ON THE APPLICATION OF THE SENSITIVITY ANALYSIS TO THE DESIGN OF SPATIAL BAR STRUCTURES

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The paper deals with some aspects of structural sensitivity analysis and its application in the engineering practice. The considerations are illustrated by numerical results concerning an industrial steel room modelled as a 3-D structure.

1. Introduction

Many failures and malfunctions of typical steel shops and roofs which have been designed over the years (for some examples see Nawrot et al [1], [2], Łosicki, Niedostatkiewicz, Szymczak [3]) are of necessity the reason for taking a closer look at the problem. To better understand the behaviour of such structures it seems useful to perform certain nonstandard structural analyses of the systems. For instance, in the paper by Bródka, Garncarek and Grudka [4] a structure of a steel shop was considered to be a 3-*D* system. Such analyses will be considered as nonstandard ones that are not a part of the everyday practice of design offices and consulting firms. Sensitivity analysis is also an example of such nonstandard procedures.

In the sensitivity analysis a variability of some functionals is investigated that characterize the behaviour of structural systems and depend on a number of design variables. All the magnitudes which affect the structural behaviour of a system under consideration can serve as the design variables, for instance the cross-sectional areas and lengths of particular elements, Young's moduli of used materials etc. These functionals can in general depend on the current states of displacements and stresses as well as on their admissible values called the design constraints. One of the first papers on the sensitivity analysis was written by Campbell and Zienkiewicz [5] followed over recent years by an increasing number of papers devoted to this subject. Relatively many papers have been published in the aeronautical periodicals (Arora, Cardoso [6], Haftka and Mróz [7]). The sensitivity analysis has found its main applications in optimization problems, though. Algorithms for the calculations of the sensitivity of inelastic structures were programmed in the ADINA system (Arora, Haririan, Ryu, Wu [8], Arora, Cardoso, Haririan [9], Arora, Cardoso [10]). However, some workers in the field maintain that it is rather difficult to appropriately interpret and employ the results of the sensitivity analyses in the design practice. The goal of this paper is to indicate suitability of such analyses in the state-of-the-art structural design and inspection, in particular, of steel structural systems.

In this paper a linear sensitivity analysis of bar structures subjected to static loads is presented under the constraints imposed on both the nodal displacements and the elements stresses; comprehensive aspects of computer implementation is discussed.

Suitable algorithms have been prepared and programmed in the POLSAP system (Hien, Kleiber [11]) and all the computations have been made in this system. An example will be presented how to analyse the sensitivity of a certain steel structure of a roof. The approach is hoped to be useful for the structural designers of real systems.

2. Formulation of the problem

In the displacement model of the finite element method (Zienkiewicz [12], Kleiber [13]) a structure is represented by means of its stiffness matrix, loading and nodal displacement vectors as well as its boundary conditions. In the sensitivity analysis it is, in addition, a function characterizing the structural behaviour that enters the picture together with certain constraints imposed by the designer. These constraints can be expressed in terms of displacements or stresses and thus are related to the limit states of serviceability and load-carrying capacity of structures as given in suitable design codes [14] as well as in recommendations and guides (Filipowicz, Łubiński, Żółtowski [15]). Some constraints may also follow from reasons other than structural ones, e.g. manufacturing requirements.

The structural response functions can be defined as

$$\Phi = \Phi[\mathbf{r}(\mathbf{b}), \mathbf{b}], \tag{2.1}$$

where $\mathbf{b}$ is an E-dimensional vector of the design variables, $\mathbf{r}$ is a N-dimensional vector of generalized displacements which depends on these design variables in an implicit way (Choi, Haug, Komkov [16]). The sensitivity analysis is aimed at the investigation of variability of the function (2.1) caused by the variations in the design parameters. Differentiation of Eq. (2.1) with respect to $\mathbf{b}$ yields the sensitivity gradient coefficients of Φ in the form

$$\underset{1\times E}{\frac{d\Phi}{d\mathbf{b}}} = \underset{1\times E}{\frac{\partial\Phi}{\partial\mathbf{b}}} + \underset{1\times N N\times E}{\frac{\partial\Phi}{\partial\mathbf{r}}\frac{d\mathbf{r}}{d\mathbf{b}}}. \tag{2.2}$$

For structures made of elastic material and undergoing small displacements the system of FEM equations has the general form

$$\underset{N\times N}{\mathbf{K}}(\mathbf{b})\,\underset{N\times 1}{\mathbf{r}}(\mathbf{b}) = \underset{N\times 1}{\mathbf{R}}(\mathbf{b}), \tag{2.3}$$

where $\mathbf{K}$ stands for the structural stiffness and $\mathbf{R}$ denotes the loading vector; both $\mathbf{K}$ and $\mathbf{R}$ depending on the design variable vector $\mathbf{b}$. The system (Eq. 2.3) is assumed to satisfy the kinematical boundary conditions. Assuming that the overall configuration of structure is fixed let us now calculate the derivatives of the generalized displacements $\mathbf{r}$ with respect to the design variables $\mathbf{b}$. Differentiating the Eq. (2.3), we get

$$\underset{N\times N}{\mathbf{K}}\underset{N\times E}{\frac{d\mathbf{r}}{d\mathbf{b}}}=\underset{N\times E}{\frac{\partial \mathbf{R}}{\partial \mathbf{b}}}-\underset{N\times E}{\frac{\partial \mathbf{K}}{\partial \mathbf{b}}}\underset{N\times 1}{\mathbf{r}}. \tag{2.4}$$

Resulting from Eq. (2.4) displacement derivatives with respect to design variables and substituting the resulting equations into Eq. (2.2) the expression for the sensitivity coefficients is obtained as

$$\underset{1\times E}{\frac{d\Phi}{d\mathbf{b}}}=\underset{1\times E}{\frac{\partial \Phi}{\partial \mathbf{b}}}+\underset{1\times N}{\frac{\partial \Phi}{\partial \mathbf{r}}}\underset{N\times N}{\mathbf{K}^{-1}}\left(\underset{N\times E}{\frac{\partial \mathbf{R}}{\partial \mathbf{b}}}-\underset{N\times N\times E}{\frac{\partial \mathbf{K}}{\partial \mathbf{b}}}\underset{N\times 1}{\mathbf{r}}\right). \tag{2.5}$$

The above manner to compute the sensitivity of a structure is called the direct differentiation method.

