Graph based algorithm for large discrete structural optimization problems

B.Błachowski and W.Gutkowski

Warsaw, Poland

WCSMO-8, June 1-5, 2009,
Lisbon, Portugal

Outline of presentation

- Introduction
- Assumptions of the method
- Statement of the problem
- The idea of the algorithm
- Hybrid algorithm and graph representation
- Numerical examples
- Conclusions

Introduction

- A discrete structural optimization (DSO) algorithm assigning to structural members, sections from a list of available cross section areas, is presented.

- It is assumed that combination numbers *n* arising from j^0 number of structural members and k^0 number of catalogue are of the order 10^{10}.

$$n = \left(k^0\right)^{j^0}$$

- In such cases the direct enumeration is not applicable.

Introduction *continued*

In DSO, mostly stochastic methods are applied, among them are Genetic Algorithm (GA) and Evolutionary Optimization (EO).

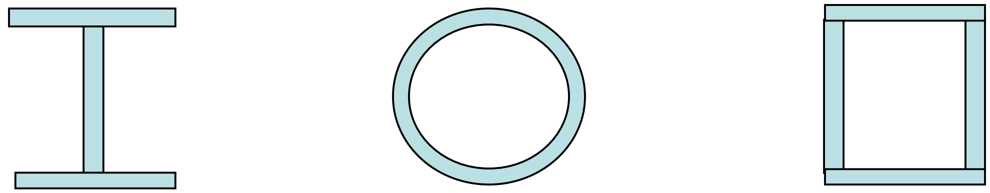
Disadvantages of GA and EO are very large numbers of analyses and needed experience in evaluating parameters.

Assumptions of the method

- A structure, of a given lay-out, is composed of a number of j^0 elements j made of linear elastic material with $j=[1,2,...,j^0]$.

- The minimum of the structure weight *V*, found in Continuous Structural Optimization (CSO) constitutes a lower bound of a DSO.

Among parameters are:



cross section areas A^k and moments of inertia I^k of beams with $k=[1,2,...,k^0]$

The following notations are assumed:

A_j-CSA of j-th structural member, discrete design variable

C_j-CSA of j-th structural member, continuous value

k_j - number of CSA assigned to j-th structural member

$A_j^{k_j}$ - k_j-th CSA from list assigned to j-th design variable

Statement of problem

Find minimum of the structural weight

$$f = \rho \sum_{j=1}^{j^0} A_j l_j$$

Equality constraints are:

for statics

$$\mathbf{K}\mathbf{u}_q - \mathbf{Q}_q = \mathbf{0} \qquad q = 1,2,\ldots,q^0$$

q^0 the number of static loading conditions,

for eigenfrequencies

$$(\mathbf{K} - \omega^2 \mathbf{M})\mathbf{\Phi} = \mathbf{0}$$

$$\mathbf{M} - \text{mass matrix}, \omega - \text{frequency}, \mathbf{\Phi} - \text{normal mode}$$

Inequality constraints are:

