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Local damage identification in frequency domain based on substructure isolation method

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ABSTRACT

This paper proposed a frequency domain method of substructure identification for local health monitoring. The substructure isolation method (SIM) consists of two steps: the first is the construction of isolated substructure which is the key of the method, and the second is damage identification of substructure. The isolated substructure is a virtual and independent structure, and it have the same physical parameters of the real substructure with the additional virtual supports on boundary, which is realized by operating the measured response. This paper extends the SIM method to frequency domain, which could make the method employ more measured response and compute more efficiently. A mass-spring numerical model is used to verify the theory of the SIM method, and a cantilever beam is experimented to test the method. The method preformed efficiently and accurately in the both numerical model and experiment.

INTRODUCTION

Structural Health Monitoring (SHM) is a hot researched field in civil engineering, and the damage identification is the theoretical basic of SHM. The damage could be detected directly by signal processing of the measured response without model, like wavelet analysis[1], time series[2], or be estimated by optimizing the FEM model using flexibility matrix[3], nature frequencies and mode shapes[4], time domain responses[5] or frequency response[6]. Some of the methods are well developed, but sometimes it is still hard to use for the large and complex structure to be identified globally and accurately. The structures of civil engineering, like bridges, tall buildings and dams, usually have some uncertain factors, such as boundary conditions, nonlinear components. Furthermore, sometimes the sensors are not enough to estimate the complex structure accurately, and the response of the structure is not sensitive to the local damage. While the substructuring method could reduce the entire structure to the local critical substructure, and only use the local according the responses of the few senses placed on the substructure.

The substructure is a local part of the global structure, and it is not independent of the global structure. In order only to focus on the substructure, mostly methods separate the substructure from the global structure, and then the interface force will be exposed. They estimated the substructure based on the motion of the equation of the substructure, so most substructuring methods belongs to time domain methods. 2003, Koh et al.[7] employed genetic algorithms; 2005, Tee et al.[8]developed a divide-and-conquer for identification at the substructure level in first-order model and second-order model; 2006, Yuen et al. [9] presented a bayesian frequency-domain approach; 2011, Trinh et al.[10] embedded a simple numerical integration scheme to obtain interface response; 2011, Xing et al.[11] use ARMAX to identification. Most of the literature [7-11] use shear structure to verify the proposed substructuring method. However, more structures are complex than shear structure in real application, and the according substructure and the boundary contains more Dofs. Then the time domain substructuring method above will be time-consuming and hard to converge.

To overcome this drawback, Hou et al.[12] have proposed the substructure isolation method, of which the core idea is different. The isolation method firstly operate of the measured responses to constructed new responses which belonged to isolated substructure, which is a virtual and independent substructure and have the same elements stiffness and mass matrixes with the real substructure. Secondly, the damage identification of the substructure could be carried out equivalently and flexibly by any of the existing methods aimed originally at global identification using the constructed responses, so there is no demand or limitation on the size or Dofs of substructure.

The key step of isolation method is the construction of isolated substructure. The literature [12] constructed the response in time domain. The SIM in time domain need compute the invers of constraining matrix, of which the computing time is in proportion to time steps and usually it is time-consuming, so usually the measured time could not be very long. Therefore, this paper will extends the method to frequency domain, which could employ more measured response and compute more efficiently.

SUBSTRUCTURE ISOLATION METHOD

The isolation method in frequency domain

The equation of the motion of the structure can be written as:

$$\mathrm{M}\ddot{\mathrm{x}}(\mathrm{t}) + \mathrm{C}\dot{\mathrm{x}}(\mathrm{t}) + \mathrm{K}\mathrm{x}(\mathrm{t}) = \mathrm{f}(\mathrm{t}) \tag{1}$$

The Fourier transfer of the Equation (1) is:

$$(-\omega^2\mathrm{M} + \mathrm{j}\omega\mathrm{C} + \mathrm{K})\mathrm{X}(\omega) = \mathrm{F}(\omega) \tag{2}$$