Another approach, connected with the so-called adjoint variables and consequently adjoint structure, will also be shown and used in considerations to follow. The reason for this is an intuitively easy interpretation of this approach for the structural designer. Namely, he shall deal with an identical structure subjected to an adjoint load determined from the design constraints. Let the term in front of parentheses in Eq. (2.5) be denoted by λ,

$$\underset{1\times N}{\lambda^T}=\underset{1\times N}{\frac{\partial \Phi}{\partial \mathbf{r}}}\underset{N\times N}{\mathbf{K}^{-1}}. \tag{2.6}$$

As the stiffness matrix K is symmetric and positive definite, the above relationship can be transformed to become

$$\underset{N\times N}{\mathbf{K}}\underset{N\times 1}{\lambda}=\underset{N\times 1}{\left(\frac{\partial \Phi}{\partial \mathbf{r}}\right)^T}, \tag{2.7}$$

where λ is a vector of the so-called adjoint displacements. This implies that

$$\underset{1\times E}{\frac{d\Phi}{d\mathbf{b}}}=\underset{1\times E}{\frac{\partial \Phi}{\partial \mathbf{b}}}+\underset{1\times E}{\lambda^T}\left(\underset{N\times E}{\frac{\partial \mathbf{R}}{\partial \mathbf{b}}}-\underset{N\times N\times E}{\frac{\partial \mathbf{K}}{\partial \mathbf{b}}}\underset{N\times 1}{\mathbf{r}}\right). \tag{2.8}$$

Consider a linear function describing the structural behaviour

$$\Phi=\frac{|r|}{r^a}-1\leqslant 0, \tag{2.9}$$

where $r^a>0$ is a prescribed admissible displacement. So, adjoint load can be expressed as

$$\underset{1\times E}{\frac{\partial \Phi}{\partial \mathbf{r}}}=\operatorname{sign}(r)\left[0, \ldots, 0, \frac{1}{r^a}, 0, \ldots, 0\right]. \tag{2.10}$$

Consider a stress-dependent function to describe the behaviour of a structure, i.e. a function related to its ultimate limit state,

$$\Phi = \frac{\sigma}{\sigma^a} - 1 \leqslant 0, \tag{2.11}$$

where σ is a stress component on which a constraint is imposed and σ^a denotes a certain limit stress. For truss systems the limit stress is different for tension and compression and can be selected according to the current codes of practice.

Let us calculate a magnitude of adjoint load for an element of a truss in the presence of the function Eq. (2.11). The well-known relationship between the stresses and the nodal displacement $\mathbf{r}$ has the form

$$\underset{1\times 1}{\boldsymbol{\sigma}} = \underset{1\times 1}{\mathbf{D}}\;\underset{1\times 2}{\mathbf{B}}\;\underset{2\times 1}{\mathbf{r}}, \tag{2.12}$$

where $\mathbf{D}$ is the stiffness matrix and $\mathbf{B}$ is such a linear operator that $\mathbf{Br}$ denotes a corresponding strain. The relevant derivative with respect to displacements takes the form

$$\underset{1\times 2}{\frac{\partial \boldsymbol{\sigma}}{\partial \mathbf{r}}} = \underset{1\times 1}{\mathbf{D}}\;\underset{1\times 2}{\mathbf{B}}. \tag{2.13}$$

The adjoint load vector being the derivative of the stress-dependent function Φ with respect to displacements, becomes, using Eq. (2.13)

$$\underset{1\times 2}{\frac{\partial \Phi}{\partial \mathbf{r}}} = \operatorname{sign}(\sigma)\frac{1}{\sigma^a}\underset{1\times 1}{\mathbf{D}}\;\underset{1\times 2}{\mathbf{B}}. \tag{2.14}$$

Remembering the relationship Eq. (2.2) and calculating the adjoint variables, the sensitivity gradients of the structure considered can thus be evaluated.

3. COMPUTER IMPLEMENTATION

The two methods of evaluating the design sensitivity gradient lead to different procedures of their computer implementation (HIEN, KLEIBER [17]). The sensitivity analysis is usually made for a certain number L of loading patterns and for a certain number C of design constraints. In the direct differentiation method Eq. (2.3) must be solved for L loading cases and next each of the obtained loading vectors is used to calculate S right-hand sides of Eq. (2.4). After the derivatives $\mathbf{dr}/\mathbf{db}$ are found, the gradients $d\Phi/d\mathbf{b}$ can be calculated for each of C design constraints. It appears that the number of algebraic equations to be solved amounts to $L\,S$ where S is the dimension of vector $\mathbf{b}$.

The methods of adjoint variable requires the system of Eqs. (2.3) to be solved for $\mathbf{r}$ for L right-hand sides and, in addition, the Eq. (2.7) to be solved for $\boldsymbol{\lambda}$ for C design constraints. Finally, the magnitudes of revelant functions must be found with the help of Eq. (2.8).

It is worth noting that in the case of the response functions (Eqs. (2.9), (2.11)) both the primary and adjoint equations may be solved simultaneously. The global stiffness matrix must be triangularized only once. Determination of the adjoint loads for the function (2.9) reduces to the calculation of a reciprocal of the design constraint magnitudes and thus setting the right-hand side. Each design constraint leads to one vector of the right-hand side. Thus, the number of algebraic equations to be solved is clearly $L+C$. Adjoint loads can be introduced in the same way as the concentrated loads.

When the relevant function is related to the stress constraint Eq. (2.11), the vectors of adjoint loads should be found in a different manner. The right-hand side vector is generated at the element level. The product **DB** is calculated at the element level and then divided by σ^a. This magnitude is obtained in the local coordinates and then transformed into the global system also at the element level. Composition of the right-hand side vectors is not necessary. It suffices to locate the adjoint load components in the right-hand side vectors by using the element allocation vectors in which the occurrence of the design constraint was shown.

