12th INCF Workshop on Node Communication
and Collaborative Neuroinformatics
Warsaw, April 16-17, 2015

Analyzing Neuroscience Signals using Information Theory and Complexity

Shannon Communication Approach

Janusz Szczepanski

Co-Authors:

Maria Victoria Sanchez-Vives with Laboratory (M. M. Arnold, N. Montejo)

B. Paprocki, A. Pregowska, J. M. Amigó, E. Wajnryb

Spanish-Polish Scientific Cooperation Programs CSIC – PAS

IDIBAPS Institut D'Investigacions Biomèdiques August Pi i Sunyer

IPPT PAN

UNIWERSYTET KAZIMIERZA WIELKIEGO W BYDGOSZCZY

UNIVERSITAS Miguel Hernández

Lines of Research

Experiments	Modeling Numerical simulations	Information Theory Complexity
▪ Prof. M. V. Sanchez-Vives IDIBAPS Institut D'Investigacions Biomèdiques August Pi i Sunyer ▪ Prof. E. Kaplan Mount Sinai Hospital Ph. D. Alexander Casti ▪ Nencki Institute nencki institute of experimental biology	▪ M. Sc. B. Paprocki ▪ Ph.D. A. Pregowska Prof. E. Wajnryb IPPT PAN ▪ Prof. J. Szczepanski Prof. J. M. Amigó IPPT PAN UNIVERSITAS Miguel Hernández	▪ Prof. J. Szczepanski Prof. E. Wajnryb M. Sc. B. Paprocki Prof. J. M. Amigó Ph.D. A. Pregowska *Prof. J. Karbowski* IPPT PAN UNIVERSITAS Miguel Hernández

Content

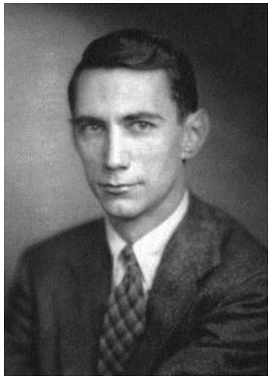
- Fundamental questions/ General statement of the problem
- Shannon Communication Approach – **Information Theory**
- **Mutual Information** and Shannon Fundamental Theorem (Decoding)
- **Entropy** (Rate) and (**Lempel-Ziv**) Complexity
- ***Experiments***
  - **Intracellular recordings *in vivo* and *in vitro***
  - ***Signals classification via Complexity/Entropy***
  - Relative Mutual Information -
    measuring transmission **efficiency**
  - Redundancy - experimental results
    measuring neurons population **collaborations**
- ***Brain-inspired networks***
- Model of neuron: **Levy-Baxter** idea
- **Brain**-inspired networks **efficiency** criteria - results
- Conclusions

Fundamental questions

Shannon C. E., ***The Bell System Technical Journal***, **1948**

- What is **Information**?
- How we can define and measure the quantitative information?

Shannon Communication Theory



- For which objects it is possible to define the information?

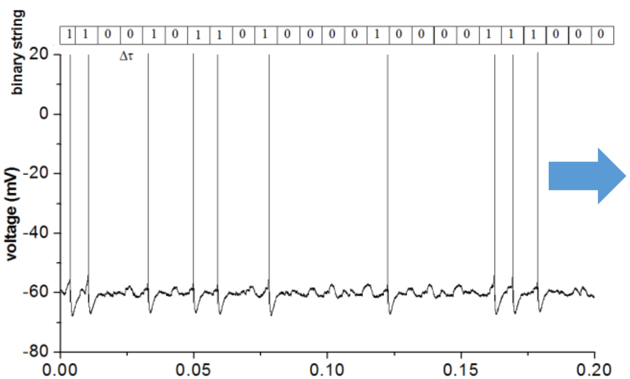
Uncertain phenomena… mathematically **random variables, stochastic processes**

