New efficient adaptive control of torsional vibrations induced by sudden nonlinear disturbances

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Abstract

The aim of this paper is to present a novel adaptive technique of control of the vibrating drilling systems. The algorithm constitutes an adaptive linear quadratic regulator that uses direct measurements of the disturbance to synthesize its linear dynamic approximation. This approach allows to generate control law that includes the impact of the friction on the system dynamics and optimally steers the system to the desired trajectory. The effectiveness of the algorithm is validated via comprehensive numerical simulations of the control of the simplified drilling model. The results are compared to these obtained with the use of the Linear Quadratic Gaussian regulator.

Keywords: vibration control, drillstring, adaptive control, auto-regressive model

1. Introduction

Torsional vibrations induced in drilling systems are detrimental to the condition of machine and to the effectiveness of the engineering process. Successful attenuation of such vibrations would increase reliability of the aforementioned process. Proposed solutions in literature include active damping system based on a feedback control [3], strategy based on optimal state feedback control [1], H_∞ controller [5].

This paper deals with the adaptive control of the drilling systems with changing characteristics of the disturbance. Disturbance is assumed to consist of the torsional friction and random noise representing summary of uncertainties affecting the controlled system. Dependence of torsional friction on the angular velocity of drillstring is highly nonlinear and changes for different layers of a soil or a rock. The control algorithm is constrained to systems governed by the dynamical equations depending linearly on the state of the system, control and disturbance.

The proposed algorithm is concurrent to the classical linear quadratic regulator (LQR), although it is based on it. The proposed novelty is that the control law adapts to changing disturbances. The algorithm does not need the knowledge of the friction characteristics to operate.

The scope of the research is to develop the control scheme and numerically verify its effectiveness on simplified drillstring model in comparison to the Linear Quadratic Gaussian (LQG) regulator.

2. Model description

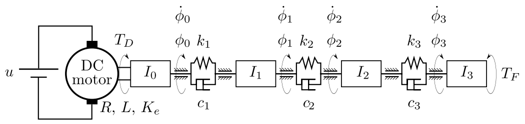

Figure 1: The scheme of the controlled object.

Let us consider the system depicted in Figure 1. It consists of the DC motor with the resistance $R = 0.472\,\Omega$, the inductance $L = 7.85\,\mathrm{mH}$ and the electromotive force constant $K_e = 4.9\,\mathrm{N\,m\,A^{-1}}$. The motor generates the torque T_D. The shaft of the motor is firmly connected to the first body, the moment of inertia of this coupling is $I_0 = 5.56 \cdot 10^{-2}\,\mathrm{kg\,m^2}$. The next three bodies are serially coupled together with the torsion springs that exhibit torsional stiffness $k_{1-3} = 200\,\mathrm{N\,m\,rad^{-1}}$ and the torsion dampers with torsional damping coefficients $c_{1-3} = 1\,\mathrm{N\,m\,s\,rad^{-1}}$. The nonlinear torsional friction $T_F(\dot\phi_3)$ acts on the last body. The setpoint angular velocity to be tracked by the drillstring is $\omega_s = 10\,\mathrm{rad\,s^{-1}}$. The motor supply voltage u is the control input.

The friction torque is defined as in [4] and consists of the phases of static and decreasing friction. Two possible characteristics are presented in Figure 2.

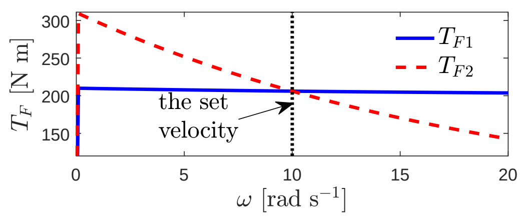

Figure 2: Two friction torque - angular velocity characteristics.

The dynamics of the system can be represented by first order time-variant differential equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{D}T_F(\dot\phi_4), \qquad (1)$$

where the state $\mathbf{x}$ is the vector of the angular positions, angular velocities of the bodies and torque generated by the motor. The initial point of the system considered in the simulations is assumed in the origin of the state-space. Described control scheme uses discrete equivalent of (1) calculated for set sampling time T_s

$$\mathbf{x}_{i+1} = \mathbf{A}\mathbf{x}_i + \mathbf{B}u_i + \mathbf{D}T_{F,i}. \qquad (2)$$

The goal of the control is to track reference trajectory $\mathbf{r}(t)$ and reference input u_r associated with setpoint angular velocity ω_s. It is achieved by minimization of the quadratic functional (3) which penalizes state deviation from the reference trajectory $\boldsymbol{\epsilon} = \mathbf{x} - \mathbf{r}$ and control deviation from the reference input $u_\epsilon = u - u_r$. Such definition of the control goal is typical for reference-tracking

LQR control.

$$J = \sum_{i=0}^{S} \left[\epsilon_i^T \mathbf{Q} \epsilon_i + u_{\epsilon,i}^T R u_{\epsilon,i} \right] \tag{3}$$

The matrices are assumed as follows $\mathbf{Q} = \mathbb{I}$ and $R = 10^{-4}$. The value of R is selected to assure the control value is bounded, i.e. $u \leq 200\,\mathrm{V}$.

