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Experimental investigation of lateral deflection of columns with intermediate slenderness submitted to axial compressive loads

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ABSTRACT: The buckling phenomenon of prismatic column is experimentally investigated in this work, being major structural design concern. A number of tests were performed on aluminum made prismatic columns with slenderness index λ_{ef} remaining within range of 15 to 460 submitted to programs of multiple in time compressive loads – force and displacement controlled. The experimental evidence obtained here for stocky and slender columns confirm already well known information that column failure results from structural material plastic flow deformation or lateral buckling, respectively. A new, interesting observation has been made for intermediate slenderness columns that "drop" of buckling load takes place (reaching in some cases over 20% of early critical force value) after the very initiation of elasto-plastic buckling process. The observation is in contradiction to adopted in elasto-plastic buckling models assumption of constant (Engesser model) or growing (Shanley model) buckling load during buckling process. The obtained experimental evidence indicates that mysterious scatter in buckling loads for apparently identical columns with intermediate slenderness index can be attributed to minute residual stresses, which easily and in hardly controlled manner can be introduced during industrial production processes.

1 INTRODUCTION

1.1 *Elastic buckling model*

Engineering structures must comply to a number of requirements in order to exhibit required from them functional features. They must be strong enough to withstand applied load, and at the same time they must be sufficiently stiff not to deform beyond prescribed limits. Sometimes load and initially acceptable by itself deflections may lead in combined action to a situation when structure warps and undergoes changes leading to catastrophic failure. Spectacular example of such a situation is phenomenon called buckling when a structure, due to increasing axial load, looses its stability by depletion of its capacity to withstand a lateral loading. The comprehension of the *physical reason* for losing structural stability by axially loaded column, i.e. depletion of its lateral stiffness to zero value led Euler, in his masters thesis from 1744, to formulation of a fundamental at present criterion for elastic buckling – $P_n = n\pi^2 EI/L^2$, n = 1, 2, 3... We will be here interested only in the lowest value of critical load (n = 1), commonly at present called Euler load or buckling load:

$$P_E = \pi^2 E\,A/\lambda_{ef}^2, \quad \sigma_E = \pi^2 E/\lambda_{ef}^2, \quad \lambda_{ef} = L_{ef}/r,$$
$$L_{ef} = k \cdot L, \quad I = A \cdot r^2, \qquad (1)$$

where P_E = Euler buckling load, E = Young modulus, A = column cross section area, λ_{ef} = column effective slenderness, σ_E = Euler buckling stress, L = column length, k = index depending on column support conditions (for clamped-clamped ended column k = 1/2, I = column cross section moment of inertia (for rectangular prismatic column $I = bh^3/12$; b = cross section width, h = cross section height), r = radius of inertia.

Euler formulated his criterion under the general assumption that everything is proportional, i.e. he assumed linear material behavior, linear geometrical effects – there appear only small deflections, and small strains. An interesting outcome from Euler *linear mathematical model* formulation of the buckling phenomenon has been observation that the instant of total physical depletion of column capacity to withstand even slightest lateral loading is equivalent to the appearance of second (multiple) eigenvalue and eigensolution of underlying mathematical boundary value problem, expressing balance of flexural moments. This resulted in one another name for buckling phenomenon, i.e. buckling bifurcation. Later there appeared the third very fruitful approach to investigating stability problems of structures, i.e. *energy method* consisting in study of energy landscape of some neighborhood of specific equilibrium state of a structure. All three methodological approaches of investigating structural stability are successfully used at present, see Simitses & Hodge (2006).