Assuming *reasonable* axioms Shannon derived the formulae:

$$I(x_i) = -\log P(x_i)$$

General statement of the problem

Shannon C. E., ***The Bell System Technical Journal***, 1948

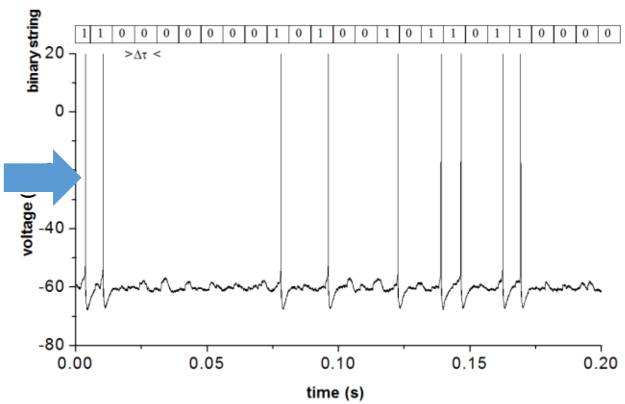


INPUT
Spike trains
seqences of bits

Shannon
Communication Channel

Neuronal Network
(Neurons)

Unfortunately, noise
is inherent property
of biological systems



OUTPUT
Spike trains
sequences of bits

Decoding scheme; Optimal ??? - Shannon.Fund.Theorem

Communication Channel

Neurons, neural networks ---> Communication channel

conditional probabilities

general formulae $p_n(\underbrace{y_1, y_2, \ldots, y_n}|\underbrace{x_1, x_2, \ldots, x_n}, s) \leftarrow$ s - states

$y_1, y_2, \ldots, y_n$ - **output symbols** $x_1, x_2, \ldots, x_n$ - **input symbols**

Memoryless channel

$$p_n(y_1, y_2, \ldots, y_n|x_1, x_2, \ldots, x_n, s) = p_1(y_1|x_1), p_1(y_2|x_2), \ldots, p_1(y_n|x_n)$$

Fundamental channel characteristic

Channel capacity $C := \max_{p(x)} \underbrace{MI(X,Y)}_{\text{Mutual Information}}$

Shannon Fundamental Theorem

Decoding opportunities

TARGET: EXISTENCE OF ***DECODING SCHEME*** FOR A GIVEN PROBABILITY ERROR ε

Given a discrete memoryless channel with capacity *C*>0 and a positive number *R*<*C* there exists sequence of codes

$$A_n \Rightarrow \left(\left[2^{nR} \right], n, \lambda_n \right)$$

where

n **– is the length of word associated to a given symbol to be transmitted**

$\left[2^{nR} \right]$ - **number of symbols from a given alphabet {a,b,c, …} to be encoded by sequences of bits of length n**

$\lambda_n \to 0$ **probability of error** $\lambda_n < \varepsilon$

The point is *R must be less than C !!!* then, there exists decoding scheme for given error

Mutual Information

Mutual Information formula

$$\boldsymbol{MI(X,Y) \coloneqq H(X) - H(X|Y) = H(X) + H(Y) - H(X,Y)}$$

$H(X) \coloneqq -\sum_{i\in I_s} p(X=i)\log p(X=i)$ entropy of the INPUT

$H(Y) \coloneqq -\sum_{j\in O_s} p(Y=j)\log p(Y=j)$entropy of the OUTPUT

$H(X,Y) \coloneqq \coloneqq -\sum_{i\in I_s} p(X=i) H(Y|X=i)$ JOIINT entropy

To be estimated -
tacitly assumed ergodicity

Conditional entropy

$$H(X|Y) = \sum_{y\epsilon Y} p(y) H(X|y) = -\sum_{y\epsilon Y} p(y) \sum_{x\epsilon X} p(x|y)\log p(x|y)$$

Entropy and (Lempel-Ziv) Complexity

Lempel A., Ziv J., ***IEEE Transactions on Information Theory***, 1976

- Entropy = Average Information (measure of uncertainty)

