# NUMERICAL INVESTIGATION OF SOME STEADY STATE WEAR PROBLEMS

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## 1. Introduction

In many practical industrial applications it is very important to predict the form of wear shape, contact stresses. Usually, the contact shape evolution is simulated numerically by integrating the wear rate expressed in terms of relative slip velocity and contact pressure. A steady state is then predicted by the incremental integration procedure with account for contact shape and pressure variation. However, much more effective procedure can be developed by postulating minimization of the contact response functional. It was shown in the previous our papers [1-2] that the total wear dissipation power at the contact interface provides the steady wear regimes by applying the stationary conditions. In the later case the stationary of the functional gives the contact stress distribution and the rate of the rigid body movement.

In the work by Páczelt and Mróz [1-2] the optimal shapes generated by wear process were analyzed by postulating minimization of the wear dissipation power. It was shown that the contact shape evolution tends to a steady state satisfying the minimum principle of the wear dissipation rate.

The specific modified Archard wear rule is assumed for wear rate in normal direction on contact surface. Very important, that in general contact conditions the vector of wear rate is not normal to the contact surface and has tangential component. A fundamental assumption is now introduced, namely, *at the steady state the wear rate vector is collinear with the rigid body wear velocity of body which has rigid body like displacement.* It is demonstrated that the wear dissipation power at the contact surface is minimal in the steady state of the wear process and in many cases corresponds to the uniform wear rate. In the normal direction the Signorini contact conditions are valid. The Coulomb dry friction models are investigated. The temperature effects and heat generated at the frictional interface in our investigation is considered.

It is assumed that the displacements and deformations are small, the material of the contacting bodies are elastic. The discretization of the contacting bodies was performed by the displacement based on *p*-version of finite elements [2] assuring fast convergence of the numerical process and accurate specification of geometry for shape optimization.

## 2. Contact optimizations problems

Without the restriction of generality, let us consider the contact problem of two elastic bodies $B_\alpha, (\alpha = 1{,}2)$ with the usual boundary and loading conditions. The boundary portion $S_c^{(\alpha)}$ will be called the potential zone of contact. In this part of the bodies the shape may be modified. In the normal direction the Signorini contact conditions are valid. The Coulomb dry friction models are investigated. In the analysis of wear problem, usually the elastic portion of relative tangent velocity is much smaller than the rigid body motion induced velocity, thus the effect of elastic component of tangent relative velocity can be neglected in the wear analysis. The temperature effects and heat generated at the frictional interface in our investigation is considered [3]. The contact conditions are checked at the Lobatto integration points of the contact elements during the solution process.

Assume the isotropic wear rule in the form [1]

$$\dot{w}_i = \beta_i (\tau_n)^{b_i} \|\dot{\boldsymbol{u}}_\tau\|^{a_i} = \beta_i (\mu p_n)^{b_i} \|\dot{\boldsymbol{u}}_\tau\|^{a_i} = \beta_i (\mu p_n)^{b_i} v_r^{a_i} = \widetilde{\beta}_i p_n^{b_i} v_r^{a_i}, \quad i = 1{,}2$$

The material parameters $\beta_i, a_i, b_i$ specify the wear rates of two contacting bodies and $\widetilde{\beta}_i = \beta_i \mu^{b_i}$, $v_r = \|\dot{\boldsymbol{u}}_\tau\|$ is the relative velocity between two bodies, $\mu$ is the coefficient of friction. In general contact conditions the vector of wear rate is not normal to the contact surface and has tangential components. This vector specifies the shape transformation and tangential motion of the worn material. To analyze this transformation, let us define first the contact stress of interaction of bodies $B_1$ and $B_2$, thus

$$\boldsymbol{p} = \boldsymbol{p}_1 = -\boldsymbol{p}_2 = -p_n(\boldsymbol{n}_c \pm \mu \boldsymbol{e}_{\tau 1}) - \mu_d \, p_n \, \boldsymbol{e}_{\tau 2} = -p_n \widetilde{\boldsymbol{n}}_c ,$$