1.2 *Elasto-plastic buckling model*

Since the pioneering work of Euler a lot of effort and systematic works have been continuously undertaken aimed at expanding the scope of knowledge on structural stability. Excellent report on achievements in that respect embracing not only columns but other structural members like frames, plates or shells with complications in the form of various types of nonlinearities geometrical, material or in boundary conditions can be found in a book by Bažant & Cedolin (2003). While considerable understanding and insight has been gained in the field, it is the present authors believe that still even the "simplest" case of prismatic column buckling deserves further research attention. The reason for that statement is considerable scatter (reaching several dozen or so percent) of experimentally registered values of critical loads at which apparently identical columns are buckling, whenever their effective slenderness index remain within the range of intermediate values. The above inspiration is further intensified by existence of numerous buckling criteria for loaded in compression structural members. Some of these criteria are predominantly experimental practice oriented, like Tetmeyer-Jasinsky criterion in which case the relation between critical stress and column slenderness is approximated by three segments with straight line approximating the relation for intermediate slenderness columns, or Johnson parabola criterion,

$$\sigma_{John} = \sigma_{0.2} - (\sigma_{0.2}^2 / 4\pi^2 E) \cdot \lambda_{ef}^2, \quad 0 \le \lambda_{ef} \le \lambda_{ef2} \quad (2)$$

$$\sigma_{John}(\lambda_{ef} = \lambda_{ef2}) = \tfrac{1}{2}\sigma_{0.2} = \sigma_E(\lambda_{ef} = \pi\sqrt{2E/\sigma_{0.2}})$$

where σ_{John} = Johnson critical stress, $\sigma_{0.2}$ = conventional plastic yield stress.

The other phenomenological buckling criteria are theoretically inspired, like in the case of Engesser tangent modulus model developed for nonlinear elastic regime of the material behavior, Engesser reduced modulus or Shanley models developed for elasto-plastic buckling

$$\sigma_{EngI} = \pi^2 E_t / \lambda_{ef}^2, \quad \sigma_{EngII} = \pi^2 E_r / \lambda_{ef}^2, \quad (3)$$

where σ_{EngI} = Engesser tangent modulus critical stress, E_t = instantaneous tangent Young modulus ($E_t = d\sigma/d\varepsilon$), σ_{EngII} = Engesser reduced modulus critical stress, E_r = reduced modulus value depends on the column geometrical cross section e.g. $E_r = 4EE_t/(\sqrt{E}+\sqrt{E_t})^{1/2}$ for rectangular cross section, σ_{Shan} = Shanley critical stress. The modeling predictions of buckling stress – for fixed slenderness column, can be ordered as follows: $\sigma_{EngI} \le \sigma_{EngII} < \sigma_{Shan} < \sigma_E$. These models of column buckling assume constant or increasing value of axial compressive load in the stress states neighboring elasto-plastic buckling event, see excellent book by Gere (2001) for detailed discussion of these models.

Nowadays design practice of loaded in compression structural members, limit state design, bases on (modified) Perry formula, see Dwight (1999). The approach is a return to three segment idea of Tetmayer-Jasinsky, in which additionally "imperfections" of column-load system are taken care of by experimental curve fitting in order to properly handle observed experimentally large scatter of buckling loads for intermediate slenderness columns

$$(\sigma_{0.2} - \sigma_{Perr}) \cdot (\sigma_E - \sigma_{Perr}) = \eta \cdot \sigma_E \cdot \sigma_{Perr} \quad (4)$$

where σ_{Perr} = Perry critical stress, η = Perry factor characterizing the column-load system imperfections (initial curvature, load eccentricity, residual stresses, inhomogeneities, etc). The η is usually expressed in terms of column slenderness index λ_{ef}, $\eta = c\cdot(\lambda_{ef} - \lambda_1)/\lambda_2$. The value of c is taken depending on mode of column operation, e.g. pure compression, torsion, bending, etc. For stocky columns $0 < \lambda_{ef} < \lambda_1 \sigma_{Perr} = \sigma_{0.2}$ and for slender ones $\lambda_2 < \lambda_{ef}$ $\sigma_{Perr} = \sigma_E$. The admissible value of stress is obtained by dividing Perry critical stress with some safety coefficient.

In order to better understand the origins and dependence of buckling load values scatter observed for intermediate slenderness columns an experimental program has been planed and executed.