X-random variable
I_S- set of values to be reached by X
$p(X = i)$ – probability the random variable X reaches the value i

$$H(X) := E(I(X)) \coloneqq -\sum_{i \in I_s} p(X = i) \log p(X = i)$$

- Lempel-Ziv Complexity (1976) **Complexity converges to Entropy**

LZ: **Number of new phrases which arrived along the sequence**

Example: $Seq = 0101101000110111 0010$

New phrases: $0\ \ 1\ \ 011\ \ 0100\ \ 011011\ \ 1001\ \ 0$

$LZ(Seq) = 7$

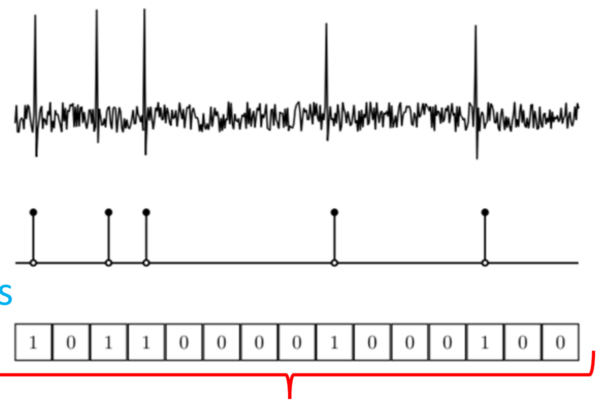
Pattern matching approach !!!
Idea: to handle short sequences

Entropy estimation

Estimation of the entropy

Lempel A., Ziv J., ***IEEE Transactions on Information Theory***, 1976

Number of symbols



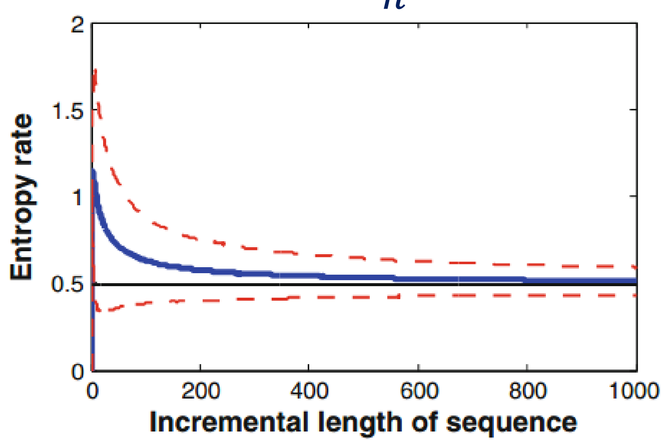
encoded spike train

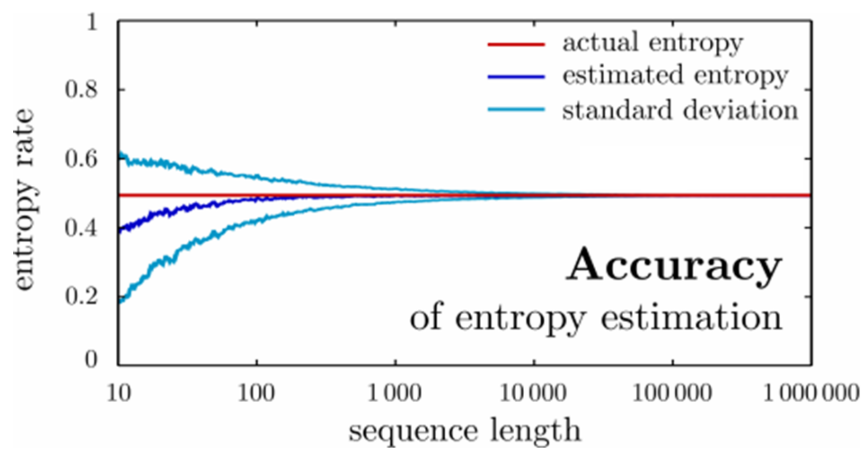
Strong S. P., et. all, ***Physical Review Letters***, 1998

Lempel-Ziv estimator

$$\hat{H}^{LZ76} = LZ(Seq)\,\frac{\log n}{n}$$

Strong estimator

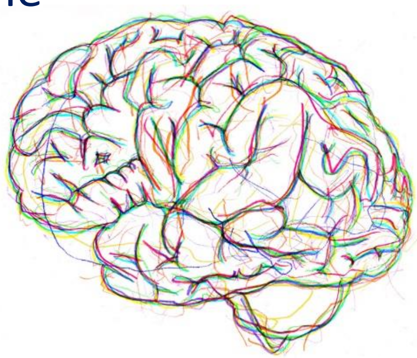




Fundamental questions

van Hemmen J. L., Sejnowski, **23 Problems in Systems Neuroscience** ***Oxford University Press***, 2006

