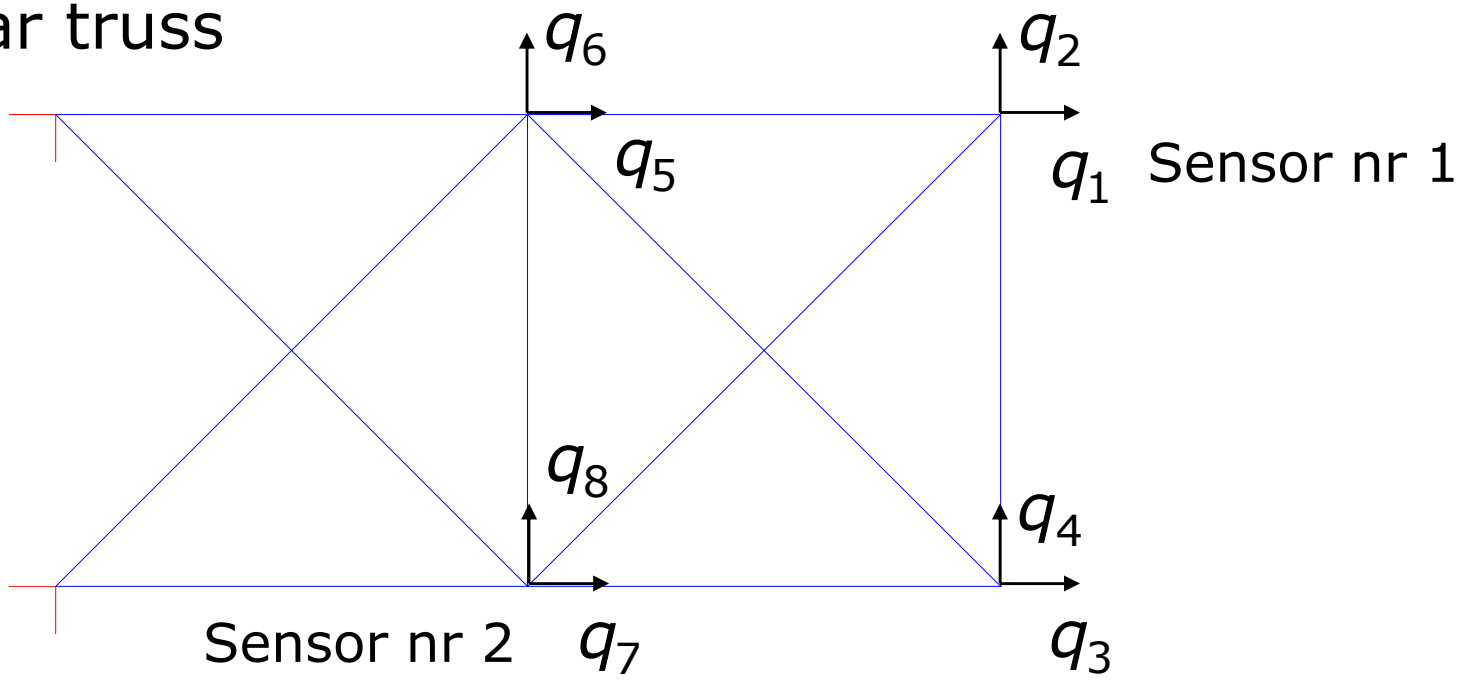


True location of impacting force is indicated by those time histories which give the same solution for the individual sensors

$$F_a(j\varpi) = \frac{Q_{s_1}(j\varpi)}{H_{as_1}(j\varpi)} = \frac{Q_{s_2}(j\varpi)}{H_{as_2}(j\varpi)}$$