- The largest and the smallest values of listed parameters

$$A^{1} \leq A_j^{k_j} \leq A^{k^0}$$

- The maximum stresses and displacements

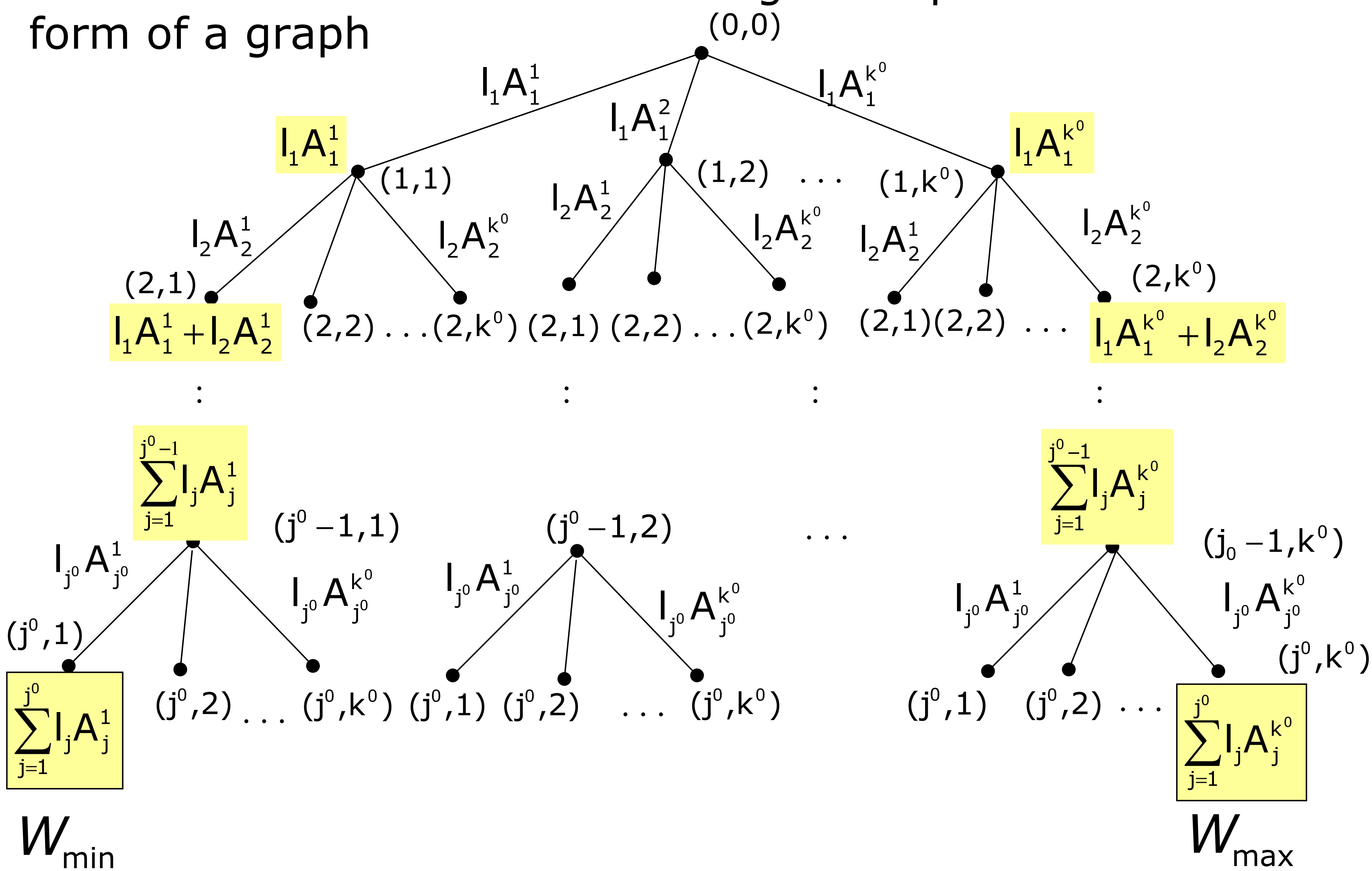
$$-\boldsymbol{\sigma}^{cr} \leq \boldsymbol{\sigma}_q \leq \boldsymbol{\sigma}^0 \qquad -\mathbf{u}^0 \leq \mathbf{u}_q \leq \mathbf{u}^0$$

