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A SCALING LAW FOR THE NECK-SHAPE IN MICROFLUIDIC FLOW-FOCUSING DEVICE

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Summary Droplets in microfluidic devices are generated by crossing flows of two immiscible fluid in a confined space. The core point of models describing droplet detachment process from a stream of the droplet phase is the formation of the so-called neck – the constriction of the droplet phase, which finally breaks, releasing a new droplet. In low Capillary number (Ca) range, the neck shrinks through a series of quasi-static shapes that were determined by the instantaneous volume behind the neck and the geometrical confinement of the junction. Surprisingly, the squeezing model predicts that the droplet size is independent of the shear. Here, we reveal that in flow-focusing device, the geometrical confinement is unable to ensure the evolution of quasi-static shapes, such that the neck becomes dramatically prone to the magnitude of shear. We provided a scaling analysis to explain this dependency.

DEPENDENCY OF AREA UNDER THE NECK TO CAPILLARY NUMBER

The squeezing model, firstly introduced in the seminal paper of Garstecki et. al.[1], is the formation of droplet at low Ca ($=\mu_C Q_C/\gamma HW$, where μ_C and Q_C are the viscosity and flow rate of continuous phase respectively, γ is interfacial tension, H and W is the height and width of channel respectively). It is understood that droplet size produced in this squeezing regime, either in T-junction or in flow-focusing device (FFD) is insensitive to Ca, as long as flow rate ratio $q=Q_C/Q_D$ (where Q_D is the droplet phase flow rate) is kept constant, as stated by its scaling law: $\ell_D=\ell_0+v_{N0}q$ (where ℓ_D is the dimensionless droplet length, with ℓ_0 and v_{N0} as constant parameters represent droplet's initial volume filling the outlet channel and total volume displaced by the neck, respectively). The insensitivity to flow rate parameters makes the squeezing regimes convenient for microfluidic application. However, earlier reports[2], [3]have shown that at some point the droplet size increases greatly as Ca is decreased further. This leads to the new leaking model[4]:

$$\ell_D=\ell_0+q\,v_{N0}+\frac{\phi}{\mathrm{Ca}}\,. \tag{1}$$

The strong dependency to Ca (see last term of Eqn. 1) is due to the leaking process, where the continuous-phase flows around the droplet through the corners of the channel (the so called gutter).

Our experimental analysis of the formation of droplet in FFD revealed that besides the effect of leaking there is another source of Ca dependence – the variation of the size of the neck. In case of T-junction the neck evolution is well localized and the break-up always occurs in the edge of T-junction. So in the case of T-junction the parameter v_{N0} associated with the volume of the continuous phase accumulated behind the neck is assumed to be constant. In contrary, in case of FFD, we observed that the neck is much longer than the width of the channel and it's final length (prior to break-up) becomes very large at very low Ca (see Fig. 1(a)). This behavior is allowed in FFD because the edges in the cross-junction does not play any role in the development of neck, supported by the observation that the position of break-up does not seems to be correlated to the edge position. As a consequence, in FFD the coefficient v_{N0} in Eqn. 1 is not constant and increases sharply as Ca decreases. This is confirmed by measurement of ℓ_D as a function q at fixed value of Q_C (i.e. constant Ca) as shown in Fig. 1(b). Assuming that v_{N0} is solely function of Ca, we can approximate it as the slope of the linear relation ℓ_D versus q . Fig. 1(c) shows the v_{N0} extracted using this assumption and taken from the experimental data of five distinct rectangular cross-section devices (shown as different aspect ratio(W/H)) and one

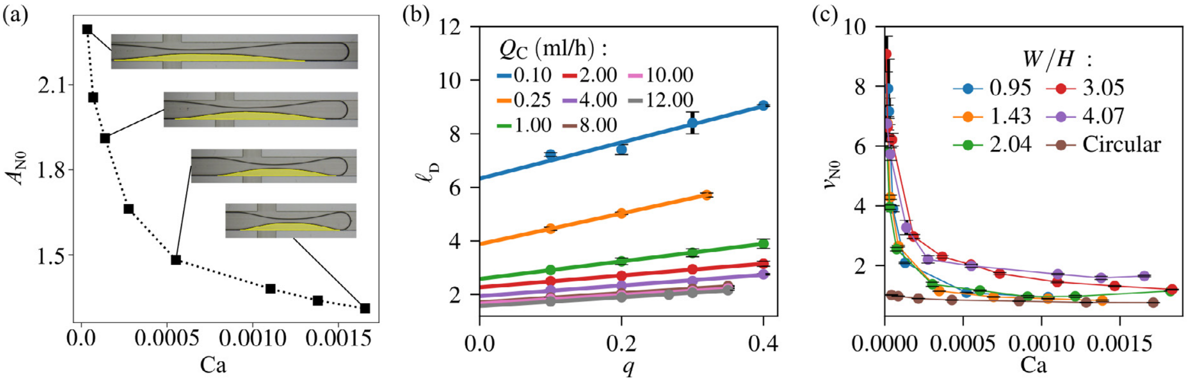

Fig. 1. (a) Visualization and measurement of neck area, A_{N0} , that drastically increases at vanishing Ca. (b) the slope of ℓ_D versus q at fixed Q_c can be use to approximate v_{N0} . The slope is higher for lower Q_c (i.e. lower Ca). Data is taken from device with W/H = 4.07. (c) Comparison of v_{N0} at low Ca from various rectangular cross-sectional devices and single circular device with radius of cross-section 0.2 mm.