4. Short description of POLSAP system

System POLSAP constitutes a considerable extension of the known program SAP-IV (Bathe, Peterson, Wilson [18]) that enables five types of analyses to be made. In POLSAP as many as fourteen types of analyses are incorporated to choose from. The system enables arbitrarily large spatial complex structures to be analysed. Both static and dynamic situations can be dealt with as well as the problem of initial stability solved. As far as the static case of sensitivity analysis is concerned, the following design variables can be used: areas of cross-sections, lengths of bars and beams, thicknesses of plates and shells, Young's moduli and densities of materials. Displacement and stress functions can serve as functionals to describe the behaviour of structures under consideration. Stochastic analysis of both static and dynamic systems is also possible by taking into account the geometric and material imperfections. Sensitivity of such systems can also be tackled.

The IBM-PC compatible microcomputer version of the system can deal with arbitrarily large problems, the only limitation being the storage capacity of disks.

5. Verification and interpretation of the results

The obtained results are compared with those of Choi, Haug, Komkov [16]). In this paper a ten-bar truss is analysed (Fig. 1). The truss constitutes a certain benchmark problem used in the verification of the results in the field of structural optimization (Arora, Haug [19]). Both displacement and stress design constraints are allowed for, cross-sectional area being the design variable. Young's modulus of all the bars is the same and amounts to $E=1.0\times 10^7$. Vertical deflection of the node 2 must not exceed $r^a=2.0$. Comparison of the results obtained with the use of POLSAP and those given in [16] are shown in Table 1. Stress constraint for the bar 5 was equal to 2.5×10^5. Corresponding results are given in Table 2.

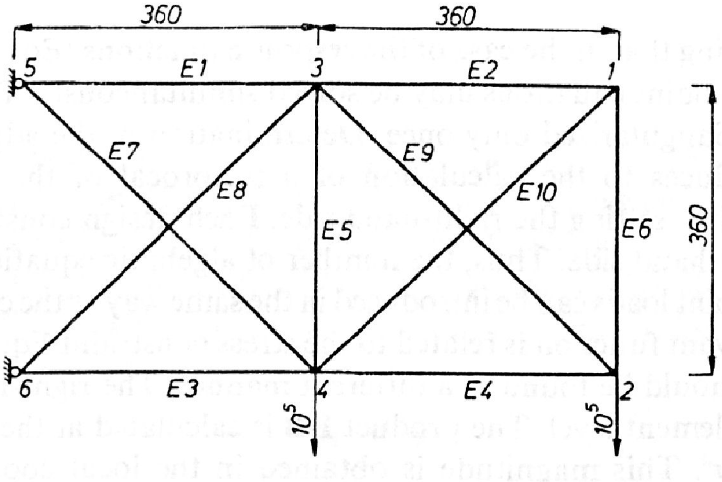

FIG. 1. Ten-bar truss.

Rys. 1. Kratownica 10-prętowa

Table 1

Displacement constraint at node 2, displacement sensitivity to variation in cross-sectional area.

Ograniczenie przemieszczeń w węźle 2, wrażliwość przemieszczeń na zmianę pola przekroju poprzecznego

element number	cross-sectional area	HAUG [16]	POLSAP
1	28.6	−0.0093	−9.2836024816D-03
2	0.2	0.0109	1.0897607678D-02
3	23.6	−0.0062	−6.2027963163D-03
4	15.4	−0.0076	−7.6322280077D-03
5	0.2	0.1402	1.4023301743D-01
6	0.2	0.0109	1.0897607678D-02
7	3.0	−0.0177	−1.7742779371D-02
8	21.0	−0.0128	−1.2775176317D-02
9	21.8	−0.0108	−1.0772705417D-02
10	0.2	0.0308	3.0823089150D-02

Table 2

Stress constraint in bar 5, stress sensitivity to variation in cross-sectional area

Ograniczenie naprężeń w pręcie 5, wrażliwość naprężeń normalnych na zmianę pola powierzchni przekroju poprzecznego

element number	cross-sectional area	HAUG [16]	POLSAP
1	28.6	−0.0082	8.2201212946D-03
2	0.2	−0.0696	−6.9587790882D-02
3	23.6	−0.0104	−1.0398823591D-02
4	15.4	−0.0006	−6.3777552420D-04
5	0.2	−2.3520	−2.3520587932D+00
6	0.2	−0.0696	−6.9587790882D-02
7	3.0	−0.8369	−8.368574147D-01
8	21.0	0.0231	2.3056270775D-02
9	21.8	0.0009	−9.0020474201D-04
10	0.2	−0.1968	−1.9682399528D-01

The notion „increase of the sensitivity" corresponds to an increasing absolute value of the derivative of the function that describes the behaviour of a structure. The physical interpretation of the fact is the following: to make any member of the structure less sensitive, for instance, in the case of cross-sectional area of an element serving as the design variable, its area should be increased in order to make the structure in this element less sensitive. In other words, in the case of a negative sign of sensitivity the decrease in displacements (or stresses) at the place where the design constraint is imposed must be accompanied by an increase in the cross-sectional area of that element for which the absolute value of the function characterizing its behaviour is at its greatest. From Table 1 it follows that, in order to diminish the vertical deflection of node 2, the cross-sectional areas of bars 1, 2, 3, 4, 7, 8, 9 should be made larger. The greatest absolute value of the displacement function derivative, hence a large sensitivity the truss, takes place in bar 7. Its cross-sectional area was increased by 50 per cent what resulted in the drop of the corresponding deflection by 3.5 per cent.

Table 2, corresponding to the stress sensitivity, shows that the truss is the most sensitive in the bar 5, whose cross-section should be increased in order to decrease its stresses. At the same time, it can be noticed that the decrease in its stresses can also be obtained by diminishing the cross-sectional area of the bar 8. Its section was decreased by 50 per cent and therefore the stress in the bar 5 was decreased by 18 per cent.

