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Reliability study of a containment shell

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ABSTRACT: Computational time for reliability analysis of realistic structural problems is very high. Improvements in efficiency are critical to allow solution of large realistic problems. The reliability analysis is usually performed using approximate First Order Reliability Method (FORM). Iterative solution procedures of FORM require substantial design sensitivity computations of high accuracy. The design of realistic structures requires computer-based numerical procedures, such as finite element analysis. For these structures, sensitivity is not available explicitly in terms of design variables. The most intensive computational task of design sensitivity computation should be carried out by highly efficient and accurate methods such as discrete design sensitivity analysis. This paper defines requirements for design sensitivity information for reliability analysis. The way of coupling reliability computation with discrete design sensitivity analysis is pointed out. The adjoint method was used to derive sensitivity information for the layered reinforcement concrete shell elements. A computational system developed allows to solve large realistic reliability problems. The reliability study of a reinforced concrete nuclear containment shell is carried out. Reliability studies show which of parameters have the highest impact on reliability of the vessel.

1. INTRODUCTION

Design of advanced structural systems should account for geometric, material and loading parameters as random variables. Furthermore, the reliability has to be evaluated as the most objective structural safety measure. Computational time for reliability analysis of realistic structural problems is very high. Improvements in efficiency are critical to allow solution of large realistic problems.

The reliability analysis is usually performed using approximate First Order Reliability Method (FORM). Iterative solution procedures of FORM require substantial design sensitivity computations of high accuracy. The design of realistic structures requires computer-based numerical procedures, such as finite element analysis. For these structures, sensitivity is not available explicitly in terms of design variables. The most intensive computational task of design sensitivity computation is often carried out quite inefficiently by the finite differences method. Recently developed, the continuum and discrete methods of design sensitivity analysis are considered as much more efficient and accurate than the finite differences. This paper shows that integration of discrete sensitivity analysis method with reliability analysis may produce an efficient system allowing to solve large realistic design problems. The paper defines requirements for design sensitivity information for reliability analysis. The way of coupling reliability computation with design sensitivity analysis is pointed out.

A reliability study of the containment structure is performed. Componental reliability indices and the design derivatives of the indices with respect to design parameters are calculated. Reliability studies show which of parameters have the highest impact on reliability of the containment.

2.RELIABILITY ANALYSIS

The first step in evaluating the reliability of a structure is usually the identification of a number of variables by which the uncertainties related to the reliability of the structure can be described satisfactorily. They are called basic variables and are modelled as random variables or, if necessary as stochastic processes. Let $\mathbf{X} = \{X_1, X_2, \ldots, X_n\}^T$ be the vector of random variables. Specific realization of the vector $\mathbf{X}$ is denoted by $\mathbf{x} = \{x_1, x_2, \ldots, x_n\}^T$. It is useful to consider the variable $\mathbf{x}$ as a point in an n-dimensional basic variable space Ω. The number of basic variables n is assumed finite. For a given set of realizations of random variables, $\mathbf{x}$, it is assumed that it is possible to determine whether the structure is in a failure state or in a safe state. In other words, the random variable space Ω is divided into two sets called the failure region Ω_f and the safe domain Ω_s. The line separating the two domains Ω_f and Ω_s is called the failure surface and is described by a failure function $g(\mathbf{x}) = 0$ defined in such a way that positive value of g indicates the safe set of random variables (the safe domain) and non-positive value of g a failure set of random variables (the failure domain), i.e.

$$\begin{aligned} g(\mathbf{x}) &> 0 \quad \textit{when } \mathbf{x} \in \Omega_s \\ g(\mathbf{x}) &< 0 \quad \textit{when } \mathbf{x} \in \Omega_f \; . \end{aligned} \tag{1}$$