- The minimum value of the first eigenfrequency

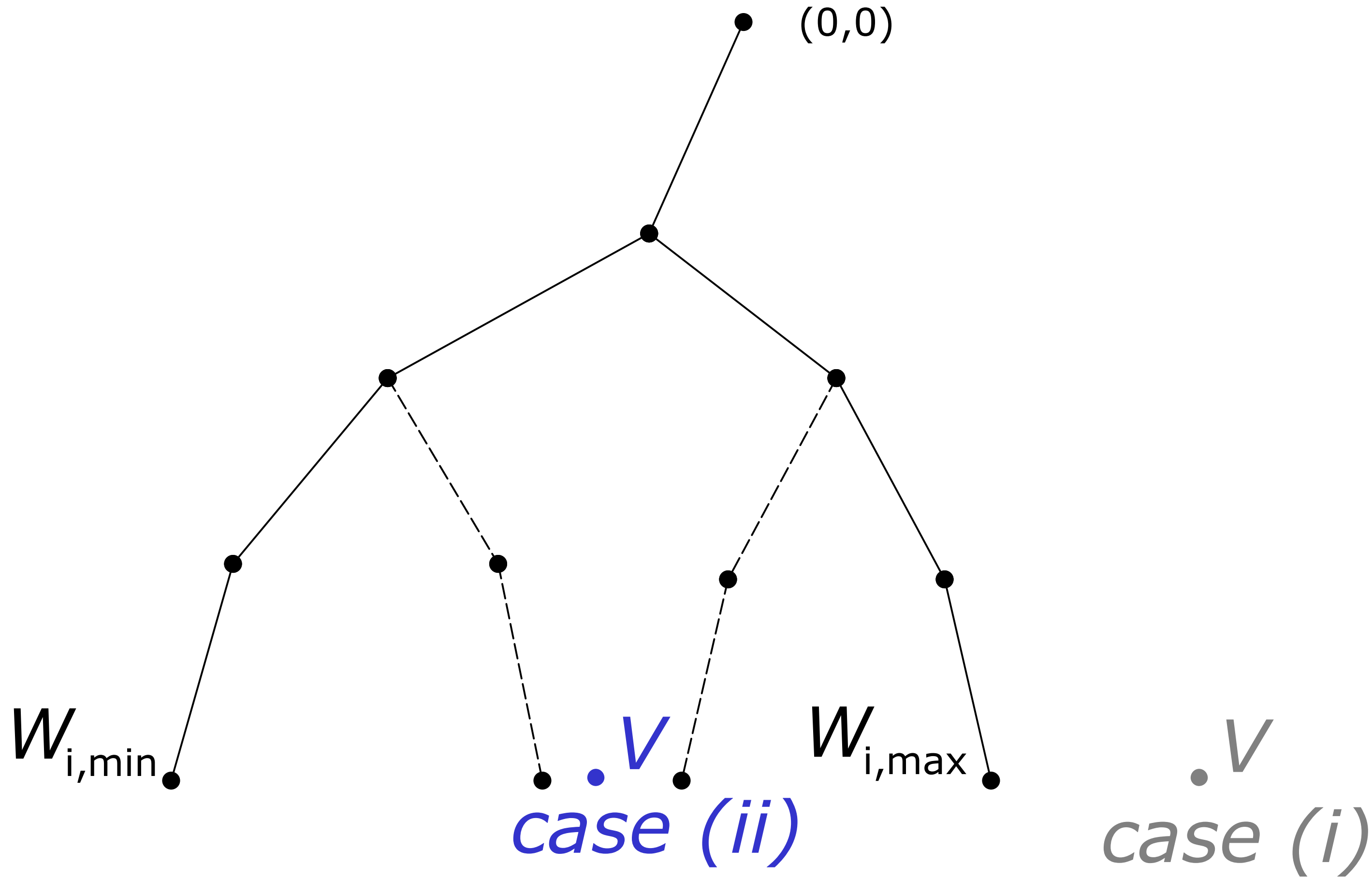
$$\omega - \omega_0 \geq 0$$

The idea of the algorithm

All combinations of structural weight are presented in the form of a graph

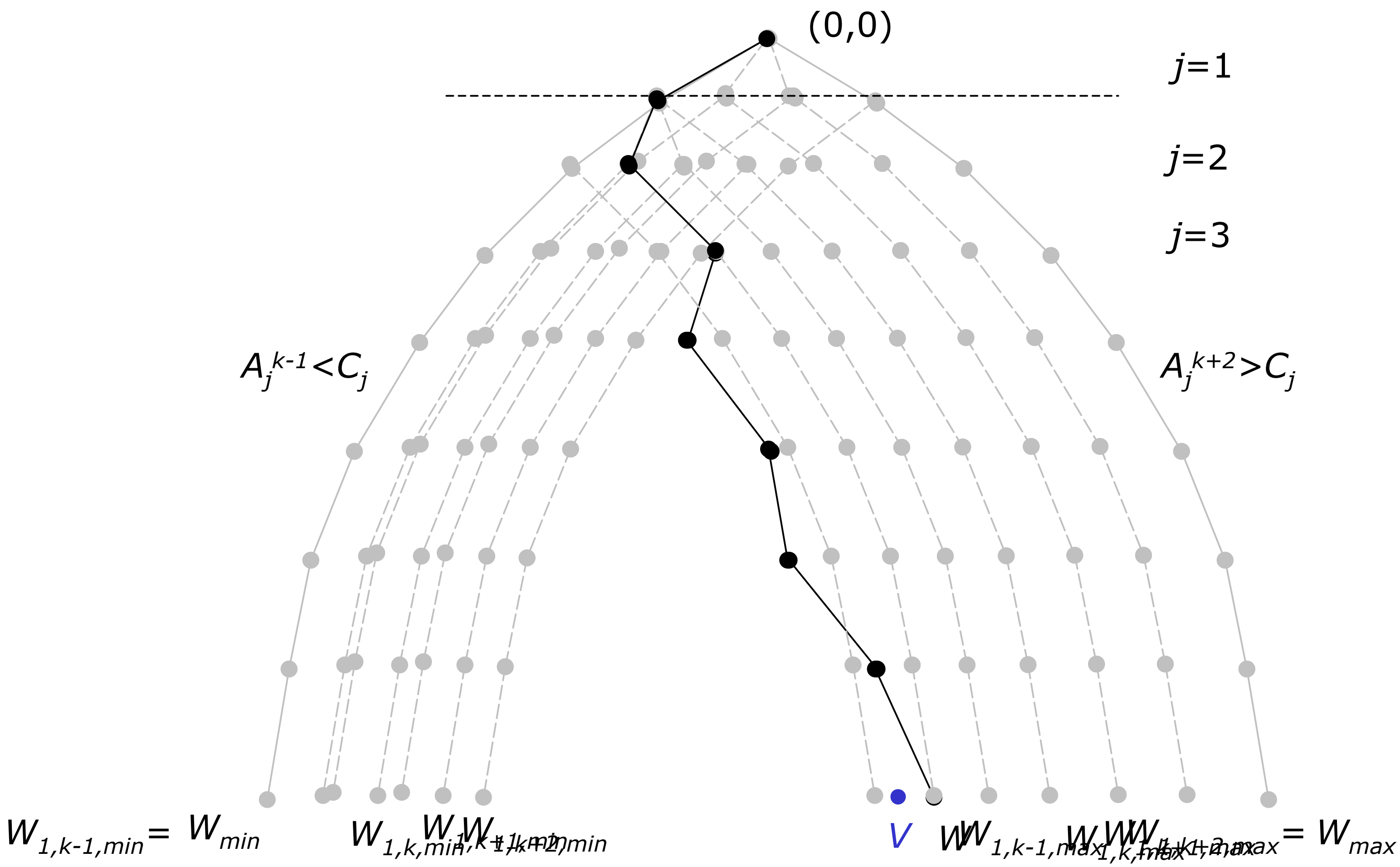


- An important graph property



$V > W_{i,max}$ allows to eliminate very large numbers of combination from farther consideration

Discrete weight close to continuous one



Hybrid algorithm

PART 1. FIND VALUES OF DISCRETE WEIGHT FUNCTION

STEP 1.1

Find cross section areas C_j of structural members, solving continuous minimum weight V problem.

STEP 1.2

Take, for each *j*-th structural member, two subsequent parameters $A_j^{k_j}$ and $A_j^{k_j+1}$, such as:

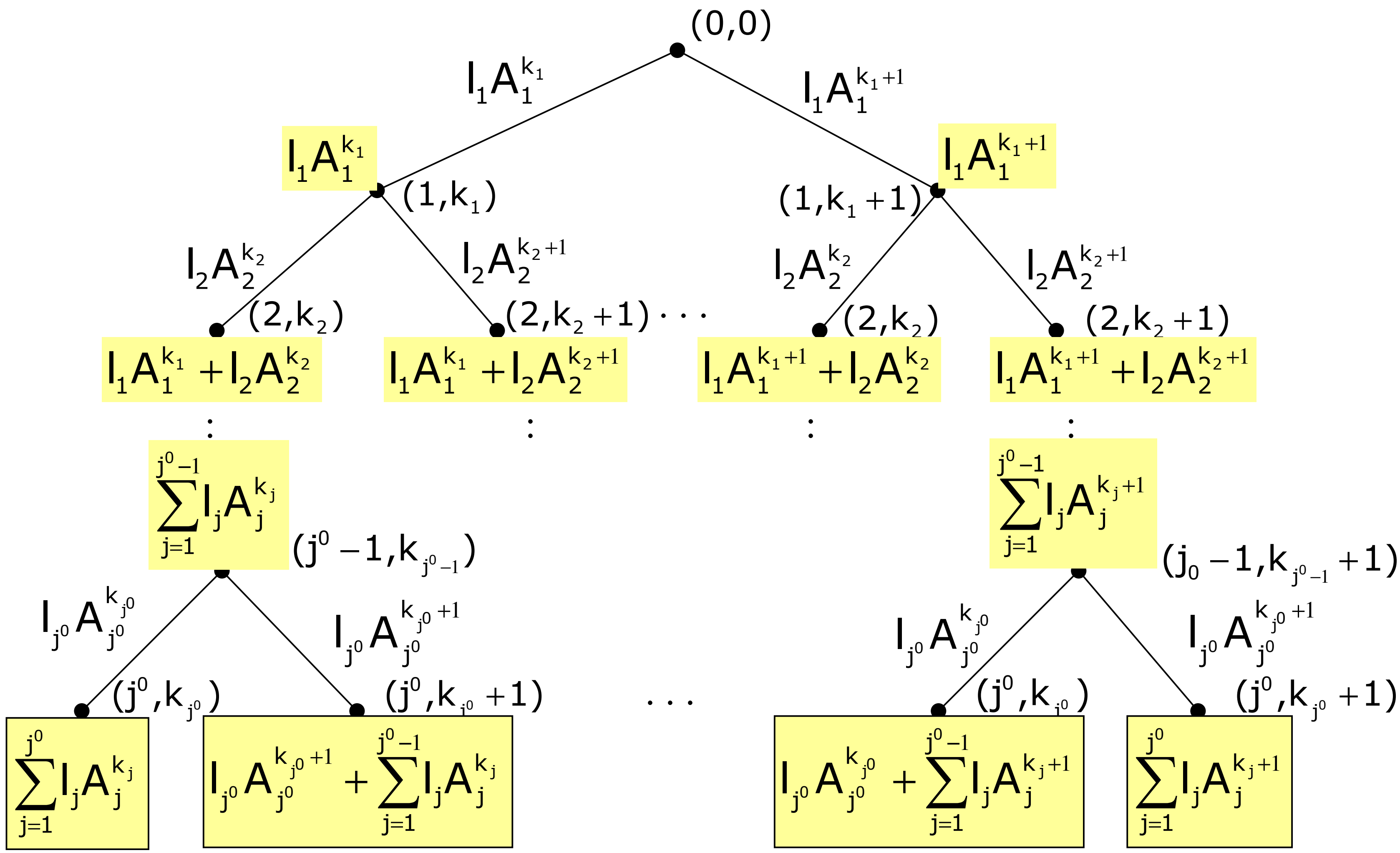
$$A_j^{k_j} \leq C_j \leq A_j^{k_j+1}$$

This gives a two parameter catalogue for each of *j*-th the structural member.

STEP 1.3

For obtained two parameter catalogue construct two branch graph.

Two branch graph



STEP 1.3 continued

The last j^0 layer contains n combinations of discrete values, of the structural weight, equal to

$$n = 2^{j^0}$$

The following two cases can take place:

(i) $$V > W_{i,max} = \rho \sum_{j=1}^{j^0} l_j A_j^{k_j+1}$$

In this case solution does not exist.

STEP 1.3 continued

(ii) $$V \leq W_{i,max} = \rho \sum_{j=1}^{j^0} l_j A_j^{k_j+1}$$

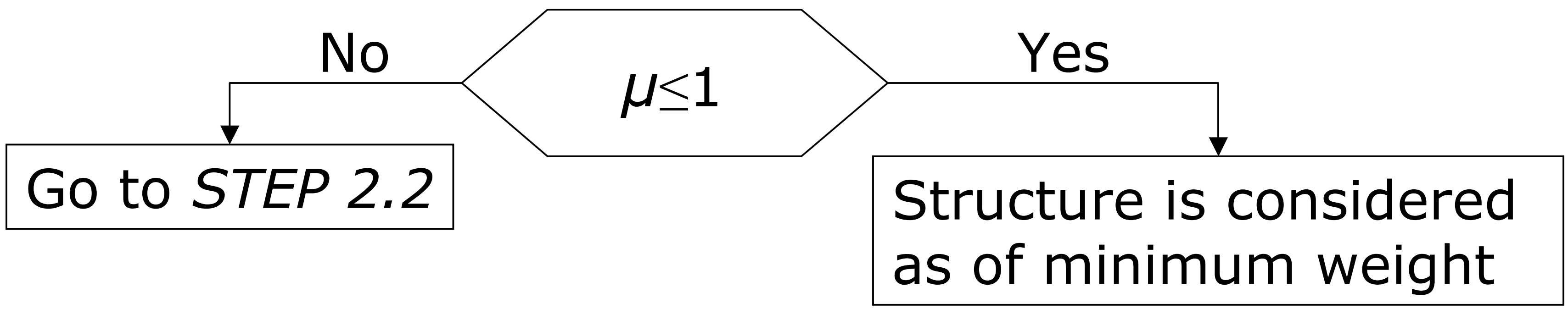
Then combination with smallest structural weight, fulfilling constraints is the solution.