To sum up, the value of the gradient of the response functional is proportional to the effect caused by a unit change of the design variable in the place in which the design constraint is imposed.

The change in cross-sectional areas of an element may appear as a consequence of small damages during transportation, assembling and process of corrosion as well. The manufactured structural members delivered from factories have their tolerances in dimensions. The investigation of design sensitivity may answer the question whether a member which may be locally damaged is of crucial importance.

6. EXAMPLE OF ANALYSIS OF A 3-D STRUCTURE

6.1. DESCRIPTION OF STRUCTURE

The analysed roof consists of four main truss girders spanning 24 m and spaced at 6 m. The structural system is stiffened with two longitudinal bracings, diagonals in the plane of roofing and some additional strengthening bars as shown in Fig. 2. Geometry and loading pattern of the main girder is shown in Fig. 3. Concentrated loads result from steel purlins on which reinforced concrete slabs rest to transmit the selfweight and snow load. Some forces are also present at the lower chord nodes. The girders are simply supported. All rolled sections are shown in Table 3. Conventional stress and displacement analysis of the roof was made by NAWROT et. al [20].

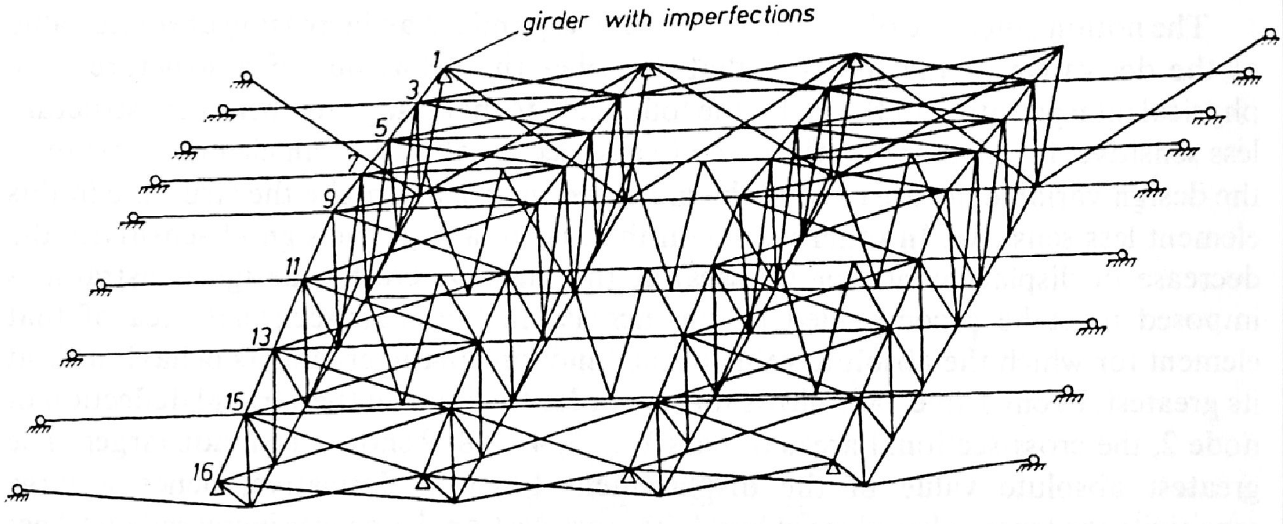

FIG. 2. Spatial structure of the roof.
Rys. 2. Przestrzenna konstrukcja hali

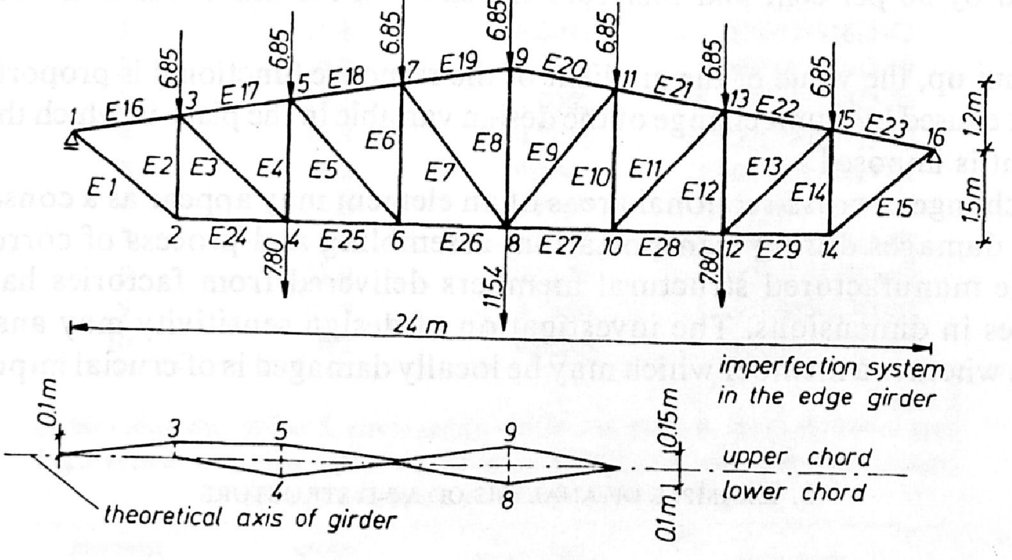

FIG. 3. Statical pattern of main girder with its imperfections.
Rys. 3. Schemat statyczny dźwigara głównego z imperfekcjami

6.2. COMPUTATIONAL MODEL

Design sensitivity analysis of the above described spatial structure was made under both displacement and stress constraints. To emphasize a number of aspects of the sensitivity analysis the structure was modelled as a frame for the displacement constraint and as a truss for the stress constraint. Hinged connections of longitudinal bracings and main girders are assumed. Full compatibility of the three displacement