2 EXPERIMENTAL PROCEDURES

2.1 *Material properties*

For purposes of the present study there have been used specimen in a form of prismatic aluminum flats, commercially produced, with rectangle cross section $A = b \times d = 5.88 \pm 0.01 \times 19.98 \pm 0.01 = 117.5[mm^2]$, cut to appropriate lengths to obtain required effective slenderness. Geometric precision of manufacturing of the used flats is high as thickness and width deviations did not exceed 0.02 mm along 1 meter length of the flats. Similarly straightness deviation has been assessed not to exceed 1 mm/m. The flats originated from commercial source and its processing history has been unknown. In view of that the material properties have been first determined by performing tests on dog bone specimen with use of MTS 858 machine – force and frame displacement control accuracy estimated to be $\Delta F \cong 2.5$ [N], $\Delta u \cong 4[\mu m]$. The loading program consisted in uniaxial tension–compression with enforced 1% plastic strain, see Figure 1. The values of material properties determined from experimental charts are as follows: $E = 63.3$[GPa], Poisson ration – $\upsilon = 0.38$, Proportionality limit – $\sigma_P = 150$[MPa] ($\lambda_P = \pi\cdot(E/\sigma_p)^{0.5} = 64.6$), $\sigma_{0.2} = 189$ [MPa] ($\lambda_{0.2} = \pi\cdot(E/\sigma_p)^{0.5} = 57.6$), $\sigma_{UTS} = 217.5$ [MPa], $\varepsilon_{UTS} = 6.91$[%], $\varepsilon_{break} \cong 11.6$ [%.].

The proportionality and yield limits have been determined from stress-plastic strain chart ($\sigma - \varepsilon_p$), where $\varepsilon_p = \varepsilon - \sigma/E$ with offsets 0.02% for determination of σ_P and 0.2% for determination of $\sigma_{0.2}$. The character of stress-strain curve and material parameters values allow to indicate the material to be Al-6000 series submitted to artificial aging, stress relieving

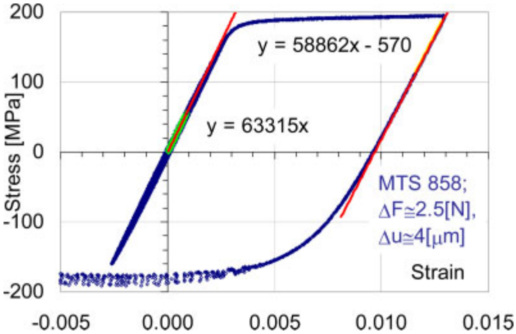

Figure 1. The uniaxial, tension-compression, nominal stress – nominal strain curve obtained for used in program of buckling tests aluminum material.

heat treatment after extrusion process (T6 the most probably).

2.2 *Experimental setup and testing program*

The program of buckling tests has been performed using MTS 810 machine with 250 KN load capacity – force and frame displacement control accuracy estimated to be $\Delta F \cong 25$ [N], $\Delta u \cong 3$ [μm], at room temperature. The specimen in the form of prismatic flats cut to appropriate length were clamped in machine grips (fixed-fixed column) to assure required effective slenderness. It was not possible to assure accuracy of gripping better than $\Delta L = \pm 0.5$ [mm] ($\Delta L_{ef} = 0.5$ [mm]). Such accuracy results in the following variation of predicted Euler loads – see formula (1): {$L_{ef} = 5$ [cm], $\lambda_{ef} = 29.5 \Delta P_E = 1718$[N]}, {$L_{ef} = 12$ [cm], $\lambda_{ef} = 70.7 \Delta P_E = 123$[N]}, {$L_{ef} = 17$ [cm], $\lambda_{ef} = 100.1 \Delta P_E = 43$ [N]}, {$L_{ef} = 40$ [cm], $\lambda_{ef} = 235.6 \Delta P_E = 3$[N]}. Lateral deflection in the middle length of buckling column has been measured with laser sensor – of estimated accuracy $\Delta f \cong 3$ [μm], mounted on a moving frame to follow vertical motion of the buckling column vertical midpoint. The column specimen with the following effective lengths has been tested $L_{ef} = 2.5, 5, 8.5, 12, 17, 24, 40$ [cm] – also with $b = 3$ [mm]. The specimen has been submitted to force controlled, displacement controlled and mixed controlled programs selected in such a way as to carefully investigate the specimen behavior in the neighborhood of buckling event, and also their post-buckling behavior. Selected, representative results from the testing program are presented in the next section.