FFT force identification-example

Ten bar truss

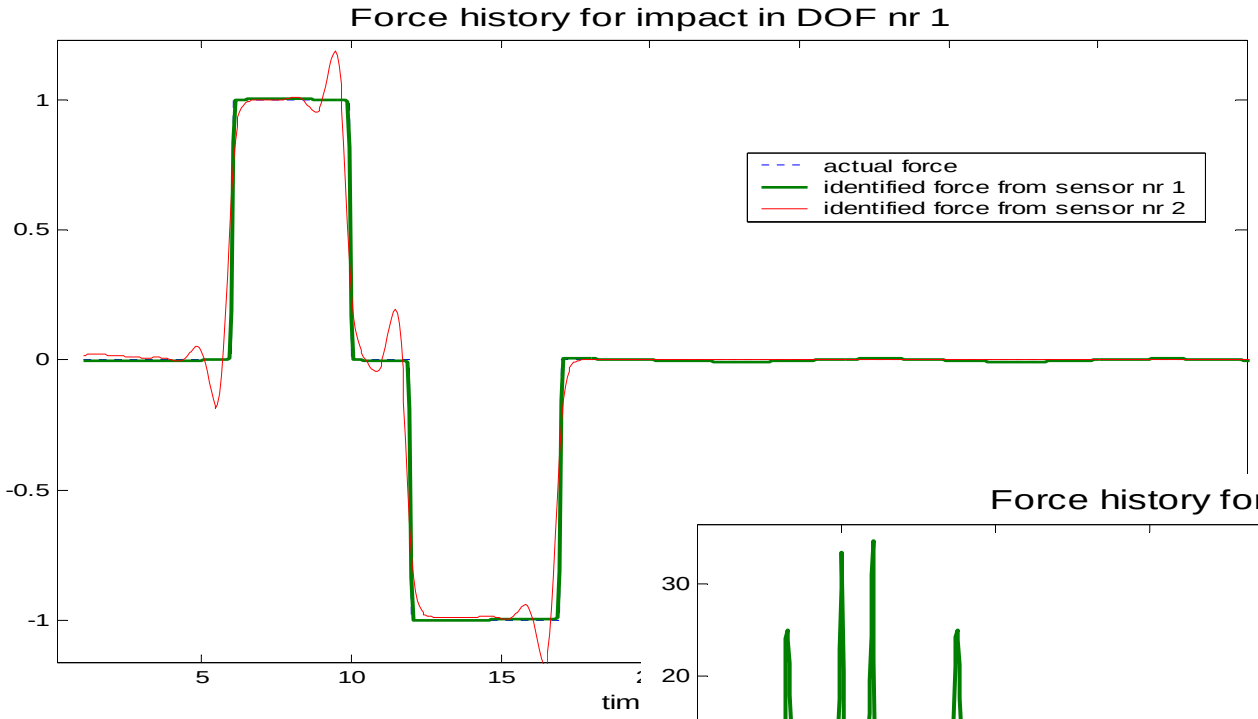


Two cases are considered

I. Impact in q_1 direction