Table 3

Catalogue of steel rolled sections.
Zestawienie profili walcowanych

Structural element	Symbol	Cross-sectional area m²	J_1 m⁴	J_2 m⁴	J_3 m⁴
upper chord	2[] 180E	1.44E-2	8.67E-8	1.23E-5	2.18E-5
lower chord	as above				
vertical bar	L90 × 90 × 8	1.40E-3	1.53E-8	4.36E-7	1.69E-6
edge diagonal bars	2JL100 × 75 × 8	2.70E-3	2.99E-8	2.70E-6	2.93E-5
diagonal bars	2LJ60 × 60 × 6	1.38E-3	1.73E-3	4.66E-7	7.83E-6
in-plane bracing	L60 × 60 × 6	6.91E-6	3.00E-9	1.50E-9	1.50E-9
upper and lower chord of bracings	[120 E	1.34E-3	2.34E-8	3.12E-7	3.04E-6
diagonal bars of bracings	L60 × 60 × 6	6.91E-6	1.53E-8	4.36E-7	1.68E-6
purlins	2[] 180E	1.44E-2	8.67E-8	1.23E-5	2.18E-5
strengthening bars	16	4.14E-5	–	–	–

components along x, y, z-axes of the node connecting the segments of purlins and the one of the girder where the purlin rests is also assumed.

To prevent the truss model from any rigid body motion some elastic supports are introduced at the nodes of the girders. Their compliances are assessed by assuming the presence of some equivalent beams spanning the nodes and subjected to unit concentrated forces. For example, the compliance of the equivalent support for the node 8 is found by calculating a deflection, caused by a unit force, of the simply supported beam spanning nodes 6 and 10.

Effect of geometrical imperfections in the edge girder is also analysed. Changes in sensitivity for a number of static patterns are evaluated. One loading pattern is assumed.

Next, some aspects of sensitivity analysis of the system are presented under the stress costraints. The typical main girders can be assembled in various 3-D systems of bracings. Eight of them are distinguished and majority of them are analysed.

To systemize the consecutive static patterns, an idea used by KAPELA [21] is here employed. The first pattern, called the basic one, consists of main girders and longitudinal bracings. The remaining patterns may be understood as sums of the basic pattern and various types of additional bracings.

The expressed symbolically static patterns are as follows:

1. Basic pattern {=} main girders {+} longitudinal bracings,
2. Pattern 1 {=} basic pattern {+} longitudinal bracings,

3. Pattern 2 {=} pattern 1 {+} strengthening bars,
4. Pattern 3 {=} basic pattern {+} strengthening bars,
5. Pattern 4 {=} basic pattern {+} purlins,
6. Pattern 5 {=} pattern 4 {+} strengthening bars,
7. Pattern 6 {=} pattern 5 {+} roofing bracings,
8. Pattern 7 {=} pattern 1 {+} purlins.

The symbols {=} and {+} stand for formal operators of equality and additivity.

6.3. Assumed design constraints

The displacement constraints are imposed on the vertical deflections of lower chord central nodes of main girders. Specifically, 0.01 m is assumed as the maximum deflection. The stress constraints are imposed according to the POLISH CODE OF PRACTICE [14]. They differ for members under tension and compression.

6.4. Analysis of results – deflection constraint

Sensitivity of a single plane truss girder was analysed. The results are shown in Fig. 4 while the values of derivatives of the response function for diagonal and vertical bars are listed in Table 4. Numbers of elements according to Fig. 3 are shown along the horizontal axis; diagonal and vertical bars are numbered from 1 to 15, upper chord bars from 16 to 23, lower chord bars from 24 to 29. Derivatives of the response function for particular elements with respect to the cross-sectional area are shown along the vertical axis. Lines that join particular point are only to help visualize the variability; no continuous change of sensitivity from element to element is meant in the picture.

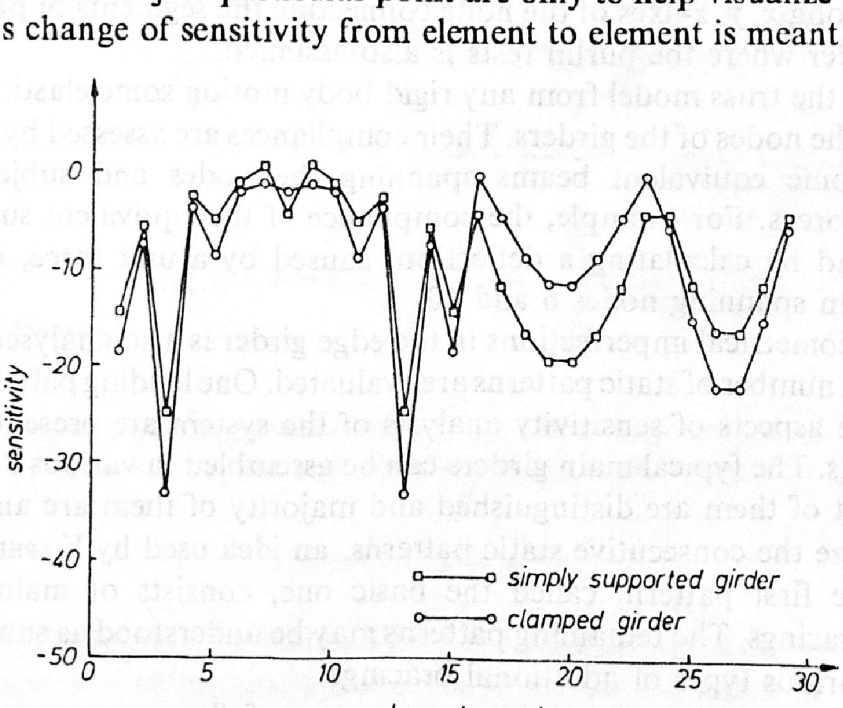

FIG. 4. Displacement sensitivity of main girder to variations in the cross-sectional area.
Rys. 4. Wrażliwość przemieszczeń na zmianę pola przekroju w elementach dźwigara głównego