3 RESULTS AND DISCUSSION

3.1 *Stocky and slender column behavior*

In Figure 2 buckling and post-buckling behavior of stocky $\lambda_{ef} = 30.3$ column behavior is shown. The column initially exhibits elastic behavior, then enters plastic flow regime to finally start buckling at strain

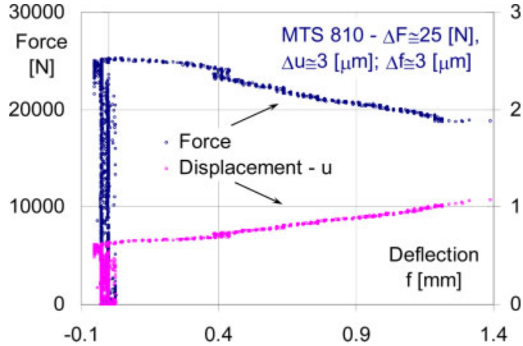

Figure 2. Buckling and post buckling behavior of column with slenderness $\lambda_{ef} = 30.3$, (L_{ef}=5.1 cm, $P_E = 84.3$ [KN], $\sigma_E = 717.2$ MPa) under displacement load control. Buckling force – $P_B = 25200$ [N] ($\sigma_B = 212$ [MPa]).

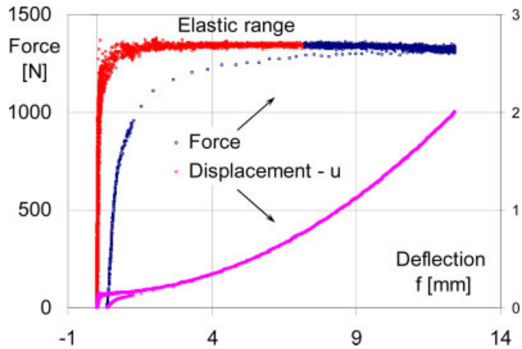

Figure 3. Buckling and post buckling behavior of column with slenderness $\lambda_{ef} = 235.6$, ($L_{ef} = 40.0$ cm, $P_E = 1321$ [N], $\sigma_E = 11.2$ MPa) under displacement load control. Buckling force – $P_B = 1342$ [N].

$\varepsilon = 0.62\%$ and stress $\sigma = 212 \cong 217$ [MPa] – ultimate stress of the material.

In Figure 3 buckling and post-buckling behavior of slender $\lambda_{ef} = 235.6$ column behavior is shown. The column behavior is perfectly as predicted by Euler elastic buckling model. The red (grey) color in fact presents several loading-unloading of column with its elastic buckling involved. Only when later deflection exceeds about 7 mm the unloading path starts differ from the loading one indicating for plastic deformation regime.

In Figure 4 buckling and post-buckling behavior of slender $\lambda_{ef} = 100.1$ column behavior is shown submitted to force and displacement controlled loading, respectively. It is interesting to note that "post catastrophic" buckling path in load controlled test follows practically the same series of states (P, f), which are followed in "non-catastrophic" way during displacement controlled experiment. In both tests the same value of buckling force is reached (within experimental error tolerance) and this being 98.9% of Euler load. No buckling load drop discussed below has been observed for column with this slenderness.