Where $\mathrm{X}(\omega) = \mathcal{F}\big(\mathrm{x}(\mathrm{t})\big) = \int_0^{\mathrm{t}} \mathrm{x}(\mathrm{t})\, \mathrm{e}^{-\mathrm{j}\omega\mathrm{t}}\mathrm{dt}$, $\mathrm{F}(\omega) = \mathcal{F}\big(\mathrm{f}(\mathrm{t})\big)$, $\mathcal{F}$ is the Fourier operator. Denote the Dofs is the Degree of freedoms (Dofs) of inner substructure, b is the Dofs of substructure boundary and r is the Dofs of outside substructure. Separate the system matrix according to 3 kinds of DOF, then the Equation (2) could be written as:

$$\left(-\omega^2\begin{bmatrix} M_{ss} & M_{sb} & 0 \\ M_{bs} & M_{bb} & M_{br} \\ 0 & M_{rb} & M_{rr}\end{bmatrix} + \omega\begin{bmatrix} C_{ss} & C_{sb} & 0 \\ C_{bs} & C_{bb} & C_{br} \\ 0 & C_{rb} & C_{rr}\end{bmatrix} + \begin{bmatrix} K_{ss} & K_{sb} & 0 \\ K_{bs} & K_{bb} & K_{br} \\ 0 & K_{rb} & K_{rr}\end{bmatrix}\right)\begin{Bmatrix} X_s(\omega) \\ X_b(\omega) \\ X_r(\omega)\end{Bmatrix} = \begin{Bmatrix} F_s(\omega) \\ F_b(\omega) \\ F_r(\omega)\end{Bmatrix} \tag{3}$$

Where $M = \begin{bmatrix} M_{ss} & M_{sb} & 0 \\ M_{bs} & M_{bb} & M_{br} \\ 0 & M_{rb} & M_{rr}\end{bmatrix}$, $C = \begin{bmatrix} C_{ss} & C_{sb} & 0 \\ C_{bs} & C_{bb} & C_{br} \\ 0 & C_{rb} & C_{rr}\end{bmatrix}$, $K = \begin{bmatrix} K_{ss} & K_{sb} & 0 \\ K_{bs} & K_{bb} & K_{br} \\ 0 & K_{rb} & K_{rr}\end{bmatrix}$, $F = \begin{Bmatrix} F_s \\ F_b \\ F_r\end{Bmatrix}$, $X = \begin{Bmatrix} X_s \\ X_b \\ X_r\end{Bmatrix}$.

If apply n+1 groups of excitation $F^0(\omega), F^1(\omega), \dots, F^n(\omega)$ on the structure respectively, the corresponding responses of the structure are $X^0(\omega), X^1(\omega), \dots, X^n(\omega)$. For convenience of later derivation, $F^0(\omega)$ is called as basic excitation, and the according response $X^0(\omega)$ is called as basic response; $F^1(\omega), \dots, F^n(\omega)$ are called as constraining excitation, and the according response $X^1(\omega), \dots, X^n(\omega)$ are called as constraining response.

Linear combination of all the excitations and the responses,

$$\begin{cases} P(\omega) = F^0(\omega) + \sum_{i=1}^{n} Z^i(\omega)F^i(\omega) = F^0(\omega) + QZ \\ Y(\omega) = X^0(\omega) + \sum_{i=1}^{n} Z^i(\omega)X^i(\omega) = X^0(\omega) + \begin{bmatrix} D \\ B \end{bmatrix} Z \end{cases} \tag{4}$$

where, $Z = [Z_1(\omega) \quad \dots \quad Z_n(\omega)]^T$ are the coefficient of linear combination, $Q = \left[F^1\ (\omega) \quad \dots \quad F^n\ (\omega)\right]$, $B = [X_b^1(\omega) \quad \dots \quad X_b^n(\omega)]$, $D = [X_s^1(\omega) \quad \dots \quad X_s^n(\omega)]$.

If the structure is linear system, then the linear combination of excitation $P(\omega)$ and response $Y(\omega)$ should be satisfy the equation of the monition of the structure:

$$(-\omega^2 M + \omega C + K)Y(\omega) = P(\omega) \tag{5}$$

The first row of the Equation (3) can be written as:

$$\begin{cases} (-\omega^2 M_{ss} + \omega C_{ss} + K_{ss})Y_s(\omega) = P_s(\omega) + P_c(\omega) \\ P_c(\omega) = (\omega^2 M_{sb} - \omega C_{sb} - K_{sb})Y_b(\omega) \end{cases} \tag{6}$$

where $P_c(\omega)$ is the coupled interface force related with the boundary response $Y_b(\omega)$.