- What is optimized in biological systems during transmission of information?
  - Mutual Information ? (in Shannon Theory sense)
  - Mutual Information per energy used ?
  - Something else ???
- How efficiency of information transmission is affected by the mechanisms formed in the process of evolution

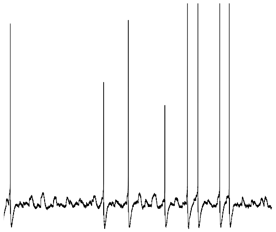


Experiments

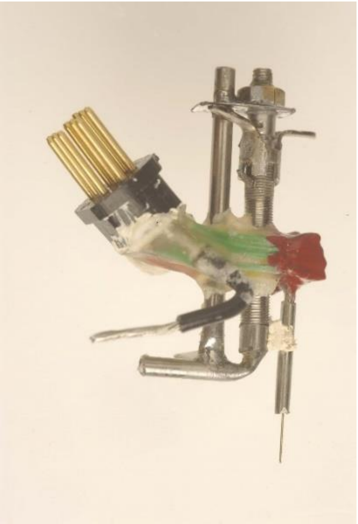
Sanchez-Vives Lab

Spike train recordings

Implanted rat/or cat



Spike



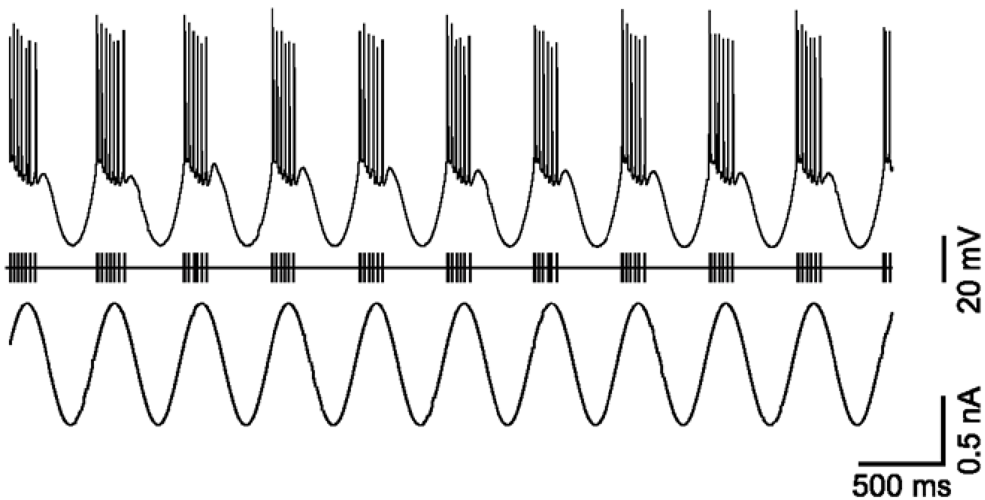
Microdrive mounted with two **tetrodes**

Szczepanski J., Arnold M., Wajnryb E., Amigó J. M., Sanchez-Vives M. V., ***Network***, , 2003
Amigó J., Szczepanski J., Wajnryb E., Sanchez-Vives M. V., ***Biosystems***, 2003
Szczepanski J., Amigó J. M., Wajnryb E., Sanchez-Vives M. V., ***Neurocomputing***, 2004
Amigó J. M., Szczepanski J., Wajnryb E., Sanchez-Vives M. V., ***Neural Computation***, 2004
Szczepanski J., Arnold M., Wajnryb E., Amigó J. M., Sanchez-Vives M. V., ***Biological Cybernetics***, 2011
Arnold M. M., Szczepanski J., Montejo N., Amigó J. M., Wajnryb E., Sanchez-Vives M. V., ***Journal of Sleep Research***, 2013