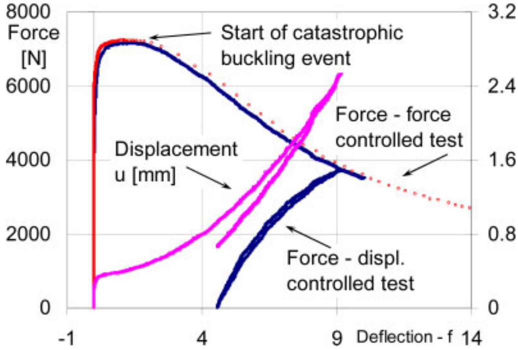

Figure 4. Comparison of buckling and post buckling behavior of column with slenderness $\lambda_{ef} = 100.1$, ($L_{ef} = 17.0$ cm, $P_E = 7320$ [N], $\sigma_E = 62.3$ MPa) under displacement and force load control. Force controlled test buckling force,- $P_B = 7240$ [N]; Displacement controlled test buckling force,- $P_B = 7180$ [N].

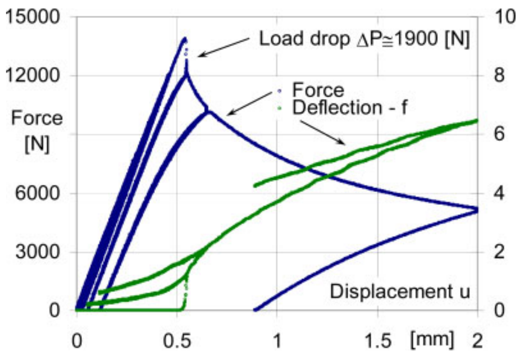

Figure 5. Buckling and post buckling behavior of column with slenderness $\lambda_{ef} = 71.9$, ($L_{ef} = 12.2$ cm, $P_E = 14200$ [N], $\sigma_E = 120.9$ MPa) – $P_B = 13870$ [N] drops to $P = 12000$ [N].

3.2 *Intermediate slenderness column behavior*

In Figure 5 buckling and post-buckling behavior of intermediate slenderness $\lambda_{ef} = 71.9$ column behavior is shown. The column after reaching buckling force load $P_B = 13870$ [N] (97.7% of Euler buckling load), remaining under displacement control, rapidly drops this load and stabilizes at loading force of $P = 12000$ [N]. Upon the column unloading residual displacement is $u_{res} = 0.06$ [mm] and residual deflection $f_{res} = 0.2$ [mm] – this last value remaining on the level of allowable deviation from straightness accepted as standard for commercially produced aluminum flats. Upon repeated loading the column attains the last "stabile" value of buckling force, i.e. $P = 12000$ [N] (84.5% of Euler buckling load) before further plastic bending, buckling takes place. The described above sequence of events allows to explain scatter of buckling load values observed by different researchers for apparently identical columns with the same slenderness index.

The character of buckling load drop process can be best observed in time chart of column loading and its response shown in Figure 6.

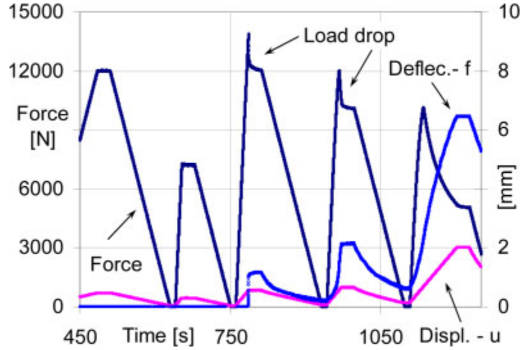

Figure 6. Loading program displacement – u controlled of column with slenderness $\lambda_{ef} = 71.9$ and its force and deflection – f response graphed in time.

4 SUMMARY

Experimental results of a program of prismatic columns buckling tests have been presented in the paper within the context of known theoretical column buckling models and currently applied design practice. The obtained experimental evidence allows to dare statement that minute residual stresses present in column member are responsible for several dozen scatter of observed buckling load value in the case of columns with intermediate slenderness index (around λ_P- here 64.6). The practical implication of the presented here research effort can be formulated as follows: It is fully justified to apply safety coefficients on the level of 1.6 (around 40% safety margin in buckling load) for intermediate slenderness columns as this guarantees that after accidental (force major) introduction of moderate residual stresses caused by these accidental plastic bending of the column member, it will still possess expected (required) in design process load capacity.

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