where $\mu$ is the friction coefficient specifying the shear stress in sliding direction and $\mu_d$ is the friction coefficient associated with transverse wear velocity. The unit vectors $\boldsymbol{e}_{\tau 1}, \boldsymbol{e}_{\tau 2}, \boldsymbol{n}_c$ constitute the local reference triad on $S_c$. Here $\boldsymbol{n}_c$ is the unit normal to the contact surface of body $B_1$, $\boldsymbol{e}_{\tau 1}$ is the tangent unit vector coaxial with the sliding velocity and $\boldsymbol{e}_{\tau 2}$ is the transverse tangent unit vector.

A fundamental assumption is now introduced, namely, *at the steady state the wear rate vector is collinear with the rigid body wear velocity of* $B_1$, so that

$$\dot{\boldsymbol{w}}_R = \dot{\boldsymbol{w}}_{1,R} + \dot{\boldsymbol{w}}_{2,R} = \dot{w}_R \, \boldsymbol{e}_R , \text{ where } \boldsymbol{e}_R = \frac{\dot{\boldsymbol{\lambda}}_F + \dot{\boldsymbol{\lambda}}_M \times \Delta \boldsymbol{r}}{\left\| \dot{\boldsymbol{\lambda}}_F + \dot{\boldsymbol{\lambda}}_M \times \Delta \boldsymbol{r} \right\|} .$$

The generalized wear dissipation power for the case of wear of two bodies

$$D_w^{(q)} = \sum_{i=1}^{2} \left( \int_{S_c} (\boldsymbol{p}_i \cdot \dot{\boldsymbol{w}}_i)^q \, dS \right)^{1/q} = \sum_{i=1}^{2} C_i^{1/q} \qquad ,$$

where $q$ is the control parameter , usually $q \geq 0$. Assume that the contact pressure $p_n(\mathbf{x})$ and the friction induced shear stress $\tau_n = \mu \, p_n(\mathbf{x})$ satisfy the global equilibrium conditions for the body $B_1$, so we have $\boldsymbol{f} = \boldsymbol{0}, \boldsymbol{m} = \boldsymbol{0}$. The Lagrangian functional at $b = b_1 = b_2$ is

$$L_{D_w}{}^{(q)} = L_{D_w}{}^{(q)}\left(p_n , \dot{\boldsymbol{\lambda}}_F , \dot{\boldsymbol{\lambda}}_M\right) = D_w{}^{(q)}(p_n) + (b+1)\, \dot{\boldsymbol{\lambda}}_F \cdot \boldsymbol{f} + (b+1)\, \dot{\boldsymbol{\lambda}}_M \cdot \boldsymbol{m}$$

and satisfying the stationary condition of the Lagrange functional, the contact pressure distribution has the next form

$$p_n = \left( \frac{\dot{\boldsymbol{\lambda}}_F \cdot \widetilde{\boldsymbol{n}}_c + \left(\dot{\boldsymbol{\lambda}}_M \times \Delta \boldsymbol{r}\right) \cdot \widetilde{\boldsymbol{n}}_c}{\left[ \left(\widetilde{\beta}_1 \, v_r^{a_1}\right)^q C_1^{\frac{1-q}{q}} + \left(\widetilde{\beta}_2 \, v_r^{a_2}\right)^q C_2^{\frac{1-q}{q}} \right]} (1 \mp \mu \tan \chi)^{-q} \right)^{\frac{1}{(b+1)q-1}}$$

where $\chi$ is the angle between $\boldsymbol{n}_c$ and $\boldsymbol{e}_R$. The given non-linear equations can be solved by applying Newton-Raphson technique. Minimization of this functional with equilibrium constraints for body $B_1$ at $q$=1 gives results for steady state wear process of arbitrary shape of contact surface

Some specific examples will be presented. It is shown that the thermal distortion effects essentially the optimal contact shape associated with the steady state

## 3. References


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