Experiments (idea)

Sanchez-Vives M. V., Nowak L. G., McCormick D., ***Journal of Neuroscience***, 2000

Sanchez-Vives Lab



Visual stimulus consisted of sinusoidal drifting grating presented in a circular patch

Intracellular recordings from a cortical cell in vivo and vitro during sinusoidal current injection

A membrane potential trace showing the trajectory while intracellular sinusoidal currant was injected. During the depolarizing phase the membrane potential value reached threshold, inducing a train of spikes or action potentials

Spikes as acquired in a separate channel to be used for the analysis

Sinusoidal current injected into the cell

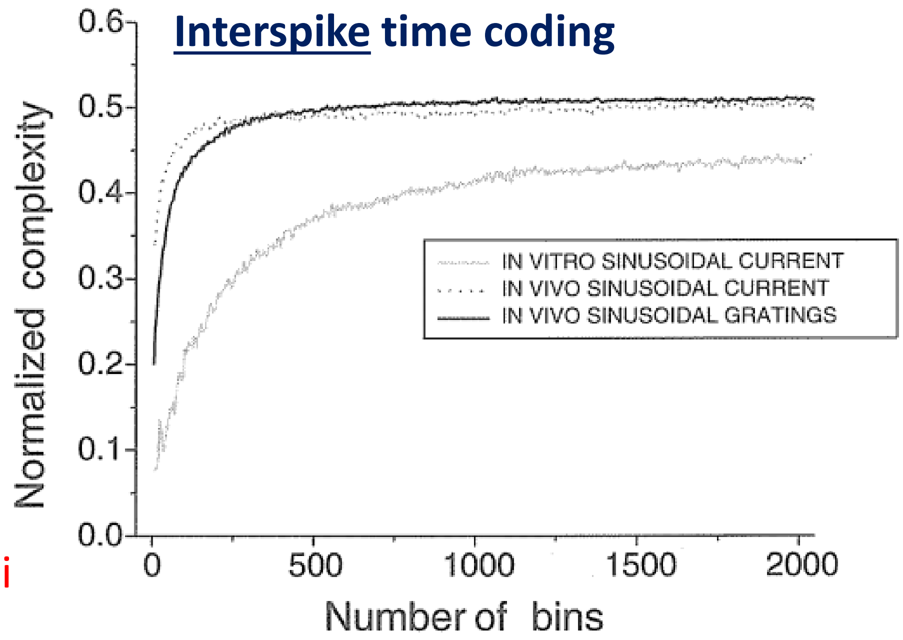
Experimental results

Intracellular recordings *in vivo* and *in vitro - classification*

Szczepanski J., Amigó J. M., Wajnryb E., Sanchez-Vives M. V., ***Network: Computation in Neural Systems***, 2003

Sanchez-Vives Lab

Interspike time coding



Current and Visual Stimuli

Complexity –
Signal **classification** for given type of Stimuli

The data was obtained from primary cortex recordings both *in vivo* and in brain slice preparations (*in vitro*)

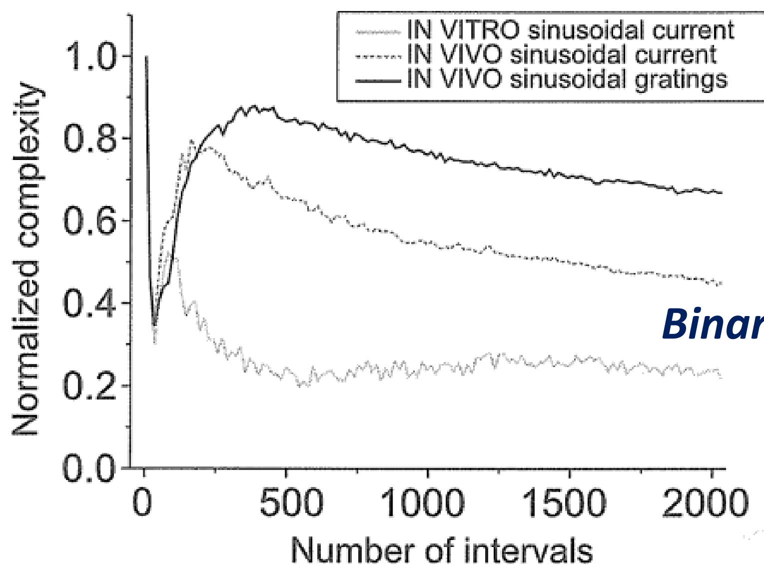
Intracellular recordings *in vivo* were obtained from anaesthetized **adult cats**

