New formulation of the discrete element method

Jerzy Rojek[1,*], **Aleksander Zubelewicz**[2], **Nikhil Madan**[1] **and Szymon Nosewicz**[1]

[1] *Institute of Fundamental Technological Research, Polish Academy of Sciences*
Pawinskiego 5B, 02-106 Warsaw, Poland
e-mail: jrojek@ippt.pan.pl

[2] *Faculty of Civil Engineering, The University of New Mexico*
New Mexico, USA
e-mail: alek@unm.edu

Abstract

This work presents a new original formulation of the discrete element method based on the soft contact approach. The standard DEM has been enhanced by introduction of the additional (global) deformation mode caused by the stresses in the particles induced by the contact forces. Uniform stresses and strains are assumed for each particle. The stresses are calculated from the contact forces. The strains are obtained using an inverse constitutive relationship. The strains allow us to obtain deformed particle shapes. The deformed shapes (ellipses) are taken into account in contact detection and evaluation of the contact forces. The numerical example shows that a particle deformation changes the particle interaction and the distribution of forces in the discrete element assembly.

Keywords: discrete element method; deformable particles; soft contact

1. Introduction

The discrete element method (DEM) is a powerful tool for predicting the behaviour of various particulate and non-particulate materials such as soils, powders, rocks, concrete, or ceramics [11, 9, 3, 10]. In the DEM, a material is represented by a large assembly of particles (discrete elements) interacting with one another by contact. Two different approaches to contact treatment in the DEM can be identified, the so-called *soft-contact* approach [2] and the *hard-contact* concept [4]. In the soft-contact DEM formulation, the particles are treated as pseudo-rigid bodies with deformation concentrated at the contact points. A small overlap of the particles is allowed and it is considered as equivalent to the particle deformation at the contact point.

The material properties in DEM cannot be prescribed directly – they emerge from the collective response of the aggregate and depend on the choice of the interparticle contact model as well as the discrete element assembly characteristics [8]. Appropriate representation of the macroscopic properties in the discrete element method is still a challenge and it is sometimes difficult or impossible to obtain a required deformation behaviour [7]. Some limitations of the discrete element method are due to the assumption of the rigidity of discrete elements. Their deformability would allow to enrich modelling capabilities of the DEM. The simplest way to introduce deformability in the discrete element method is to discretize discrete elements with finite elements [6]. This approach is computationally very expensive and it cannot be used for a large number of particles.

An alternative approach is by adding deformation modes to a rigid motion of discrete elements [1, 12]. Until now this concept has been applied to the discrete elements in the form of polygonal prisms (in 2D) or polyhedra (in 3D). The present work presents an original formulation of the discrete element method based on the soft contact approach with deformable circular discs. The developed numerical algorithm has been implemented in the author's own discrete element program. Preliminary numerical results will be presented.

2. Formulation of the discrete element method with deformable discs

We shall consider a discrete element model consisting of cohesionless or cohesive cylindrical particles. The particles are assumed to be uniformly deformed under the internal particle stress induced by the contact forces. The idea of the new formulation is shown in Fig. 1. A uniform stress is assumed in the particle. The internal particle stress $\bar{bsig}_p$ is obtained as the average stress derived from the contact forces using the following formula [5]:

$$\bar{\boldsymbol{\sigma}}_p = \frac{1}{V_p} \sum_{c=1}^{n_{p\,c}} \frac{1}{2} \left(\mathbf{s}^c \otimes \mathbf{F}^c + \mathbf{F}^c \otimes \mathbf{s}^c \right) , \tag{1}$$

where V_p is the particle volume, $n_{p\,c}$ – number of elements being in contact with the particle, $\mathbf{s}^c$ – vector, connecting the particle center with the contact point, $\mathbf{F}^c$ – contact force, and the symbol $\otimes$ denotes the outer (tensor) product. In case of a constrained particle, except for contact forces we have also reaction forces.

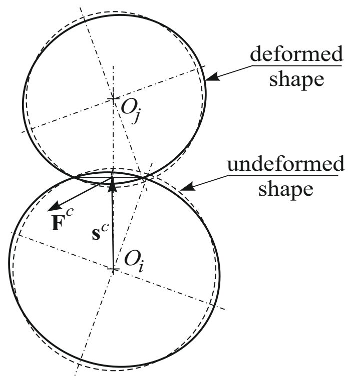


Figure 1: The idea of the deformable discrete element method

*This work has been financed from the funds of Polish National Science Centre (NCN) awarded by the decision number DEC-2015/19/B/ST8/03983.

Particle strains are calculated using an inverse constitutive relationship

$$\boldsymbol{\epsilon}_p = \mathbf{D} : \boldsymbol{\sigma}_p \tag{2}$$

where $\mathbf{D}$ is the elastic compliance tensor for the plane strain.

The circle under an uniform strain is deformed into an ellipse with its principal axes aligned with the prinicipal strain directions. The normal contact force is determined as a linear function of the overlap of such ellipses. This overlap is considered as equivalent to local deformation of the particles. Similarly, as in the standard DEM the local damping is included in the normal interaction. The tangential contact force is evaluated similarly as in the standard DEM.