The probability of failure describes the probability that the limit (failure) state will be attained, i.e. $g(\mathbf{x})$ will be less or equal zero. This is given by

$$p_f = P[g(\mathbf{X}) < 0] = \int_{g(\mathbf{X})<0} f_X(\mathbf{x})dx \tag{2}$$

where $f_X(\mathbf{x})$ is the joint probability density function of $\mathbf{X}$. The integration is performed over the failure region Ω_f. If some of the basic random variables are discrete, the integration over the corresponding densities is substituted by a summation over finite probabilities. The reliability analysis aims at evaluation of the multidimensional integral in Eqn (2). Only a few analytical and exact results are known. Standard numerical integration techniques are generally not feasible for high-dimensional problems, and in general, either Monte Carlo simulation (MCS), or the analytically based first– and second– order reliability methods (FORM/SORM) must be used. The methods, MCS and FORM/SORM should be considered as complementary methods. For example, if the basic variables are discrete, the necessary transformation between the physical x–space and the standard normal u–space does not exist and MCS methods must be used. On the other hand, for the large class of engineering problems, where FORM/SORM do apply and p_f is very small $(10^{-4} - 10^{-8})$, FORM/SORM is generally preferable to MCS. For this reason, in this paper only the FORM method is discussed. Commercial software with the computation methods implemented is available, see for instance Ref. 7. The failure function g is often specified as a set of components, and these components can be arranged into systems. For example, the state of the i–th component is described by the failure function $g_i(x)$, such that failure is defined by $g_i(x) < 0$, and $g_i(x) > 0$ identifies a safe state of the component. The basic system representations comprise series systems, parallel systems, series systems of parallel subsystems, and parallel systems of series subsystems. Since the evaluation of the system reliability can be viewed as a post-processing which uses the componental probabilities of failure, we will concentrate on the methods for componental reliability. Problems of system reliability are discussed for example in Refs 3, 8. In the FORM method, the failure function is linearized about a point which lies on the failure surface and corresponds to the maximum likelihood of failure occurrence, Ref. 6. This point is known as the most probable failure point (MPP). The reliability is measured through the reliability index β, which is defined as the distance of MPP to the origin. Although this method requires iterations to locate MPP and thus is more expensive, it is more accurate and its results are invariant to the form of the failure function. The FORM method uses the space of standard independent and normally distributed design variables $\mathbf{u}$ obtained from the basic uncertain variables by the transformation $\mathbf{X} = T(\mathbf{U})$ (e.g. the Rosenblatt transformation). In FORM method the failure function $g(\mathbf{u}) = 0$ is expanded to the first order at the β-point. This is defined as the point on $g(\mathbf{u}) = 0$ at the shortest distance from the origin of the coordinate system to the linearized failure surface in the standardized normal space. Reliability index corresponding to the β-point is evaluated by the minimization problem

$$\beta = ||\mathbf{u}^*|| = min||\mathbf{u}|| \\ subject\ \ to\ \ g(\mathbf{u}) = 0 \tag{3}$$

The β-point should be defined in the normal space. Rackwitz and Fiessler, Ref. 9, proposed a probability distribution transformation for arbitrarily distributed, independent vectors into normal vectors and found a suitable search algorithm for the β-point.

3. REQUIREMENTS FOR DESIGN SENSITIVITY INFORMATION

Application of complex reliability models in design/optimization problems requires that effective and accurate sensitivity analysis of the reliability estimates with respect to design parameters can be performed. Especially, if general nonlinear optimization algorithms are used high precision sensitivity coefficients are needed in order to assure convergence. Since fully analytical design sensitivity analysis cannot be applied to general, realistic structural design problems, the finite differences method is usually proposed. This leads to very low computational efficiency as well as unreliable numerical accuracy. Recently developed discrete and continuum methods of design sensitivity analysis have shown to be far more efficient and accurate than finite differences, while being equally general. This paper presents strategies for integration of reliability analysis with discrete design sensitivity analysis. Solution of Eqn (3) requires computation of the first derivative of the failure function with respect to the standard variables $\mathbf{u}$. The potential failure mode is described by the failure function

$$g\{(x(\mathbf{u}), \Psi[x(\mathbf{u})]\} = 0 \tag{4}$$

where Ψ is a general performance measure, e.g., a stress or a displacement component. The first derivative of g with respect to standard variables $\mathbf{u}$ can be written as:

$$\frac{dg}{d\mathbf{u}} = \frac{\partial g}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial g} + \frac{\partial g}{\partial \Psi}\frac{\partial \Psi}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}} \,. \tag{5}$$