If the response of substructure boundary $Y_b(\omega) = X_b^0(\omega) + B(\omega)Z(\omega) = 0$, then Equation (7) can be obtained from the second equation of Equation (4):

$$\begin{cases} Z(\omega) = -[B(\omega)]^+ X_b^0(\omega) \\ Y_s(\omega) = X_s^0(\omega) - D(\omega)[B(\omega)]^+ X_b^0(\omega) \end{cases} \tag{7}$$

where '+' is general inverse.

Put $Y_b(\omega)=0$ into the Equation (6), then the equation of the monition of the substructure is

$$(-\omega^2 M_{ss} + \omega C_{ss} + K_{ss})Y_s(\omega) = P_s(\omega) \tag{8}$$

As is well known, $Y_b(\omega)=0$ is the boundary condition of boundary fixed support. Furthermore, it can be seen from Equation (8) that the constructed response $Y_s(\omega)$ are only associated with the linear combined excitation $P_s(\omega)$ and substructure system matrix M_{ss}, C_{ss}, K_{ss}. Therefore, the virtual fixed support is constructed, and the substructure is successfully isolated from the structure which is called

Isolated Substructure. The constructed response $Y_s(\omega)$ belongs to the response of the Isolated Substructure, and they have no relationship with the outside of the substructure. So if the value or the feature of the excitation $P_s(\omega)$ is known, then the response $Y_s(\omega)$ could be used for the identification of the substructure.

A 2-Dofs mass-spring system is taken as an example, see Figure 1. The mass of each dof is m, and the stiffness and the damping of the each spring is k and c. so the matrix of the system is $M = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}, C = \begin{bmatrix} 2c & -c \\ -c & c \end{bmatrix}, K = \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}$.

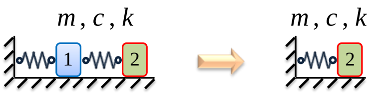

Figure 1. A 2-Dofs system

The matrix of frequency response matrix is $H(\omega)$, see Matrix (9), where $H_{iv}(\omega)$is the frequency response of i-th Dof at the coordinate ω, when apply unite impulse excitation on v-th Dofs; $A = m^2\omega^4 - 3cm\omega^3 j - (c^2 + 3km)\omega^2 + 2ck\omega j + k^2$.

$$H(\omega) = \frac{1}{A}\begin{bmatrix} -m\omega^2 + c\omega j + k & c\omega j + k \\ c\omega j + k & -m\omega^2 + 2c\omega j + 2k \end{bmatrix} \tag{9}$$

If 2-th mass and spring is substructure, and 1-th Dofs is the boundary of substructure, then according to the isolation method, the parameters in Equation (7) can be written as:

$$\begin{cases} B(\omega) = (-m\omega^2 + c\omega j + k)/A \\ X_b^0(\omega) = (c\omega j + k)/A \\ D(\omega) = (-m\omega^2 + 2c\omega j + 2k)/A \\ X_s^0(\omega) = c\omega j + k/A \end{cases} \tag{10}$$

Then the frequency response of isolated substructure can be computed using Equation (7), see Equation (11), which is the same as frequency response of the one mass-spring system. That's to say, the isolated substructure has been constructed successfully.

$$H_s(\omega) = 1/(-m\omega^2 + c\omega j + k) \tag{11}$$

The method using the FFT of measured response

In real application the measured response is discrete, so the Fast Fourier Transform (FFT) is used practically to compute the frequency response. When the time-domain signal is of a finite length and does not tend to zero during the integration time, the spectral leakage is avoidless. It will affect the accuracy of the constructed frequency response of the isolated substructure using the isolation function. Therefore, the window is used to avoid the spectral leakage. The Equation (12) is exponential window, and the coefficient η in exponential window is the damped exponential.

$$w_e(t) = \begin{cases} e^{-\eta t}, 0 \le t \le T \\ 0, \quad \text{else} \end{cases} \tag{12}$$