Table 4

Displacement sensitivity to cross-sectional area in vertical members and diagonal bars of plane girder.
Vrażliwość przemieszczeń z uwagi na zmianę pola powierzchni w elementach słupków i krzyżulców dźwigara płaskiego

Element number	Simply supported truss	Simply supported frame	Clamped truss
1	−1.4281E+01	−1.8605E+01	−1.4262E+01
2	−5.7010E+00	−7.4274E+00	−5.7303E+00
3	−2.4723E+01	−3.3262E+01	−2.3946E+01
4	−2.3954E+00	−3.4995E+00	−2.4836E+00
5	−4.7937E+00	−8.6202E+00	−4.9710E+00
6	−1.0038E+00	−1.8052E+00	−9.9423E−01
7	8.1485E−01	−1.2301E+00	6.6978E−01
8	−4.0592E+00	−1.9904E+00	−3.9501E+00
9	8.1485E−01	−1.2301E+00	6.6978E−01
10	−1.0038E+00	−1.8052E+00	−9.9423E−01
11	−4.7937E+00	−8.6202E+00	−4.9710E+00
12	−2.3954E+00	−3.4995E+00	−2.4836E+00
13	−2.4723E+01	−3.3262E+01	−2.3946E+01
14	−5.7010E+00	−7.4274E+00	−5.7303E+00
15	−1.4281E+01	−1.8605E+01	−1.4262E+01

The structure is seen to be very sensitive in diagonal bars 3 and 13 but in diagonal bars 7 and 9 which are close to the centre of the girder remain practically insensitive. The system is also relatively insensitive in the upper chord bars 16 and 23 near the supports. When the boundary conditions are changed from free to clamped the sensitivity of the structure in diagonal bars near supports increases while in the upper chord near supports the structure becomes practically insensitive. Comparison of the two support conditions may lead to the conclusion that the diagonals 3 and 13 should have larger cross-sections or the geometrical setup of the girder should be enhanced. As indicated by KAPELA, NAWROT, POSTEK in [22], errors in the construction of supports, causing the changes in boundary conditions, are frequently encountered in this type of structures. Next, a frame pattern was analysed and the conclusion was drawn that the results of sensitivity analysis for both patterns are very close to each other.

Further, effects of geometrical imperfections on the sensitivity distribution in particular elements were investigated for girders in 3-D situations. Imperfections were imposed in the edge girder treated as a framed system. A number of variants were considered to show that some often applied simplifications (such as mental decomposition into plane systems, neglect of certain elements in calculations by treating them as second-order ones) can lead to unreliable or even wrong results.

All the sensitivity analysis results for 3-D systems are reffered to the behaviour of the plane main girder. These results are given in Table 5. In the first column the numbers of elements are shown, column 0 contains the derivatives of the response function for plane girder, column 01 for the basic pattern while the remaining columns 1−6 show the respective values for the rest of patterns.

Table 5

Displacement sensitivity to cross-sectional area of plane girder compared with corresponding values in imperfect space systems.

Porównanie wrażliwości przemieszczeń dźwigara płaskiego z odpowiednimi wielkościami dla przestrzennych układów z imperfekcjami

El.	Statical pattern							
no.	0	01	1	2	3	4	5	6
1	−14.262	−13.563	−17.219	−13.878	−12.599	−14.645	−13.359	−13.621
2	−5.730	−11.881	−23.747	−7.302	−6.903	−15.483	−8.875	−7.118
3	−23.946	−24.452	−26.857	−22.857	−22.068	−24.074	−22.326	−22.730
4	−2.483	−5.619	−4.537	−2.457	−3.288	−3.443	−2.386	−2.451
5	−4.971	−7.879	−6.180	−4.537	−6.245	−5.374	−4.859	−1.767
6	−0.994	−3.011	−3.435	−1.761	−2.434	−2.211	−1.992	−1.779
7	0.669	−0.143	0.139	0.397	−0.021	0.217	0.257	0.433
8	−3.950	−26.411	−5.137	−5.033	−24.087	−8.879	−7.239	−4.491
9	0.669	0.213	0.335	0.428	0.191	0.489	0.472	0.394
10	−0.994	−1.384	−3.475	−2.197	−1.365	−2.164	−2.135	−2.459
11	−4.971	−4.100	−6.765	−5.148	−4.107	−5.639	−5.482	−5.431
12	−2.483	−2.367	−3.590	−2.380	−2.303	−2.795	−2.549	−2.354
13	−23.946	−21.143	−23.030	−22.689	−21.329	−21.666	−21.753	−22.500
14	−5.730	−5.805	−6.235	−5.697	−5.674	−5.511	−5.225	−5.600
15	−14.262	−12.727	−13.492	−13.566	−12.782	−12.709	−12.722	−13.247
16	−4.409	−6.332	−4.269	−4.258	−6.010	−4.102	−4.163	−4.164
17	−11.725	−20.238	−11.293	−11.525	−18.311	−12.335	−12.141	−11.228
18	−16.784	−22.689	−16.239	−16.659	−23.672	−15.602	−15.822	−16.229
19	−19.514	−31.737	−19.172	−19.453	−31.557	−20.292	−19.585	−18.985
20	−19.514	−31.227	−19.083	−19.354	−29.734	−20.592	−19.692	−18.938
21	−16.784	−16.339	−15.908	−16.195	−17.616	−14.949	−15.350	−15.868
22	−11.725	−10.892	−10.928	−11.117	−11.402	−10.374	−10.528	−10.899
23	−4.409	−3.976	−4.099	−4.166	−4.067	−3.908	−3.962	−4.078
24	−4.434	−9.093	−7.265	−6.460	−8.452	−7.963	−7.361	−6.294
25	−11.616	−13.949	−13.32	−16.588	−13.974	−13.400	−13.835	−16.438
26	−16.510	−52.634	−52.221	−50.485	−48.080	−56.022	−50.656	−50.311
27	−16.510	−54.750	−52.876	−48.179	−49.597	−57.131	−51.418	−47.909
28	−11.616	−11.390	−12.855	−13.862	−12.238	−11.311	−12.304	−13.648
29	−4.434	−4.338	−4.624	−4.442	−4.488	−4.169	−4.244	−4.347

In the performed calculations the basic system was analysed which means that the influence of roofing and roof braces was neglected. The basic system was treated as both ideal one and that with imperfections. A considerable increase in sensitivity of the structure in the vertical central bar 8 and the near-support bar 2 was observed. Similar increase was also noticed for the upper chord midspan bars 19, 20 as well as for the lower chord midspan bars 26, 27 (column 1).