The derivatives $\partial g/\partial \Psi$ and $\partial g/\partial \mathbf{x}$ are usually computed analytically or numerically. The advanced reliability codes compute the derivatives $\partial \mathbf{x}/\partial \mathbf{u}$ internally from the transformation $\mathbf{x} = T(\mathbf{u})$. The most difficult and time consuming task is to compute derivatives $\partial \Psi/\partial \mathbf{x}$. This should be performed by highly efficient methods of the design sensitivity analysis, Ref. 3. To compute derivatives of the reliability index β with respect to distribution parameters $\mathbf{x}^{\mu}$ and $\mathbf{x}^{\sigma}$ (mean value and standard deviation, respectively) as well as deterministic parameters $\mathbf{x}^{d}$ the following formulae should be used

$$\frac{d\beta}{d\mathbf{x}^i} = \left|\left|\frac{\partial g}{\partial \mathbf{u}}\right|\right|^{-1} \frac{\partial g}{\partial \mathbf{x}}\frac{\partial T}{\partial \mathbf{u}^i} \quad for\ \ \mathbf{x}^i = \mathbf{x}^{\mu}, \mathbf{x}^{\sigma} \tag{6}$$

where gradients $\partial g/\partial \mathbf{x}$, $\partial g/\partial \mathbf{u}$ and $\partial T/\partial \mathbf{x}^i$ are known from the reliability computations, whereas in the case of deterministic parameters $\mathbf{x}^d$

$$\frac{d\beta}{d\mathbf{x}^d} = \left|\left|\frac{\partial g}{\partial \mathbf{u}}\right|\right|^{-1} \frac{\partial g}{\partial \mathbf{x}^d} + \frac{\partial g}{\partial \Psi}\frac{\partial \Psi}{\partial \mathbf{x}^d} \tag{7}$$

where the derivatives $\partial \Psi/\partial \mathbf{x}^d$ should be computed by the efficient methods of design sensitivity analysis, Ref. 3.

4. DESIGN SENSITIVITY ANALYSIS OF REINFORCED CONCRETE SHELLS

The computational aspects of the DSA are given in Refs 1,2. For discretization of the nuclear reactor containment the layered shell elements of Ahmad, Ref. 10, 11, has been used. The stiffness matrix **K** is obtained by numerical integration according to the Simpson rule along the thickness and according to the 4 or 9 point Gauss rule on the boundary. For the reinforcement, an equivalent steel sheet layers are used with uniaxial stress field assumption. The influence of the reinforcement is encountered in the total stiffness matrix **K**.

For the design sensitivity analysis the adjoint discrete method has been used, Ref. 1. The equilibrium equation is written as

$$\mathbf{K}\mathbf{q}(\mathbf{x}^{\alpha}) = \mathbf{Q}(\mathbf{x}^{\alpha}) \tag{8}$$

where **K** is the stiffness matrix, **q** is the displacement vector, and **Q** is the external load vector. All these quantities depend on design parameters $\mathbf{x}^{\alpha}$ comprising deterministic parameters $\mathbf{x}^{d}$ and mean values $\mathbf{x}^{\mu}$ of stochastic parameters **X**. Sensitivity of the performance measure Ψ, where $\Psi = \Psi(q(\mathbf{x}^{a}), \mathbf{x}^{a})$, with respect to the design parameter $\mathbf{x}^{a}$ can be written as

$$\frac{d\Psi}{d\mathbf{x}^{a}} = \frac{\partial\Psi}{\partial\mathbf{x}^{a}} - \boldsymbol{\lambda}^{T}\left(\frac{\partial\mathbf{K}}{\partial\mathbf{x}^{a}}\mathbf{q} - \frac{\partial\mathbf{Q}}{\partial\mathbf{x}^{a}}\right) \tag{9}$$

where $\boldsymbol{\lambda}$ is obtained from the adjoint equation

$$\mathbf{K}\boldsymbol{\lambda} = \left(\frac{\partial\Psi}{\partial\mathbf{q}}\right)^{T} . \tag{10}$$

As design parameters for the shell element considered the Young modulus, thickness of the reinforcement layer and its position with respect to the midsurface are taken. As performances the stress and displacements are considered. The stiffness matrix derivatives are given in Refs 12, 13.