3. Numerical example

A uniaxial compression of a rectangular specimen discretized with bonded discs as it is shown in Fig. 2 has been simulated using the standard and new DEM formulation.

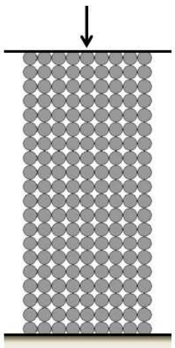
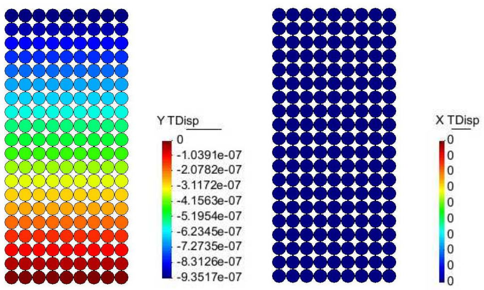
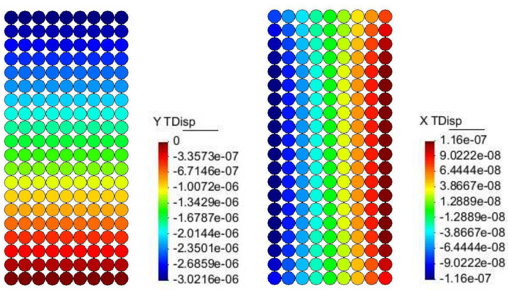


Figure 2: Uniaxial compression of a rectangular specimen – DEM model



a) b)

Figure 3: Simulation results obtained with the standard DEM formulation – contours of displacements along: a) the y-axis, b) the x-axis.



a) b)

Figure 4: Simulation results obtained with the new DEM formulation – contours of displacements along: a) the y-axis, b) the x-axis.

It has been assumed that the discrete model represents an elastic solid material. Figure 3 shows the results obtained with the standard DEM formulation in the form of the contours of displacements along the y and x axes. It can be seen that all the elements have zero x displacements. This means that the macroscopic effective Poisson's ratio is zero in this model under the loading along the y axis. Figure 4 shows the results obtained with the new DEM formulation. One can notice non-zero displacements in the x direction which implies a non-zero Poisson's ratio. This shows that the new formulation allows us to capture the Poisson's effect even in such configuration of discs. This confirms new capabilities of the proposed formulation with repsect to the standard DEM.

The shape change of the particle a nonlocal contact model. The contact interaction between two particles influences indirectly (through the change of the shape of these particles) the contact between these particle and the other particles. As a result, the non-local interactions enhances capabilities of the DEM method.

References

[1] P.A. Cundall, T. Maini, J. Marti, P.J. Beresford, N.C. Last, and M.I. Asgian. *Computer Modeling of Jointed Rock Masses*. U.S. Army Engineers Waterways Experiment Station, Technical Report N-78-4, 1978.

[2] P.A. Cundall and O.D.L. Strack. A discrete numerical method for granular assemblies. *Geotechnique*, 29:47–65, 1979.

[3] S. Hentz, L. Daudeville, and F.V. Donzé. Identification and validation of a discrete element model for concrete. *ASCE J. Eng. Mech.*, 130:709–719, 2004.

[4] D.C. Hong and J.A. McLennan. Molecular dynamics simulations of hard sphere granular particles. *Phys. A: Stat. Mech. Applicat.*, 187:159–171, 1992.

[5] S. Luding. Micro-macro transition for anisotropic, frictional granular packings. *International Journal of Solids and Structures*, 41(21):5821–5836, 2004.

[6] A. Munjiza. *The Combined Finite–Discrete Element Method*. Wiley, 2004.

[7] C. O'Sullivan, J.D. Bray, and L. Cui. Experimental validation of particle-based discrete element methods. In *Proceedings of the GeoCongress, Georgia, Atlanta*, 2006.

[8] J. Rojek, C. Labra, O. Su, and E. Oñate. Comparative study of different discrete element models and evaluation of equivalent micromechanical parameters. *Int. J. Solids and Structures*, 49:1497–1517, 2012.

[9] J. Rojek, E. Oñate, C. Labra, and H. Kargl. Discrete element simulation of rock cutting. *International Journal of Rock Mechanics and Mining Sciences*, 48:996–1010, 2011.

[10] R. Senapati and J. Zhang. Identifying fracture origin in ceramics by combination of nondestructive testing and discrete element analysis. In *AIP Conference Proceedings*, volume 1211, pages 1445–1451, 2010.

[11] L. Widuliński, J. Kozicki, and J. Tejchman. Numerical Simulations of Triaxial Test with Sand Using DEM. *Archives of Hydro-Engineering and Environmental Mechanics*, 56:149–171, 2009.

[12] J.R. Williams and G.W. Mustoe. Modal methods for the analysis of discrete systems. *Computers and Geotechnics*, 4:1–19, 1987.