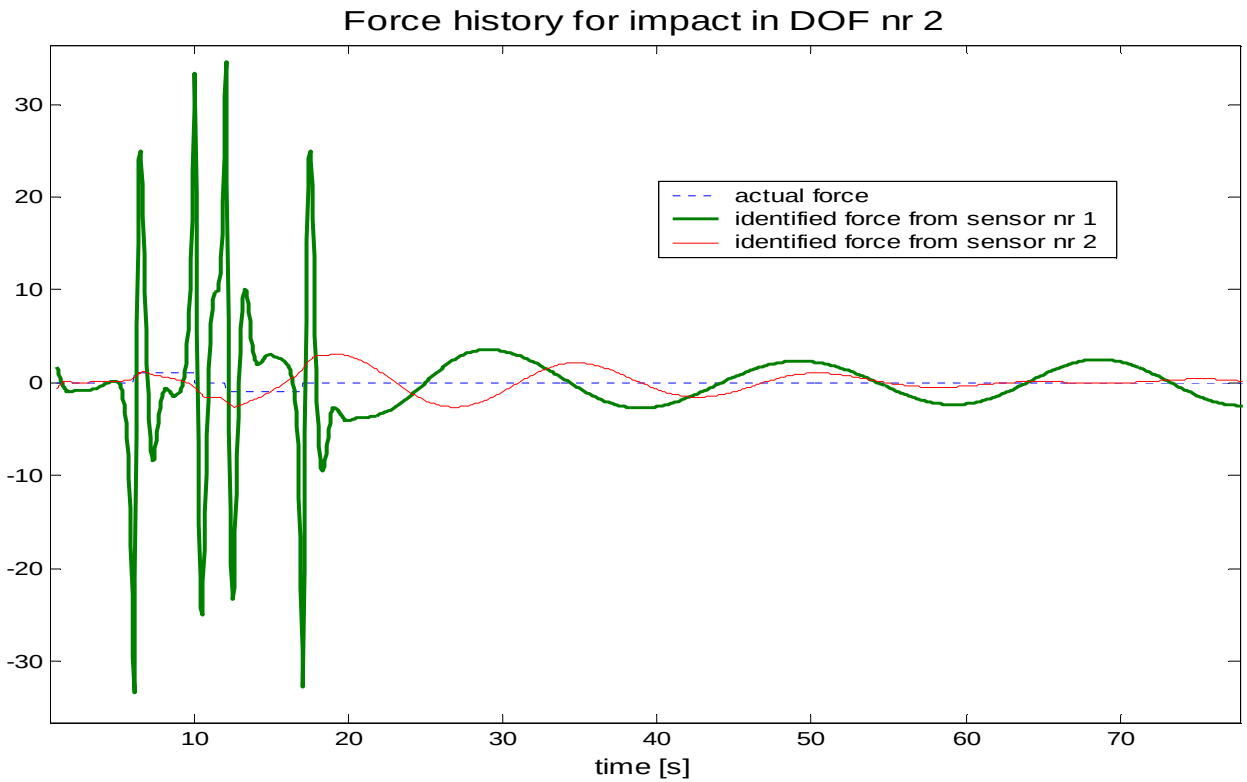
II. Impact in q_2 direction

FFT force identification-results

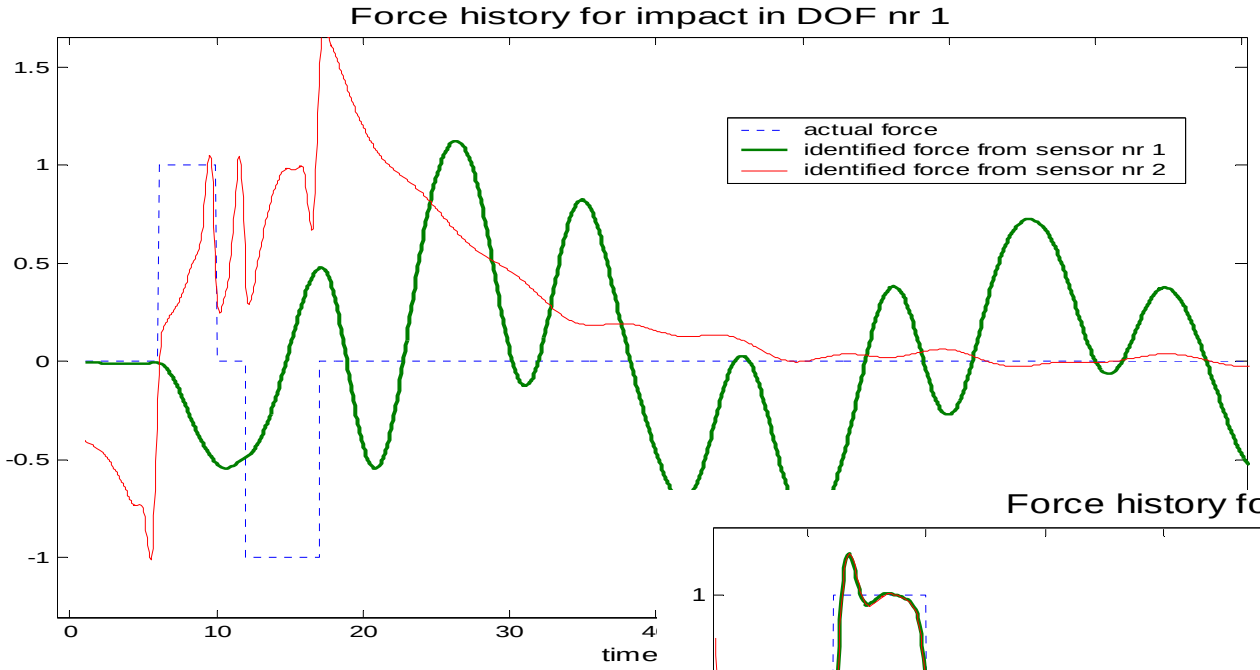


Case I:

Actual impact in q_1 direction

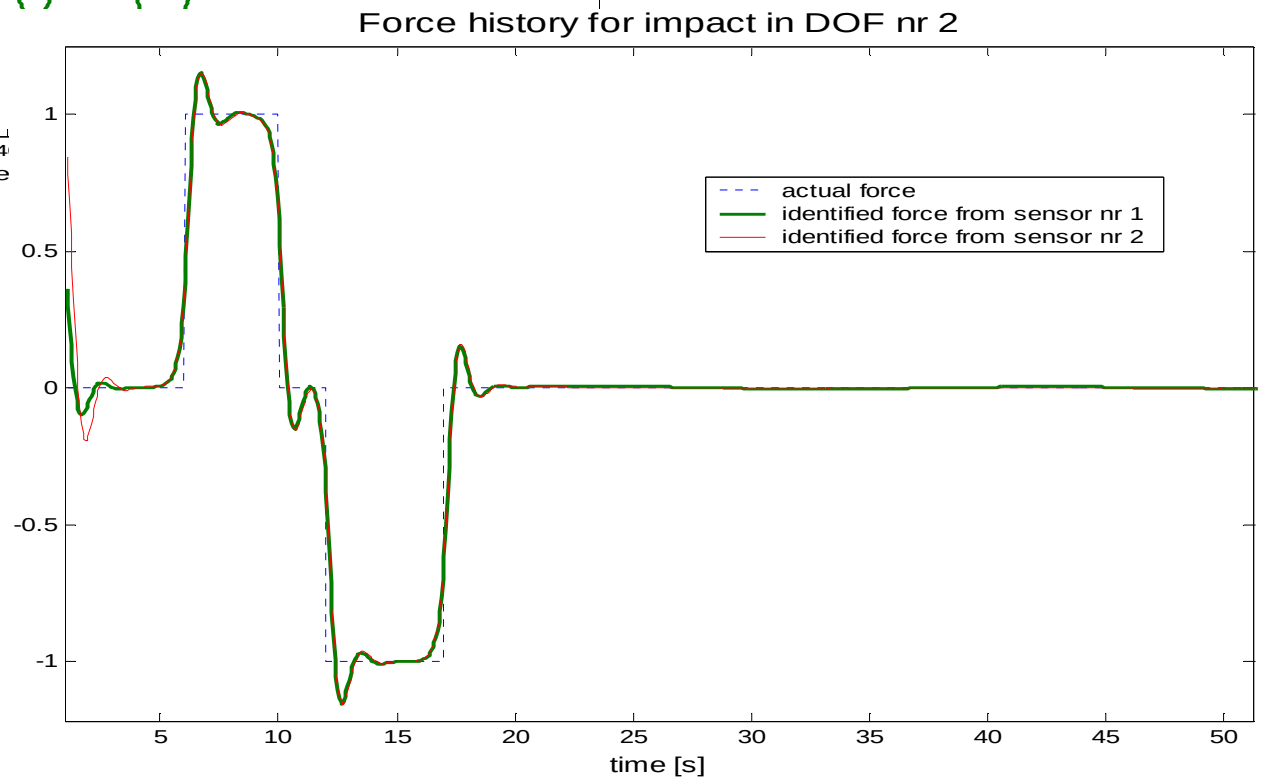


FFT force identification-results cd



Case II:

Actual impact in q_2 direction



On-line force identification

State-space representation of the structure

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{f}(t) \qquad \mathbf{x}(t) = \begin{bmatrix} \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) \end{bmatrix}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \qquad \text{(Measurement)}$$

Observer for the above system

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{f}(t) + \mathbf{L}\underbrace{(\mathbf{y}(t) - \mathbf{C}\mathbf{x}(t))}_{\text{innovation vector}}$$

Unknown input observer – main concept

Singular value decomposition (SVD) of state equations

$$\begin{cases} \dot{\bar{\mathbf{x}}}_1(t) = \bar{\mathbf{A}}_{11}\bar{\mathbf{x}}_1(t) + \bar{\mathbf{A}}_{12}\bar{\mathbf{x}}_2(t) + \bar{\mathbf{B}}_1\bar{\mathbf{f}}(t) \\ \dot{\bar{\mathbf{x}}}_2(t) = \bar{\mathbf{A}}_{21}\bar{\mathbf{x}}_1(t) + \bar{\mathbf{A}}_{22}\bar{\mathbf{x}}_2(t) \end{cases}$$

and measurement equations

$$\begin{cases} \bar{\mathbf{y}}_1(t) = \tilde{\mathbf{C}}_{11}\bar{\mathbf{x}}_1(t) + \tilde{\mathbf{C}}_{12}\bar{\mathbf{x}}_2(t) \Rightarrow \bar{\mathbf{x}}_1(t) = \tilde{\mathbf{C}}_{11}^{-1}\left(\bar{\mathbf{y}}_1(t) - \tilde{\mathbf{C}}_{12}\bar{\mathbf{x}}_2(t)\right) \\ \bar{\mathbf{y}}_2(t) = \tilde{\mathbf{C}}_{22}\bar{\mathbf{x}}_2(t) \end{cases}$$

Observer design for states independent on force

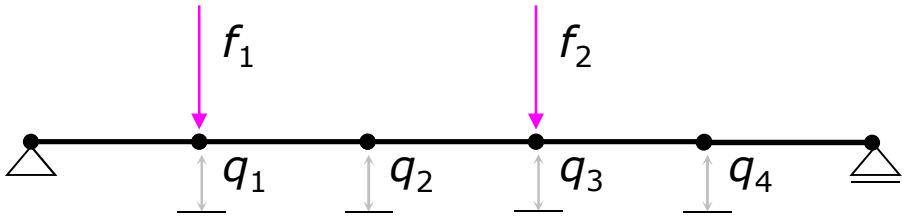
$$\dot{\hat{\bar{\mathbf{x}}}}_2(t) = \mathbf{A}_L\hat{\bar{\mathbf{x}}}_2(t) + \mathbf{B}_L\bar{\mathbf{y}}_1(t) + \mathbf{G}_L\left(\bar{\mathbf{y}}_2(t) - \tilde{\mathbf{C}}_{22}\hat{\bar{\mathbf{x}}}_2(t)\right)$$