5. RELIABILITY STUDY OF THE CONCRETE CONTAINMENT SHELL

In the unlikely event of a severe accident, the pressure inside a containment building may significantly exceed the postulated design accident pressure. Since the containment structure provides the last structural barrier against leakage of radioactive material into the environment in the event of beyond–design–basis accident, knowledge of performance of the containment structure to internal pressure and temperature transients associated with the accident is essential to determine probabilities of failures.

The reliability analysis (RA) associated with reliability sensitivity analysis (RSA) constitutes an effective investigative tool to supplement but not at all to replace the traditional deterministic approach. The RA/RSA not only grants a quantitative perspective on plant safety but also provides a more balanced, objective and realistic picture. The most important results provided are engineering insights, i.e. the identification of vulnerabilities to a high safety level of the plant, sensitivities of reliability measures with respect to design parameters and the assessment of alternative means for improvements.

This paper presents a reliability study of a reinforced concrete nuclear containment shell. The structure is modelled by 640 isoparametric, layered, composite shell finite elements, see Fig. 1. The number of degrees of freedom is about 12500. The structure consists of the cylinder (radius 40 m) and the dome (radius 40 m). The height of the structure is 64 m. The reinforcement distribution is taken into consideration.

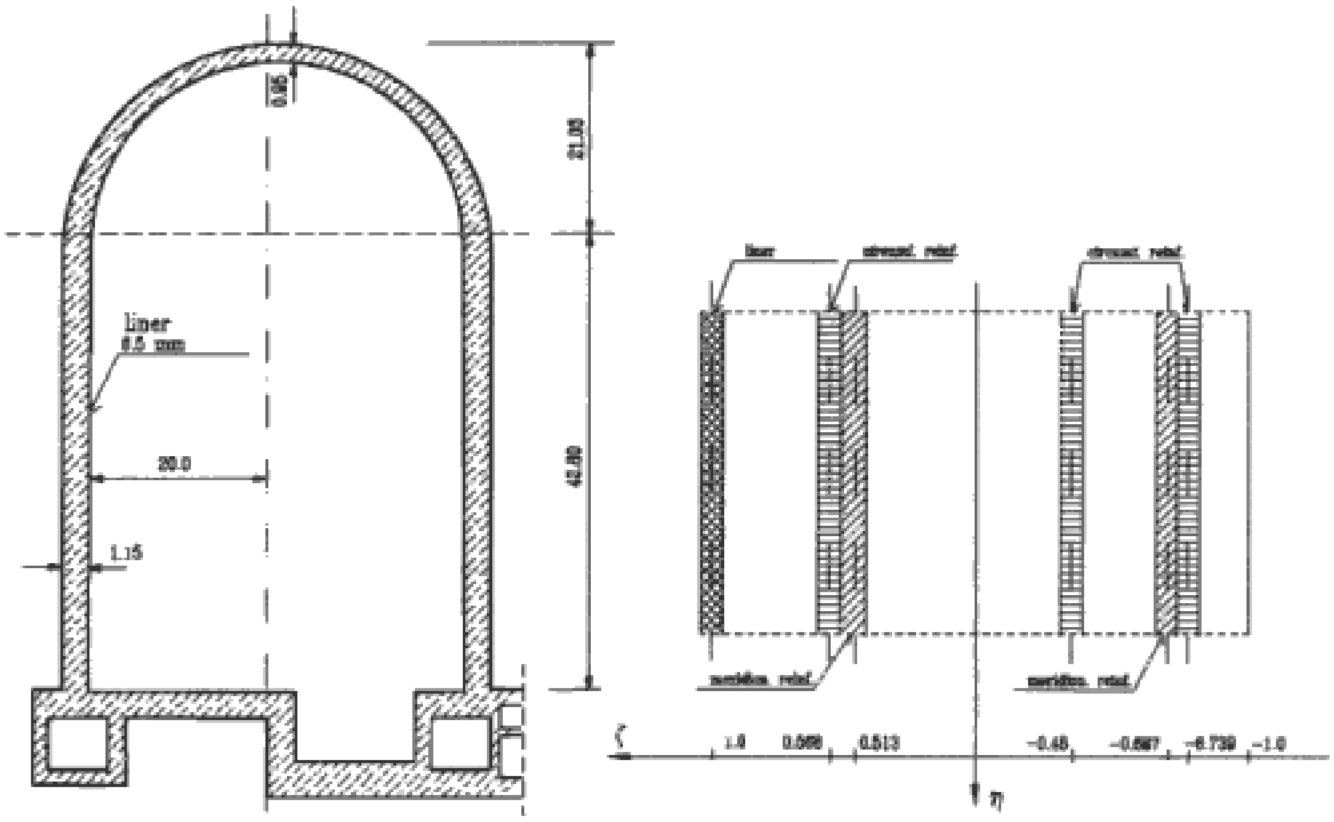

Figure. 1. The vertical cross-section of the structure.