Experimental results

Intracellular recordings *in vivo* and *in vitro - classification*

Szczepanski J., Amigó J. M., Wajnryb E., Sanchez-Vives M. V., ***Network: Computation in Neural Systems***, 2003

Sanchez-Vives Lab



Binary bin coding

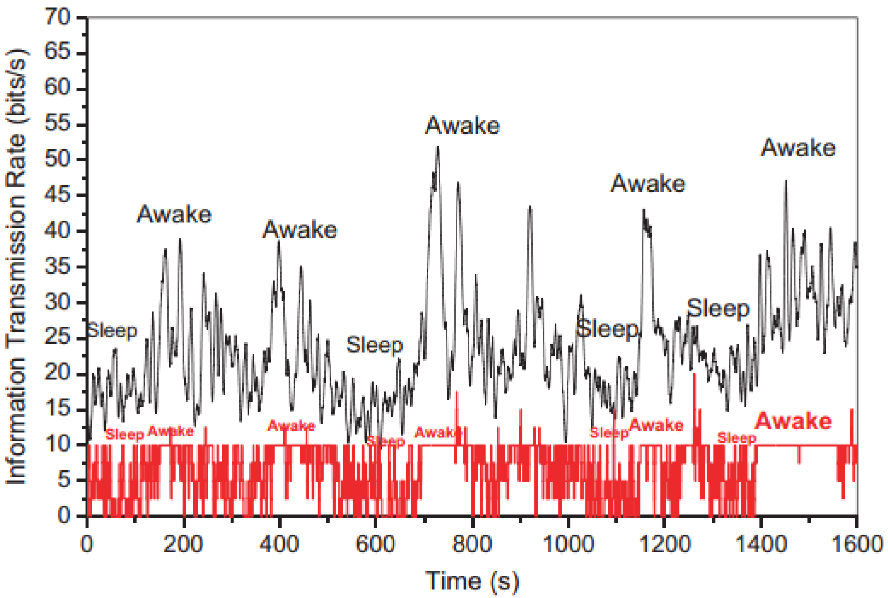
Normalized complexity versus number of intervals for periodic stimuli
More information is transmitted with binary bin coding
Significant advantage with *in vivo*

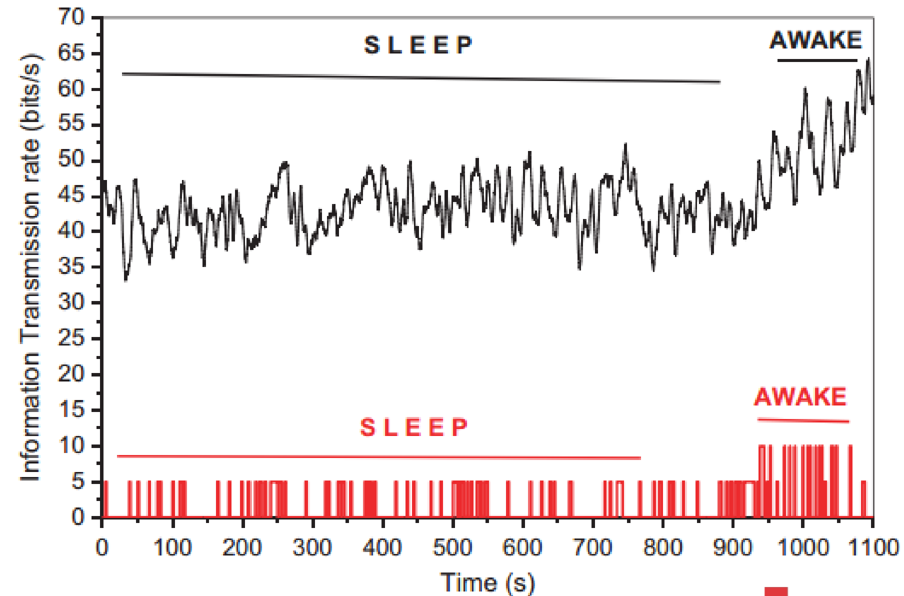
Polish National Science Centre grant N N519 646540, and by the Spanish Ministry of Science and Innovation to Maria V. Sanchez-Vives (BFU2008-01371 and BFU2011- 27094). Jose M. Amigo was supported by the Spanish Ministry of Science and Innovation, grant MTM2009-11820