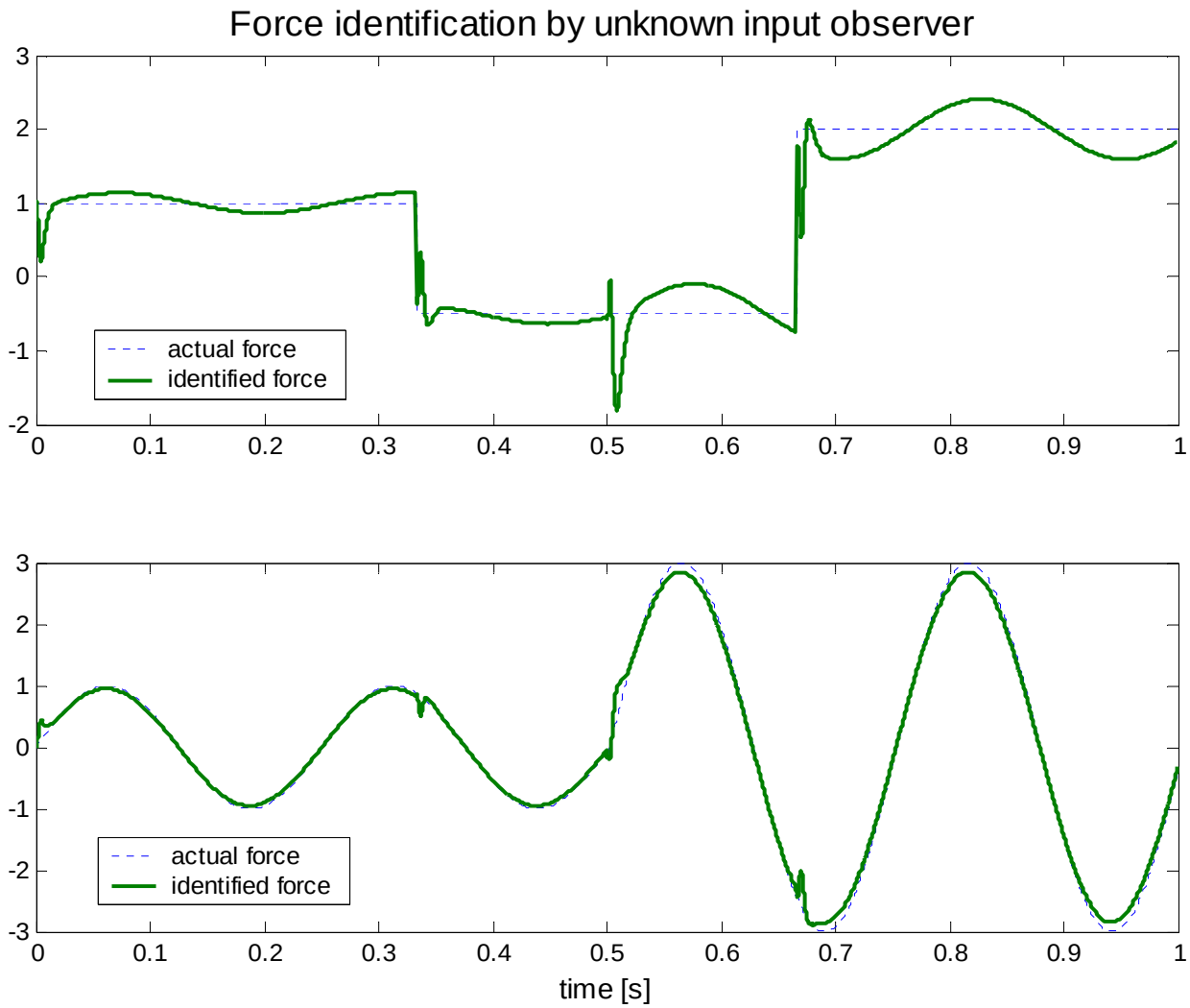
Finally, on-line estimator of unknown forces

$$\hat{\mathbf{f}}(t) = V\bar{\mathbf{B}}_1^{-1}\left(\dot{\hat{\bar{\mathbf{x}}}}_1(t) + \bar{\mathbf{A}}_{11}\hat{\bar{\mathbf{x}}}_1(t) + \bar{\mathbf{A}}_{12}\hat{\bar{\mathbf{x}}}_2(t)\right)$$