The cylindrical and dome parts of the containment are prestressed by a system of tendons. The action of tendons was approximated by an external pressure. The ultimate load combination introduced for containments usually requires a demonstration that the primary containment has the capacity to withstand a hypothetical internal pressure loading of at least twice the design pressure. It was assumed that the pressure loading obeys the Frechet distribution.

As variable design parameters the mean values of Young moduli of equivalent steel layers, thicknesses and distances of reinforcement from the midsurface of the shell are considered, see Tab. 1. The reinforcement placement and the deterministic data are shown in Fig. 2 and Tab. 2, respectively.

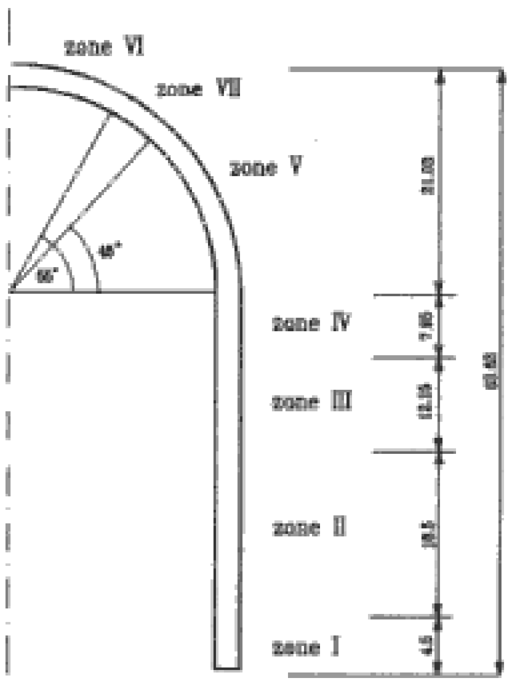

Figure 2. Reinforcement zones.

Table 1 Parameters of the vessel.

parameter	distribution	mean value	CoV
pressure	Frechet	800	0.2
thickness of the wall	N	1.15	0.02
Young modulus of concrete	N	3.1E+5	0.05
thickness of particular steel layers	LN		0.05
Young moduli of particular steel layers	N	2.1E+6	
dist. of particular steel layers	N		0.1

As the potential failure mode the gross structural failure of the containment structure is considered. Results of extensive experiments that have been carried out for the reinforced concrete cylinders loaded with cyclic loads show that the collapse is caused by failure of concrete in the bottom of the cylindrical wall, after the yield of reinforcement. From the deterministic studies the critical horizontal displacement q^* in the middle of the cylindrical wall is assumed and the associated actual displacement q taken as a representative of the overall containment deformation. Figure 3 shows design sensitivity of the selected displacement with respect to the parameters of external circumferential reinforcement.