Experimental results (brain states …)

Arnold M. M., Szczepanski J., Montejo N., Amigó J. M., Wajnryb E., Sanchez-Vives M. V., ***Journal of Sleep Research***, 2013

Sanchez-Vives Lab





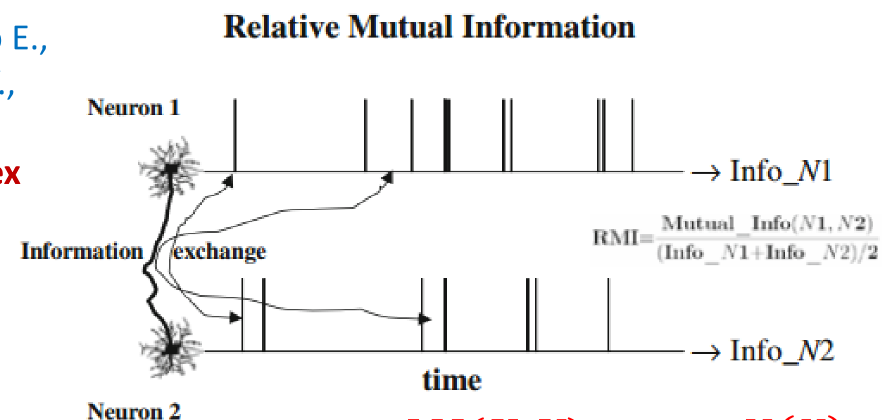
Male Listed Hooded ***Rat*** – tetrodes were implanted in ***primary visual cortex***
Typical runs of the information rate for two neurons (**spike trains**)
The awake-sleep transitions for two typical physiological states as a function of time
The rat alternated several times between the states of sleep and awake
The brain states classification by **EEG** (red line) and **behavioral** observations

IDIBAPS Institut D'Investigacions Biomèdiques August Pi i Sunyer
UNIVERSITAS Miguel Hernández
IPPT PAN

Relative Mutual Information
measuring transmission efficiency

Szczepanski J., Arnold M., Wajnryb E., Amigó J. M., Sanchez-Vives M. V., ***Biological Cybernetics***, 2011

Exp.: Rats – primary visual cortex



Relative Mutual Information concept

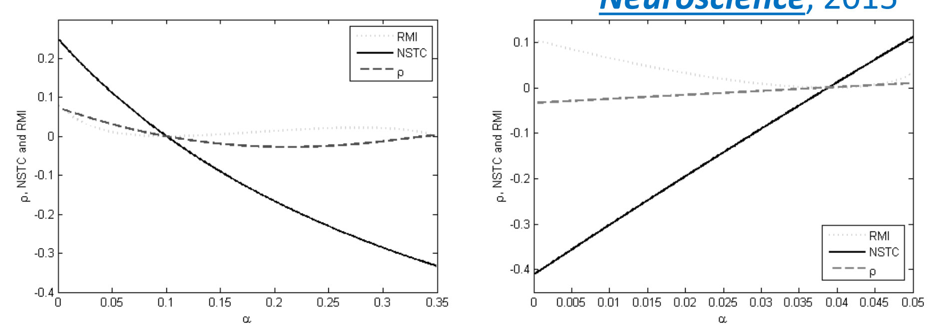
$$\boldsymbol{RMI(X,Y)} := \frac{\boldsymbol{MI(X,Y)}}{\boldsymbol{[H(X)+H(Y)]/2}} = \frac{H(X)+H(Y)-H(X,Y)}{[H(X)+H(Y)]/2}$$