3. Algorithm formulation

The control law of the autonomous system

$$\mathbf{x}_{i+1} = \mathbf{A}\mathbf{x}_i + \mathbf{B}\mathbf{u}_i, \quad \mathbf{x}_0 = \mathbf{x}_{\text{initial}} \tag{4}$$

minimizing (3) is known as the linear quadratic regulator and has the form of proportional feedback control

$$\mathbf{u}^*{}_i = -\mathbf{K}_i \cdot \mathbf{x}_i, \tag{5}$$

where time-variant feedback matrix $\mathbf{K}$ is calculated by solving associated discrete-time Riccati dynamic equation (see [2]).

Main proposition of this paper is to turn original nonlinear time-variant system (2) to the linear time-invariant form (4) by approximation of $T_F(\dot{\phi}_4)$ by linear autonomous signal f_i governed by discrete dynamics

$$f_{i+1} = \sum_{j=1}^{p} \theta_j f_{i-j+1}. \tag{6}$$

The weights θ_j of the auto-regressive (AR) model (6) are calculated by a least-square algorithm basing on measurements of past N values of $T_{F,i}$, i.e. the AR model is fitted to reproduce past evolution of $T_{F,i}$ in the least-squares sense. Equation (6) can be easily transformed to the state-space form

$$\mathbf{F}_{i+1} = \begin{bmatrix} f_{i+1} & f_i & \cdots & f_{i+2-n} \end{bmatrix}^T = \mathbf{G} \cdot \mathbf{F}_i. \tag{7}$$

The approximation of (2) is then

$$\begin{bmatrix} \mathbf{x}_{i+1} \\ \mathbf{F}_{i+1} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \begin{bmatrix} \mathbf{D} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} \\ \mathbf{0} & \mathbf{G} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{x}_i \\ \mathbf{F}_i \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \mathbf{u}_i, \tag{8}$$

for which the optimal regulator (5) can be synthesized.

4. Numerical Results

The simulation of the control achieved by the algorithm is compared to the results obtained for the Linear-Quadratic-Gaussian (LQG) regulator. The parameters of LQG regulator were chosen by trial and error to provide best simulation results.

In the first phase of the simulation $t \in [0\,\mathrm{s}, 1\,\mathrm{s})$, the friction torque has the characteristics as T_{F1} in Figure 2. At $t = 1\,\mathrm{s}$ the friction characteristics switches to T_{F2}. The simulation is terminated at $t = 9\,\mathrm{s}$.

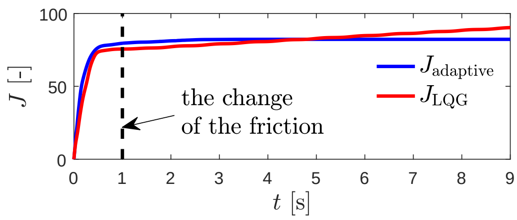

Figure 3: The objective functionals achieved by the algorithm and the LQG regulator.

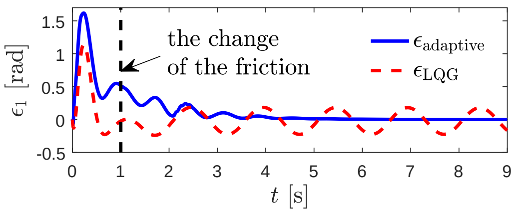

Figure 4: The error of the angular displacement of the first mass controlled with the proposed algorithm and LQG control.

The results are depicted in Figures 3 and 4. One can see that at the first phase of simulation both schemes exhibits a similar effectiveness, but for $t > 1\,\mathrm{s}$, LQG regulator fails to stabilize system and proposed algorithm successfully adapts to changed friction characteristics and steers the system to desired reference trajectory. Failure of LQG results in the lack of convergence of J depicted in Figure 3. On the other hand J associated with proposed scheme quickly converges to constant value, which implies that ϵ converges to 0. The final value of the objective functional J obtained in simulation is 82.25, for LQG control the objective value is $J = 90.28$. The results of this simulation prove the efficiency of the proposed algorithm to adapt to the sudden change of the disturbance. The algorithm was tested in other simulation scenarios, e.g., with the presence of measurement noise or multiple different frictions acting on different bodies. All of this scenarios resulted in smaller value of J achieved for proposed algorithm than for LQG control.

5. Conclusions

The optimal adaptive controller for the dynamical systems with external disturbances was studied. The proposed scheme was applied to the control problem of drillstring. The simulations proved the efficiency of the approach. The algorithm is efficient in the presence of switched and nonlinear disturbance. The numerical simulations shows that the described controller is superior to the Linear Quadratic Gaussian control in the concerned control case. The scheme is robust to the switched characteristics of the friction and under the presence of noise.

The method is versatile and can be applied to numerous engineering problems, for example stabilization of off-road vehicles subjected to sudden uneveness of the surface, stabilization of airplane wings under turbulences, masts under strong wind blows or high buildings subjected to seismic excitation.

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