Next, the pattern 1 was analysed in which the effects of roof bracings were accounted for and were found out to change the picture. Sensitivities of the structure in the upper chord are close to those for ideal system, bracings have no influence on the sensitivities in the lower chord bars whereas the sensitivity in the vertical bar 2 near support increases considerably and in the diagonal bar 3 (column 2) increases lightly. Efficacy of the strengthening bars, suggested by a consulting engineer as means to make the structure more reliable, (pattern 2) was investigated. It appeared hat the sensitivities in all the bars become close to those for ideal system, except in the ower chord bars 26, 27 (column 3). The strengthening of the structure is, however, ffective only if sufficient interaction of roofing braces is ascertained. This was shown y analyzing the pattern 3. The strengthening system reinstated the nearly ideal ehaviour of the vertical bar 2 and the diagonal bar 3 but resulted in no changes in the nidspan bars 26, 27 of the lower chord (column 4). Analysis of the pattern 4 was imed at verifying whether the purlins alone can replace the roofing braces. Such an ssumption is often made in practice when a relatively in-plane rigid reinforced oncrete slabs are used to span the system of purlins. The presence of purlins lecreased the sensitivity in central vertical bar 8 while increased that in the vertical ar 2. Sensitivities in the upper chord bars 19, 20 were decreased while the behaviour f the lower chord bars 26, 27 remained unchanged (column 5). The question was next sked whether the joint system of strengthening bars and purlins is able to replace the ystem of roofing braces (pattern 5). The answer is no since the sensitivity in both ertical bars 2 and 8 is larger than for the ideal system in the same elements (column). Lastly, a system was analysed consisting of roofing braces, strengthening bars and urlins (pattern 6). If turned out that sensitivities in all bars except in the lower chord ars 26, 27 are close to those for the ideal system (column 7). Thus a different system f strengthening bars should be used to reinstate the ideal situation.

The above results have shown that the sensitivity analysis is a very efficient tool to nvestigate variations in the behaviour of complex spatial structural systems.

Changes in the displacements at the places where the constraints were imposed did ot exceed several milimeters and nevertheless, the sensitivity analysis enabled the edistribution of effects to be followed and the right conclusions to be drawn as to fficacy of the proposed structural remedies or the methods of calculations.

6.5. ANALYSIS OF RESULTS – STRESS CONSTRAINT

Similar sensitivity analysis can be performed for the stress constraints. In what follows he normal stress constraint will be imposed in the element 7 of the roof bracing system, Fig. 5. The stresses, called here the admissible ones, are calculated according to the POLISH CODE OF PRACTICE [14]. Their values are: 2.15×10^5kN/m^2 for tension and 3.40 $\times 10^3$kN/m^2 for compression. Both ideal systems and those with imperfections were analysed. Sensitivity of bracing bars 1–15 was analysed. Systems 1, 7, 6, 2 were dealt with and the results are shown diagrammatically in Fig. 6. In system 1 no nteraction of purlins with the structure was taken into account and sensitivity in racing bars was found to be negligibly small. In system 7 an interaction of purlins was ccounted for – thus the roofing was considered in an indirect manner. The purlins cause a remarkable increase in the sensitivity in the elements 7 and 9. After an addition of the strengthening bars (system 6) the structure becomes insensitive in the elements gain.