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circular cross-section channel. v_{N0} increases drastically at decreasing Ca in rectangular cross-section devices but not in a circular cross-section device. This confirms the importance this effect in the leaking regimes of commonly used rectangular cross-section microfluidic devices. Note that although dependency of v_{N0} to flow rate parameters were indicated in previous T-junction[5] and FFD[6]literature, the changes of v_{N0} is not very significant at higher Ca flows (squeezing or dripping regime), such that it's effect is usually neglected. The non-negligible dependency of v_{N0} to Ca in the leaking regime has lead us to revise the previous assumption and modify the scaling law.

SCALING ANALYSIS OF v_{N0}

To explain the observations of v_{N0} variation we explored the simplified model of the evolution of the cosine-like profile of the neck (see Fig. 2). The analysis of the distribution of the capillary stress on such a profiles reveals that such a shape tends to increase its width for the relaxation of the stored potential energy. We Assume that the magnitude of capillary action depends on the maximal amplitude of the curvature - λ/ω^2 (where λ and ω are the height and the width of the profile respectively). The motion of the side edge points of the profile is supposed to be slowed by the viscous friction proportional to the speed of the edge propagation. Finally, balancing both interactions acting on the profile we obtained the following relation $d\omega/dt = c\lambda/\omega^2$, where $c = \xi\gamma/\mu_C$ and ξ is a constant coefficient of the length dimension corresponding to the height of the channel. Further analysis revealed Ca-dependency of neck shape as:

$$v_{N0} \propto \mathrm{Ca}^{-1/2} . \tag{2}$$

The scaling law was able to describe the exponential increase of v_{N0} at low Ca regime, as shown in Fig. 3 (a). Introduction of this relation into Eqn. 1 gives the extended leaking model, which is in very good agreement with the experiment results, as shown in Fig. 3 (b).

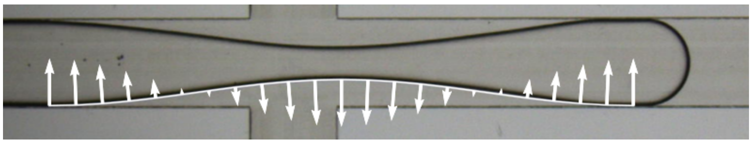

Fig. 2. The cosine-like neck profile shown as the white curve. The arrows are the $\kappa\hat{\mathbf{n}}$ vectors, where κ is the instantaneous curvature at any point along the curve and $\hat{\mathbf{n}}$ is the unit normal vector.

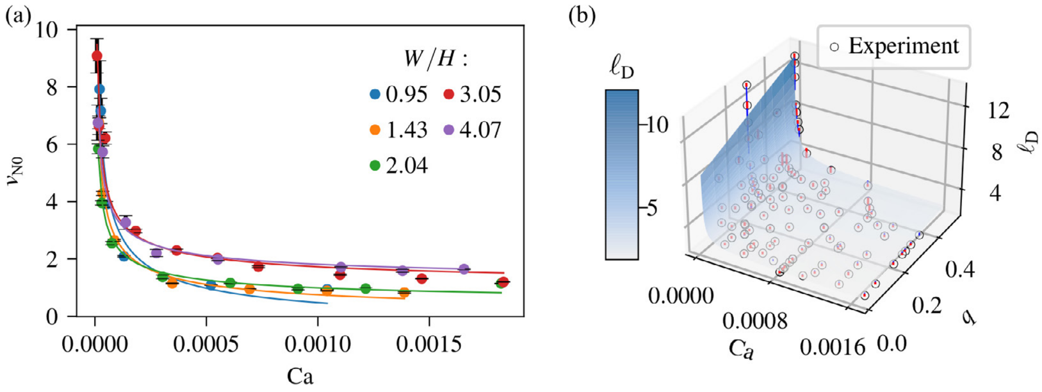

Fig. 3. (a) solid lines is the fitting results of v_{N0} based on Eqn. 2. (b) Theoretical prediction of ℓ_D by modified model of Eqn.1 is shown as the surface plot. Data shown is from device with $W/H = 4.07$.

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