The free response of a N-dofs structure could be written as in Equation (13), and the free response with adding exponential window is Equation.(14). ϕ_i is i-th mode shape, ξ_i is damping ratio, $\omega_{di} = \omega_i\left(1 - \xi_i^2\right)^{0.5}$, ω_i is the i-th natural frequency. Compared with the Equation (13) and (14), the exponential window only adding the damping radio η/w_{di} of free response, which doesn't change the frequency information of the response. Therefore, the constructed response $Y_s(s)$ belongs to isolated substructure.

$$x(t) = \sum_{i=1}^{n} A_i \phi_i e^{-\omega_{di}\xi_i t} \sin(\omega_{di} t + \varphi_i) \tag{13}$$

$$x(t) w_e(t) = \sum_{i=1}^{n} A_i \phi_i e^{-\omega_{di}(\xi_i + \eta/\omega_{di})t} \sin(\omega_{di} t + \varphi_i) \tag{14}$$

SUBSTRUCTURE DAMAGE IDENTIFICATION

The above derivation indicates that the substructure damage identification can be performed equivalently via the identification of Isolated Substructure, since they have the same system matrix as the substructure. If the basic excitation $P_s(t)$ is impulse excitation, then the constructed frequency response $Y_s(\omega)$ could be used for identifying the natural frequencies of the isolated substructure, of which the *i*-th natural frequency is denoted as ω_i^m. If the natural frequencies of FEM model of isolated substructure is $\omega_i^F(\mu)$, where μ_i is the *i*-th damage extent, then the damage extent could be identified by minimizing the following objective function:

$$\Delta(\mu) = \sum_i \left| \frac{\omega_i^F(\mu) - \omega_i^m}{\omega_i^m} \right|^2 \tag{15}$$

EXPERIMENT

An aluminum cantilever beam is experimented for verification the proposed substructure isolation method in frequency domain, see Figure 2. The upper part of the beam is the considered as the substructure. The damage with the length 10.2 cm is made by cutting even notches near the fixed end, which is decreased to 42% of its original stiffness .Three strain sensors (Y1, Y2, Y3) are placed on the substructure, of which one (Y3) is placed on the boundary. The boundary velocity (Y4) is also measured by laser vibrometer. The single virtual pinned support is constructed by constraining the boundary responses of Y3 & Y4 sensors to zero, see Figure 7. The virtual support separated the substructure from the global structure to be the independent isolated substructure, see Figure 3.The substructure is divided into five parts, and the real damage extents of the five parts are [1 0.42 1 1 1].