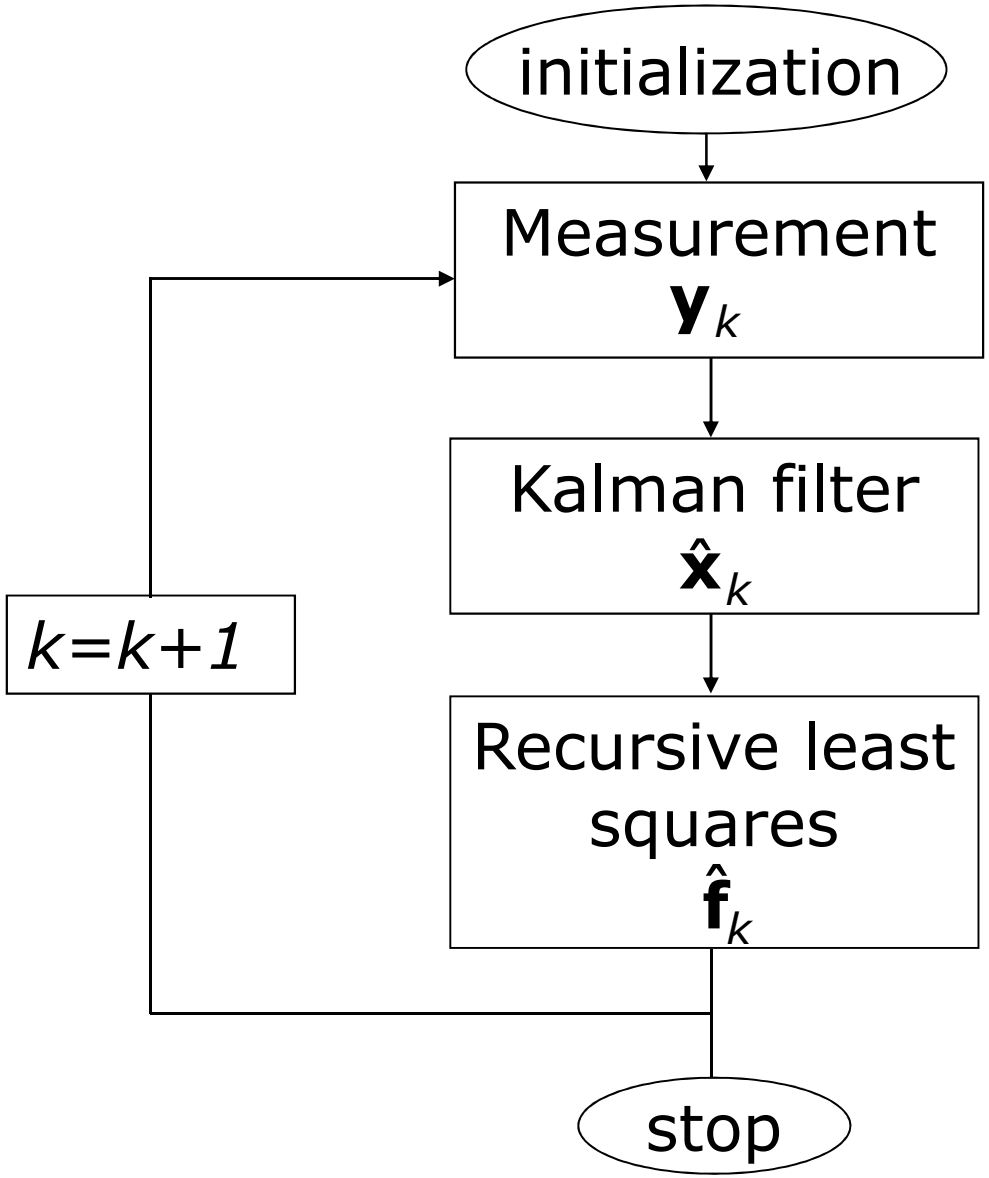
Unknown input observer – example

$$\mathbf{C} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$



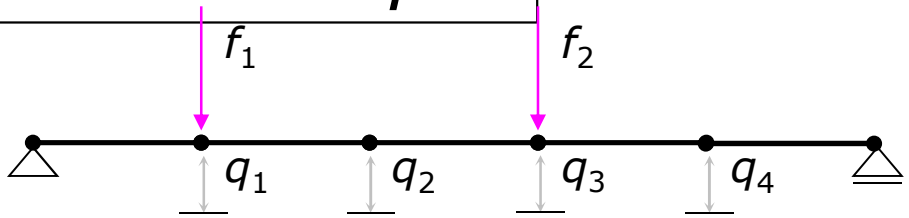


Recursive input estimation algorithm

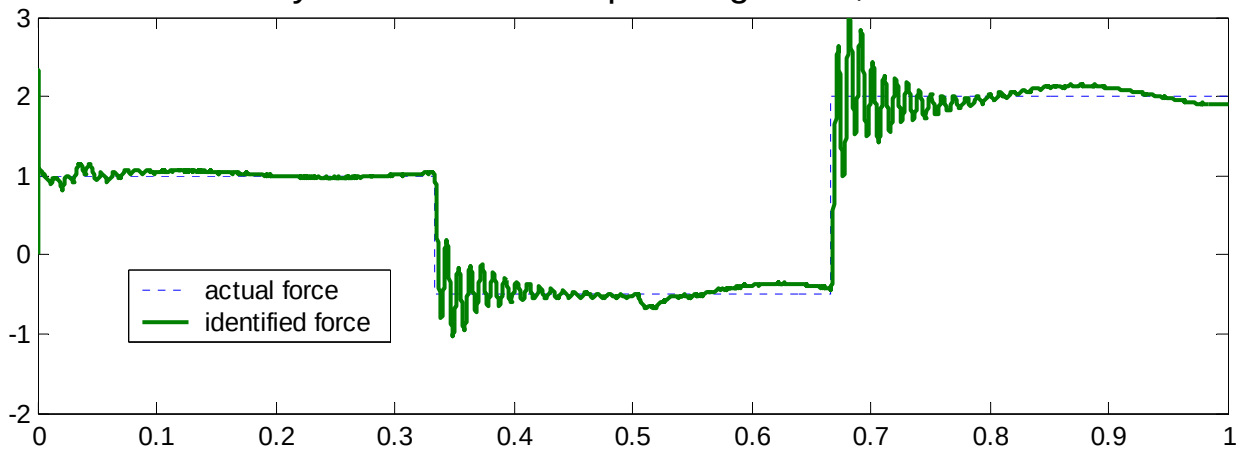


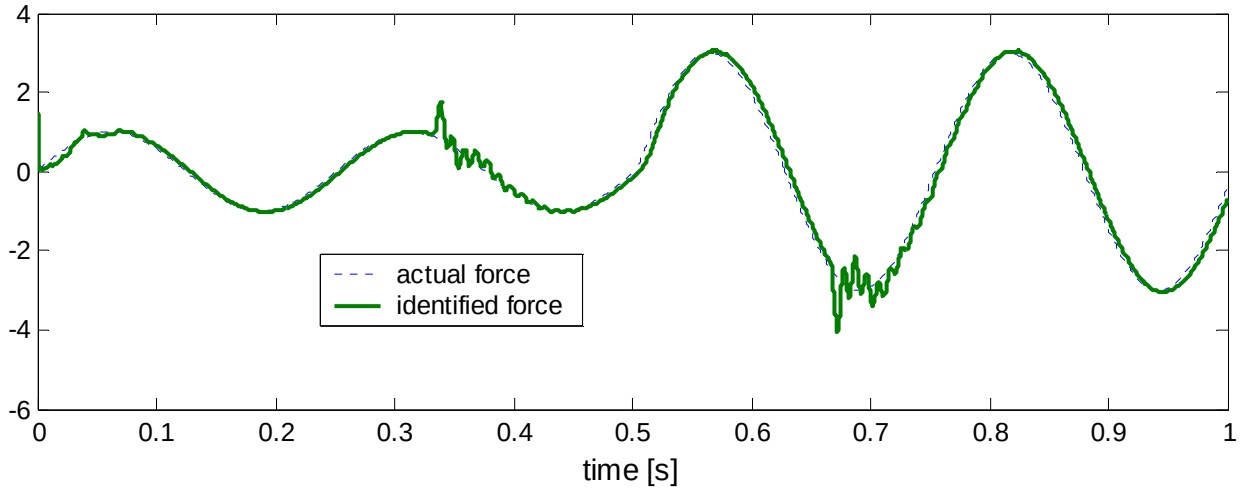
Recursive input estimation - example