PART 2. VERIFY CONSTRAINTS VIOLATION

STEP 2.1

For the smallest $W_{i,max}$ from *STEP 1.3* find a parameter

$$\mu(m) = \max\,(u_i / u^0;\ \sigma_j / \sigma^0) \qquad m=1$$

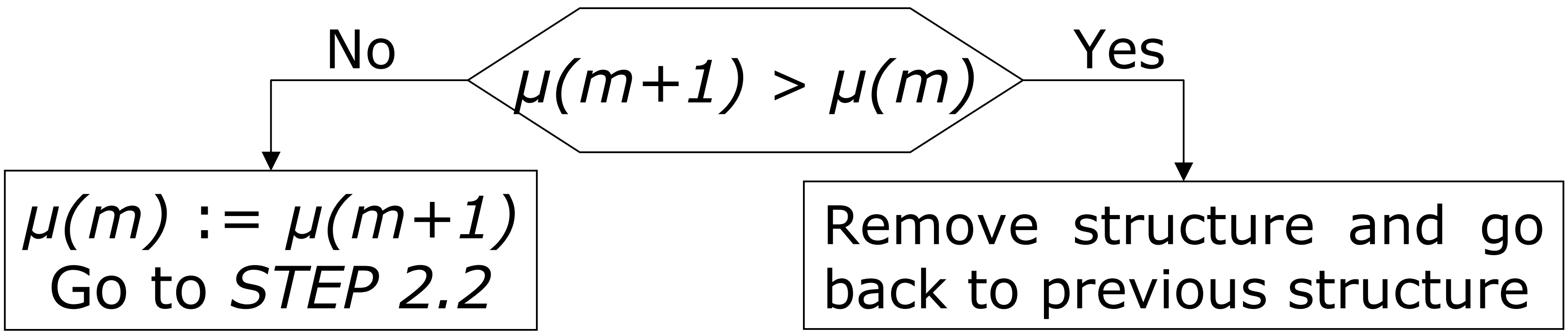


STEP 2.2

For $\mu>1$ choose two structural members. In one of them, decrease its CSA by assigning to next smaller value. In the second chosen member increase CSA to next larger value.

STEP 2.3

Perform structural analysis for new set of structural members and find $\mu(m+1)$.



STEP 2.4

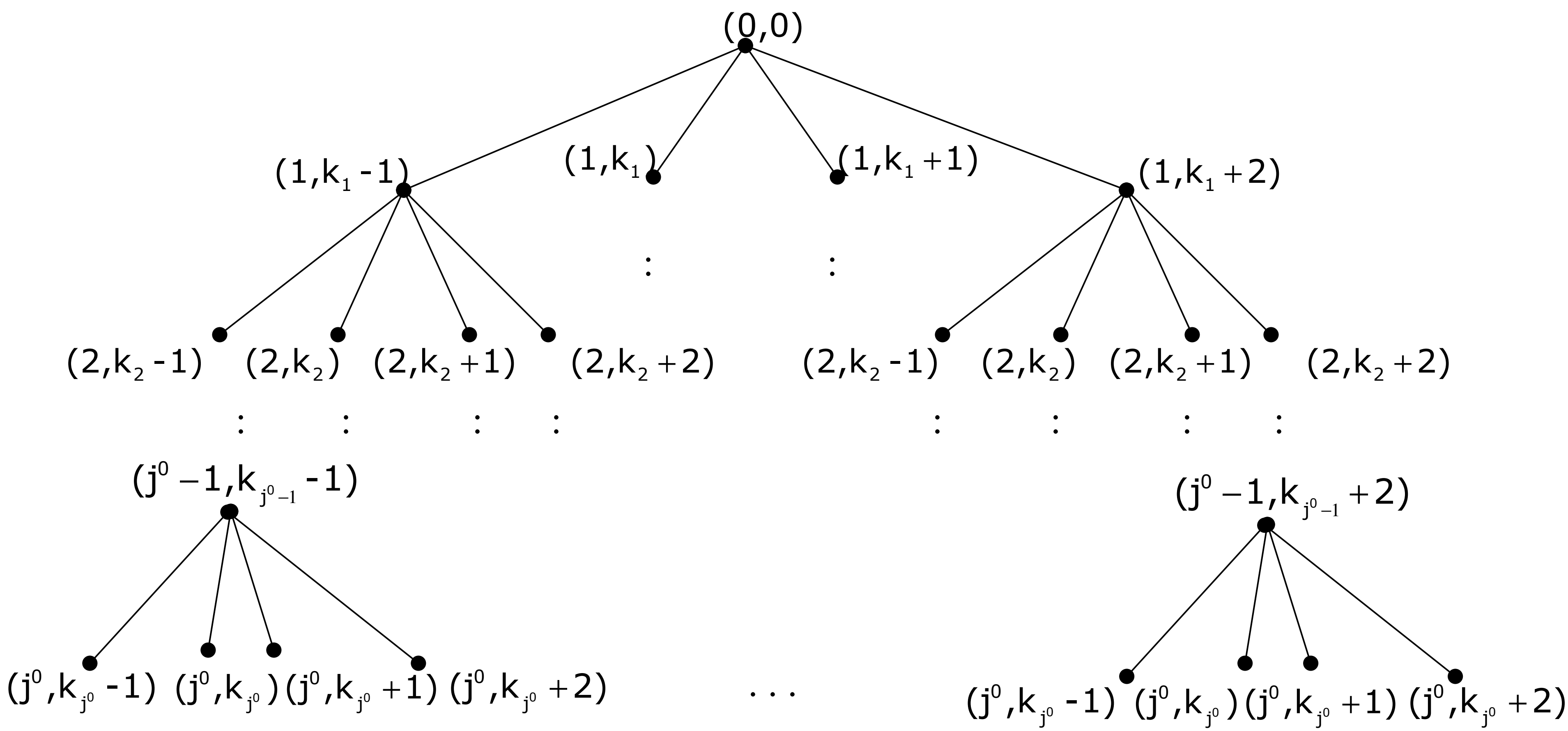
Repeat *STEPs 2.2* and *2.3* until μ reaches value equal or smaller than one.