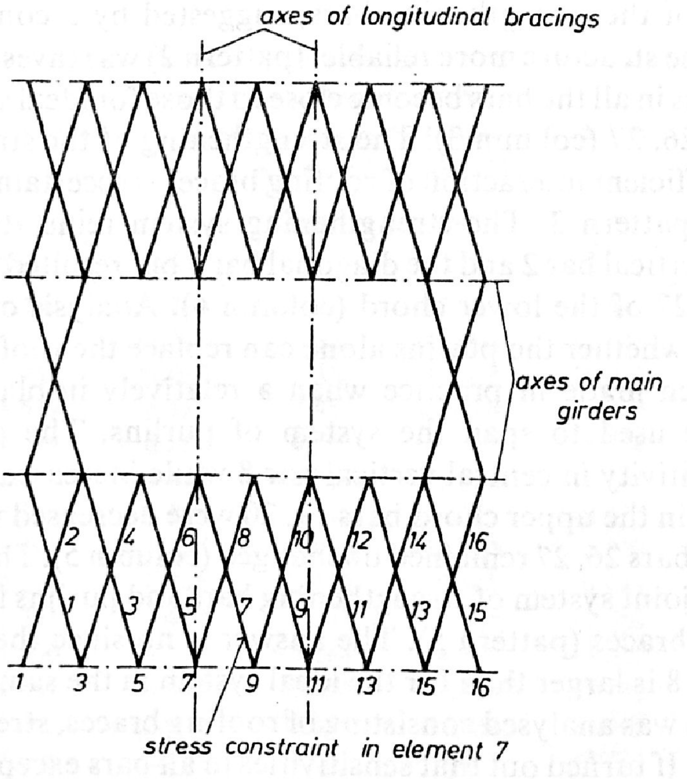

FIG. 5. Roof bracing system.
Rys. 5. Schemat układu stężeń

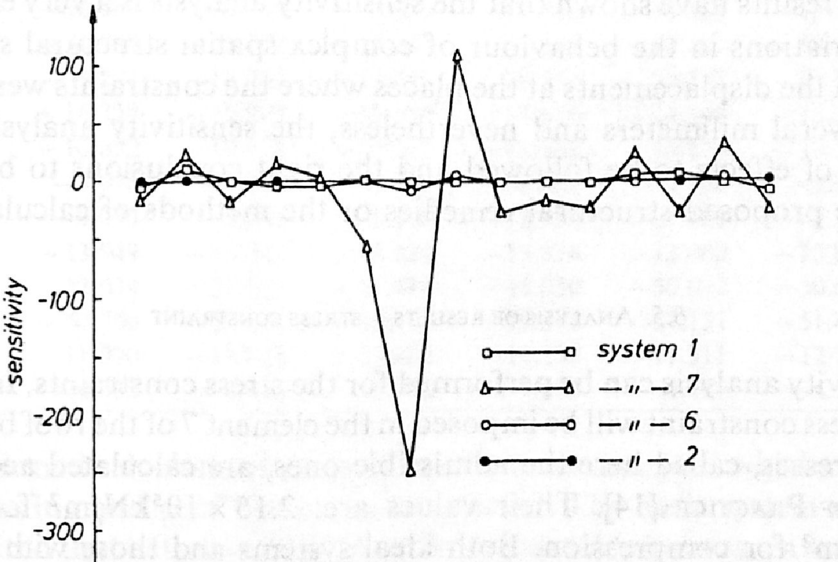

FIG. 6. Stress sensitivity to variations in the cross-sectional area of roof bracing ideal systems.
Rys. 6. Wrażliwość naprężeń normalnych na zmianę pola przekroju poprzecznego w elementach stężeń układów idealnych

Imperfections are found to change the picture. All the analysed systems are very sensitive in the element 7. An addition of the strengthening bar system (system 7) results in slight decrease of sensitivity at this place but constitutes no decisive factor in

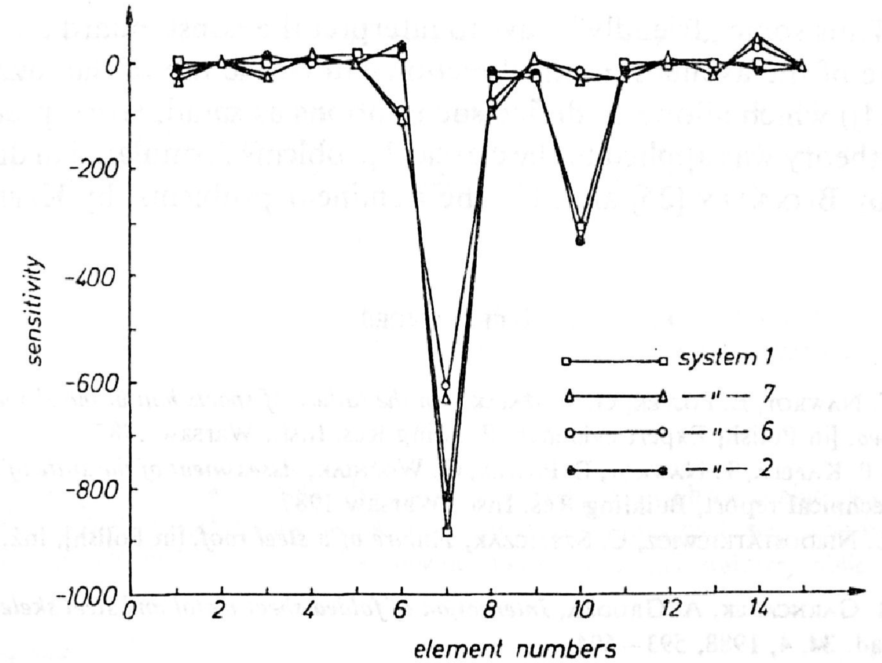

FIG. 7. Stress sensitivity to variations in the cross-sectional area of roof bracing systems with imperfections.
Rys. 7. Wrażliwość naprężeń normalnych na zmianę pola przekroju poprzecznego w elementach stężeń. Układy z imperfekcjami

he whole behaviour. Since bar 7 is a compressed one, to decrease sensitivity in this element one should increase admissible stresses, i.e. decrease the buckling lengths of bracing bars.

7. CONCLUSIONS

Static sensitivity analysis of 3-D bar systems is presented in the paper under both the displacement and the stress constraints. Some aspects of the computer implementation are also discussed.

Two types of analyses of a 3-D system are shown that enabled to investigate the changes occuring in complex structures. The conclusions drawn are directed to the practicing structural designer and consulting expert. Thus the classical methods can be supplemented with the sensitivity analysis which has otherwise been treated as a tool in the problems of structural optimization.

Possibility to study sensitivity with respect to variations in Young's moduli and bar lengths makes the POLSAP system suitable for investigating changes in the structures during fire and due to the dimensional tolerances, respectively. Further studies are planned to allow for various types of stress functions and for a nonlinear behaviour. Usefulness of classical geometrically nonlinear analysis of systems similar to that analysed in this paper was shown for example, in KAPELA, POSTEK [23].

Displacement and stress state analyses have been deeply rooted in the designer's mind. Less firmly established are the sensitivity analysis methods and some difficulties can be encountered in the proper interpretation of results obtained in the form of rows of numbers. Thus some „friendly" ways to interpret the nonstandard analyses should be found. One of the avenues in this direction can be the use of the fuzzy set theory (KACPRZYK [24]) which allows to define such notions as small, large, pretty large etc. The fuzzy set theory was applied to the classical problems formulated in displacements and stresses by BLOCKLEY [25] and, for the nonlinear problems, by KLEIBER [26].

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OCENA PRZYDATNOŚCI ANALIZY WRAŻLIWOŚCI KONSTRUKCJI NA PRZYKŁADZIE ZŁOŻONYCH UKŁADÓW PRĘTOWYCH

Streszczenie

Przedstawiono niektóre aspekty analizy wrażliwości konstrukcji prętowych zwracając uwagę na możliwości wykorzystania jej wyników przez projektantów oraz ekspertów zajmujących się oceną stanu konstrukcji. Rozważania przeprowadzono na przykładzie pewnej konstrukcji hali stalowej obliczanej jako układ przestrzenny.

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