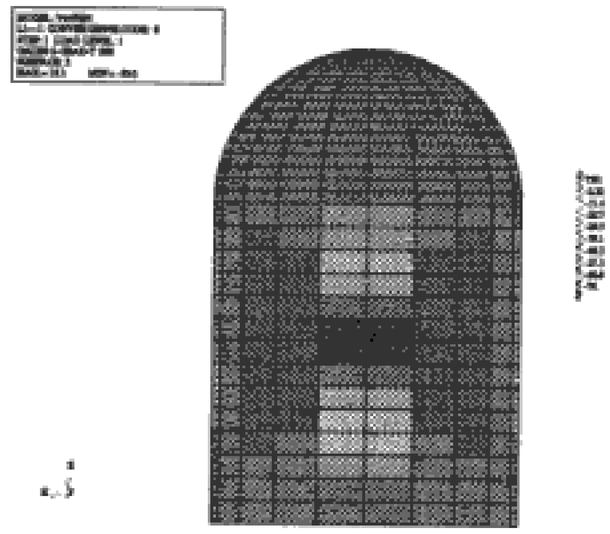

Figure 3. Design sensitivity of the selected displacement with respect to the thickness (as well as Young modulus) of external circumferencial reinforcement

The global failure function is described by an excessive displacement

$$g(x) = q^* - q \ . \tag{11}$$

To perform the reliability study of the structure, the reliability analysis code COMREL, Ref. 6, has been integrated with the finite element code POLSAP-RC for analysis nonlinear analysis of reinforced concrete shells. Design sensitivity capabilities has been added, Refs 12, 13.

The componental reliability index corresponding the failure function Eqn (11) as well as sensitivities of reliability index with respect to design parameters has been computed. Additionally, for reliability studies, these sensitivity values were normalized into so called elasticities. Moreover, so called α-sensitivities were computed indicating importance and influence of random variables for the obtained reliability index. Reliability studies show which of design parameters have the highest impact on reliability of containment shell. Detailed results will be shown at time of the conference.

Table 2. Reinforcement parameters.

Zone	Distance from the midsurface	Equivalent thickness m^2	Description
I	1.000	0.650E−2	liner
	−0.739	0.761E−2	circ. ext.
	0.513	0.264E−2	circ. internal
	−0.687	0.940E−2	meridional ext.
	0.565	0.974E−2	meridional internal
	−0.480	0.900E−2	prestressed (circ.)
II	1.000	0.650E−2	liner
	−0.739	0.761E−2	circ. ext.
	0.513	0.264E−2	circ. internal
	−0.687	0.530E−2	meridional ext.
	0.565	0.309E−2	meridional internal
	−0.480	0.900E−2	prestressed (circ.)
III	1.000	0.650E−2	liner
	−0.739	0.761E−2	circ. ext.
	0.513	0.264E−2	circ. internal
	−0.687	0.530E−2	meridional ext.
	0.565	0.193E−2	meridional internal
	−0.480	0.900E−2	prestressed (circ.)
IV	1.000	0.650E−2	liner
	−0.739	0.761E−2	circ. ext.
	0.513	0.264E−2	circ. internal
	−0.687	0.414E−2	meridional ext.
	0.565	0.193E−2	meridional internal
	−0.480	0.900E−2	prestressed (circ.)
V	1.000	0.650E−2	liner
	−0.725	0.471E−2	circ. ext.
	0.537	0.275E−2	circ. internal
	−0.725	0.448E−2	meridional ext.
	0.537	0.217E−2	meridional internal
	−0.480	0.900E−2	prestressed (circ.)
VI	1.000	0.650E−2	liner
	−0.725	0.328E−2	circ. ext.
	0.537	0.204E−2	circ. internal
	−0.725	0.328E−2	meridional ext.
	0.537	0.204E−2	meridional internal
	−0.480	0.900E−2	prestressed (circ.)
VII	1.000	0.650E−2	liner
	−0.725	0.328E−2	circ. ext.
	0.537	0.275E−2	circ. internal
	−0.725	0.328E−2	meridional ext.
	0.537	0.217E−2	meridional internal
	−0.480	0.900E−2	prestressed (dome)

6. CONCLUSIONS

This paper proposed a way of integration of the discrete design sensitivity analysis with the reliability analysis and the reliability sensitivity analysis for significant improvement of the computational efficiency. Requirements for design sensitivity information were defined. The adjoint method of discrete design sensitivity analysis was used to derive sensitivity information for the layered reinforcement concrete shell elements. The computational system developed allows to solve large realistic reliability problems. The reliability study of a reinforced concrete nuclear containment building was carried out. Reliability studies showed which of parameters had the highest impact on reliability of the containment.

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