$$\mathbf{c} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

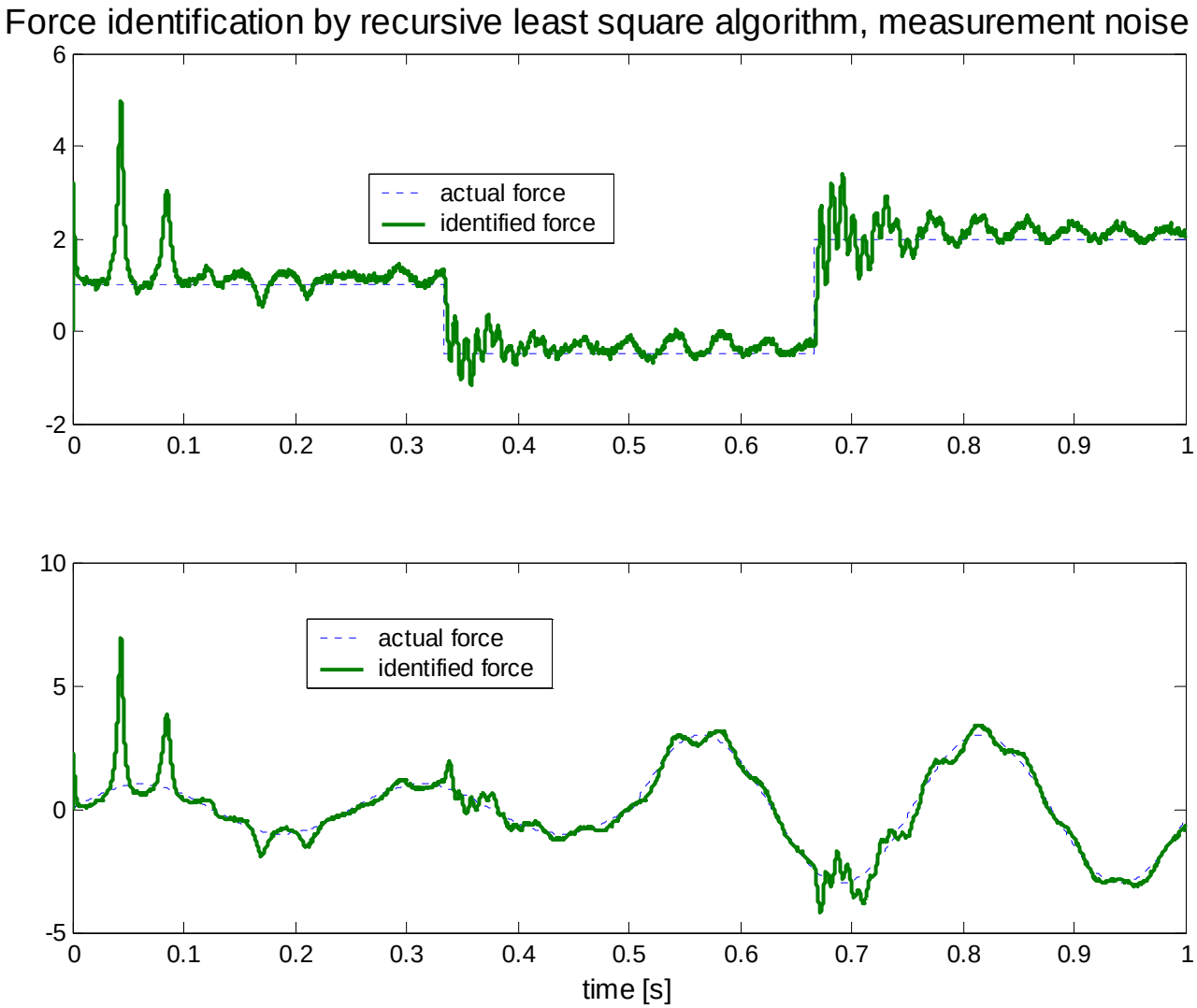


Force identification by recursive least square algorithm, measurement error 0.1 %

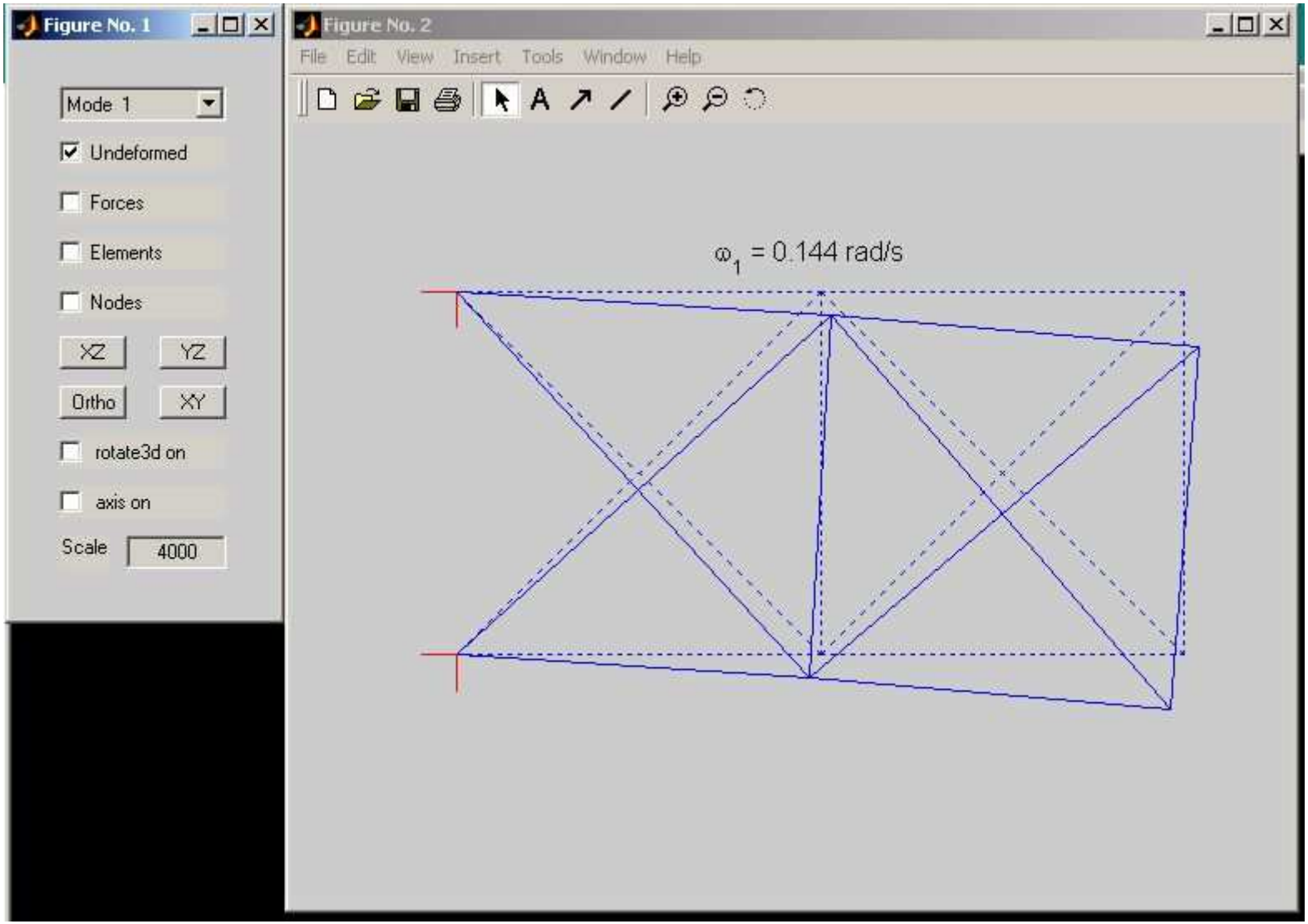




Recursive input estimation – example cd



MATLAB Toolbox for load ID



Conclusions

- An FFT based method for location of impacting forces was presented
- Two observer based methods for unknown input identification were proposed
- Applicability of the methods was evaluated on examples of simply supported beam and ten bar truss
- Numerical examples shows good accuracy and convergence
- MATLAB Toolbox for load ID was developed