If such a value is not obtained, go to PART 1 and enlarge lists of available CSA to four positions.

STEP 2.4 *continued*

Graph constructed by taking two additional parameters

$A^{k_j - 1}$ and $A^{k_j + 2}$.



Example 1
160 bar space truss

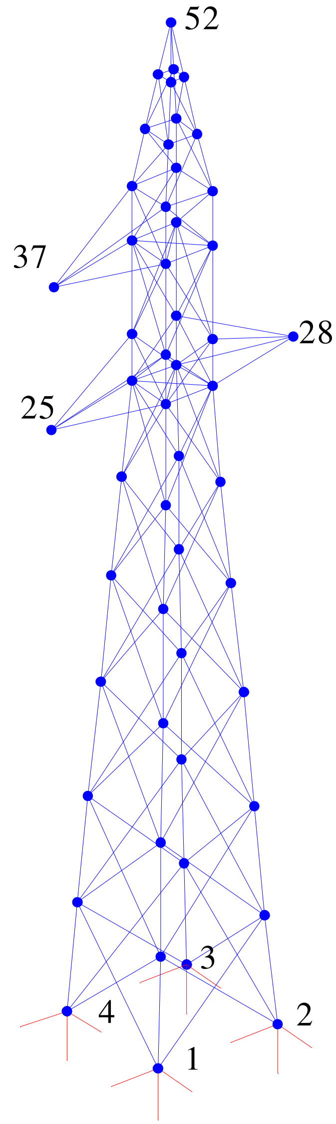
38 linking groups $j^0=38$

42 catalogue parameters $k^0=42$

Number of possible combination

$42^{38}=4.82*10^{61}$

Constraints imposed on:
-stress limit
σ^0 = 1500 kg/cm^2
-and buckling
σ^{cr} = 1300-$S^2/24$ kg/cm^2, for $S<120$
σ^{cr} = $10^7/S^2$ kg/cm^2, otherwise
where S - slenderness



Results – Example 1

2 parameter graph

Solution hasn't been found.

Subsequent iterations end with μ>1.02

4 parameter graph

Optimal weight W=1341.4 kg

Constraints violation μ=1.019

	**Groenwold solution*	***Juang solution*	*Present method*
Weight	**1359.78 kg**	**1331.75 kg**	**1341.4 kg**
Number of analyses	**16+387**	**170**	**8+28**