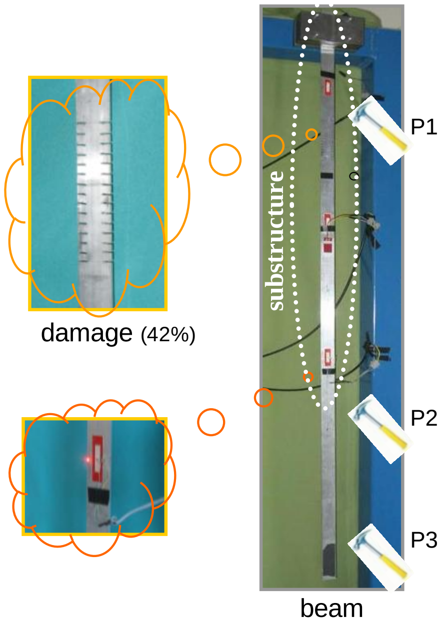

Figure 2. Cantilever beam

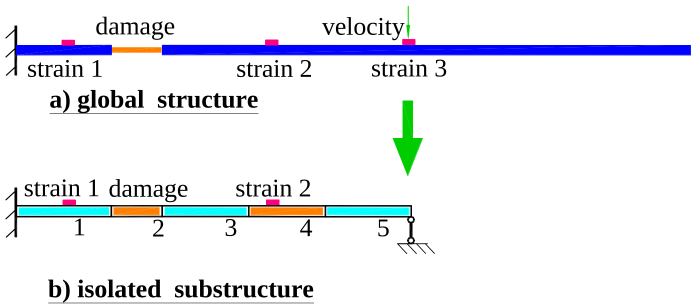

Figure 3. Isolation of the substructure

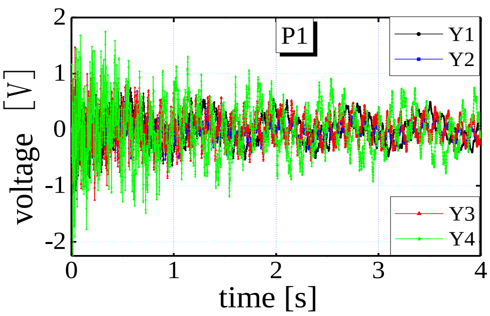

Figure 4. The measured basic responses

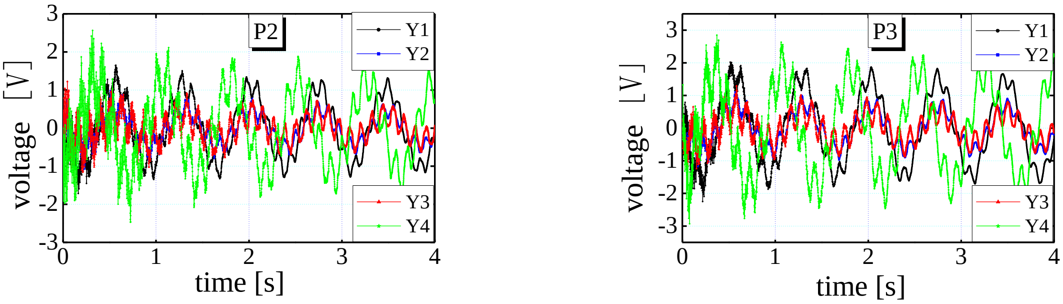

Figure 5. The measured constraining responses

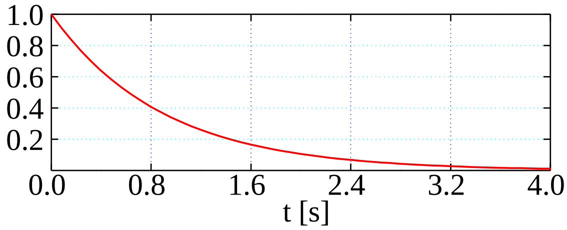

Figure 6. The exponential window

The three groups of responses excited by the hammer which is applied on the P1, P2 and P3 of beam respectively (see Figure 2) are shown in Figure 4 and Figure 5. Do FFT of the windowd measured response with the exponential window (see Figure 6), and put the frequency response into Equation (7), then the frequency responses of the isolated substructure can be computed, seen in Figure 7. The nature frequencies of the substructure were easily obtained via the peaks of its constructed frequency responses (Figure 7). The damages of the substructure were then identified by minimizing the square distance between the constructed nature frequencies of the isolated substructure and the nature frequencies computed using its Finite Element model, see Figure 8. Both the isolation and the identification steps are performed very well using the experimental data.

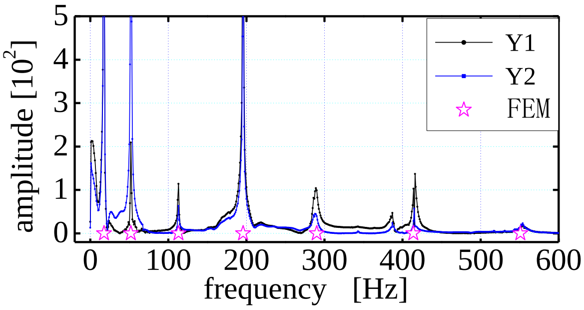

Figure 7. Constructed frequency responses

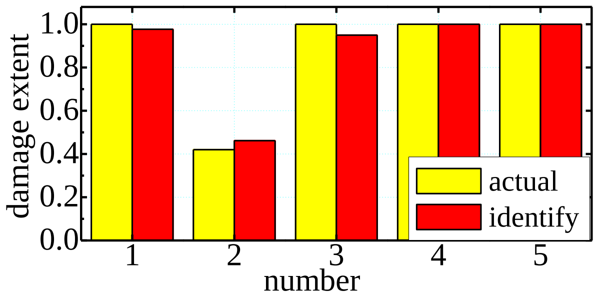

Figure 8. Damage extent

CONCLUSION

The paper extended the substructure isolation method in frequency domain. It could be using for a large scale substructure, and only focus on the identification of the damage extends of substructure stiffness, and don't regard to the interface force, damping coefficient, state vectors and some other parameters in the optimization. The mass-spring numerical model and beam experiment have verified that the proposed substructring method is efficient and accurate.

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