* Groenwold A.A, Stander N., (1997), *Structural Optimization*, **14**, 71-80

** Juang D.S., Chang W.T., (2006), *Struct Multidisc Optim*, **31(3)**, 211-223

Example 2
25 bar space truss

8 linking groups

Case 1

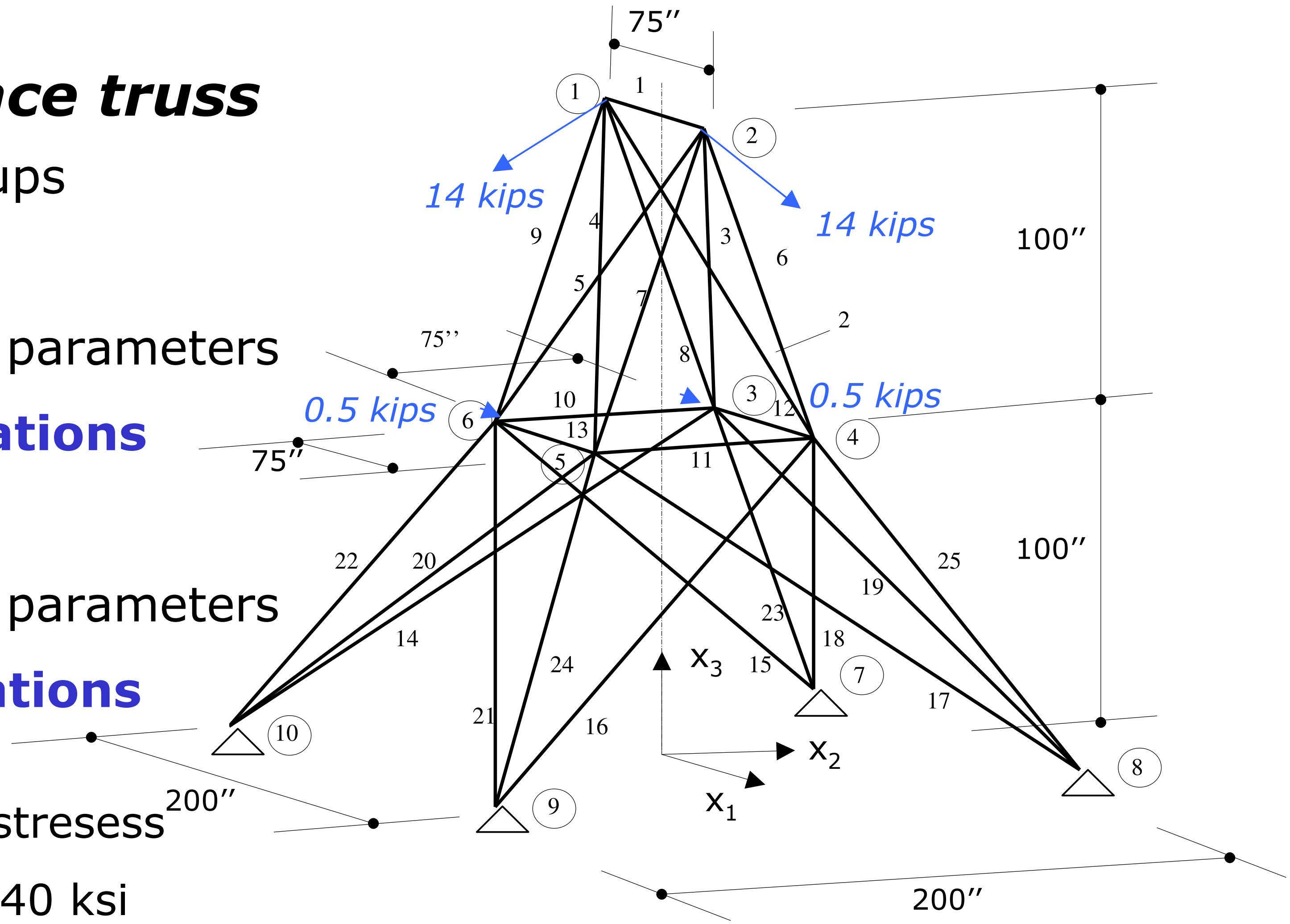
30 catalogue parameters

30^8 combinations

Case 2

16 catalogue parameters

16^8 combinations



Constraints on stresess

$-40 \text{ ksi} \le \sigma \le 40 \text{ ksi}$

and displacement

$-0.35 \text{ in.} \le u \le 0.35 \text{ in.}$

K.S.Lee, Z.W.Geem, S.Lee, K.W.Bae, *Engineering Optimization*, 2005, **37**(7), 663-684.

Results –Example 2

(2 parameter graph)

Design variables A_i (*in.*^{2})	HS algorithm by Lee et al. (*Case 1*)	Present algorithm	HS algorithm by Lee et al. (*Case 2*)	Present algorithm
Weight (lb) μ	**484.85**	**485.04 (0.998)**	**560.59**	**564.85 (0.992)**
Number of structural analyses	**14163**	**16 (cont.) +1 (disc.)**	**27847**	**44 (cont.) +9(disc.)**

(4 parameter graph)

Design variables A_i (*in.*^{2})	HS algorithm by Lee et al. (*Case 1*)	Present algorithm	HS algorithm by Lee et al. (*Case 2*)	Present algorithm
Weight (lb) μ	**484.85**	**484.85 (1.000)**	**560.59**	**551.60 (1.001)**
Number of structural analyses	**14163**	**16 (cont.) +5 (disc.)**	**27847**	**44 (cont.) +31(disc.)**

Conclusions

- ❑ A very simple and robust algorithm for designing a minimum weight of a structure composed of prefabricated elements, is presented. The structure can be subjected to several static loads and constraints imposed on eigenfrequency.

- ❑ The algorithm is based on two main assumptions:
  - The structural weight obtained from continuous minimum design constitutes a lower bound for discrete minimum weight.
  - The graph representation of structural volume allows to reject, from considerations, large numbers of unfeasible discrete values.

Conclusions *continued*

- The algorithm is numerically very efficient. It requires very small number of equilibrium equation solutions, and a number of additions of structural element volumes.

- In the example 2, numbers of equilibrium equations solved applying the presented method are: **21** (case 1) and **75** (case 2) .

The same problem, solved by the harmony search (*Lee et al.*) numbers of equilibrium equations solutions required are: **14163** (case 1), and **27847** (case 2).

Conclusions *continued*

- The algorithm is very friendly for designers. They don’t need to know any thing about genes, ants, swarms and harmony search. The only knowledge required to find a discrete minimum is FEM and simple additions.

Thank you for your attention.