Pregowska A., Szczepanski J., Wajnryb E., ***BMC Neuroscience***, 2015

Relative Mutual Information against Correlations

The quantitate Neuroscience Spike-Trains Correlation (NSTC) coefficient

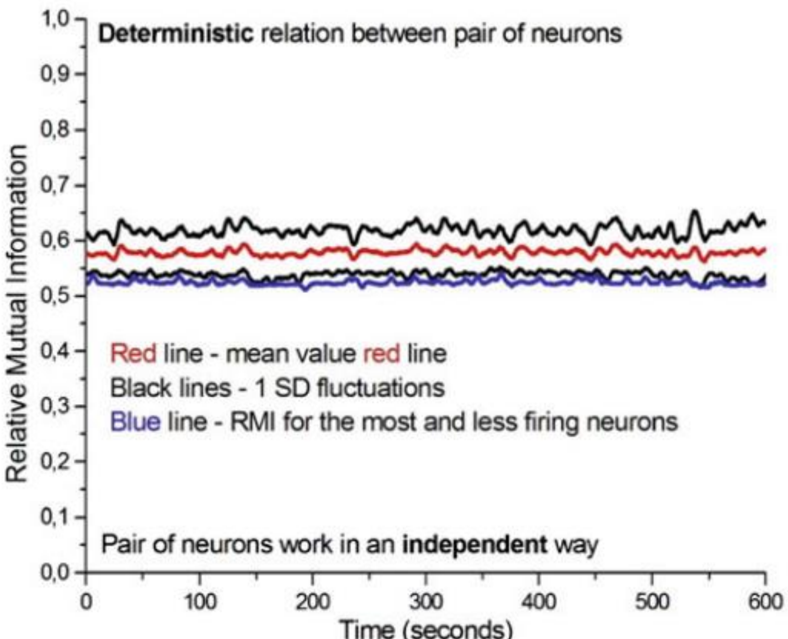
$$NSTC(X,Y) := \frac{p_{11} - p_1^X \cdot p_1^Y}{p_1^X \cdot p_1^Y}$$

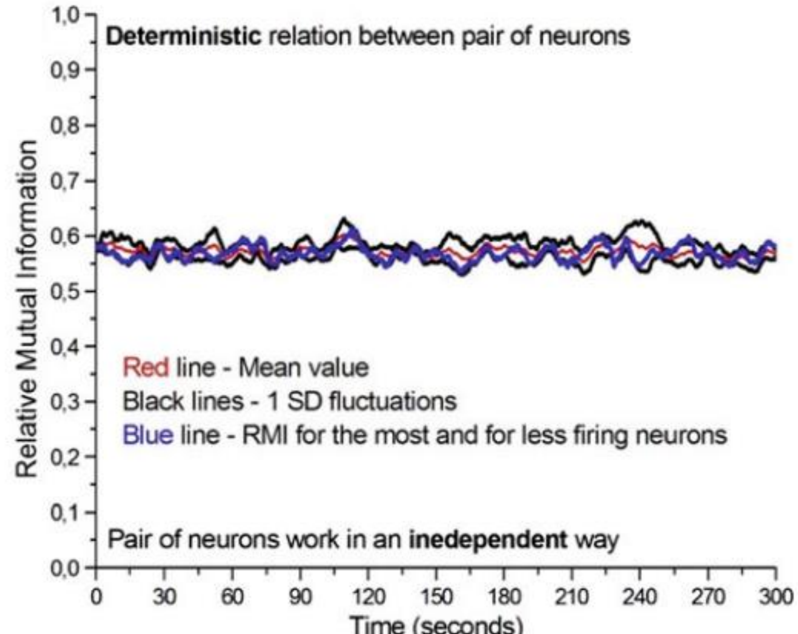


Relative Mutual Information - experimental results

Szczepanski J., Arnold M., Wajnryb E., Amigó J. M., Sanchez-Vives M. V., ***Biological Cybernetics***, 2011

Sanchez-Vives Lab





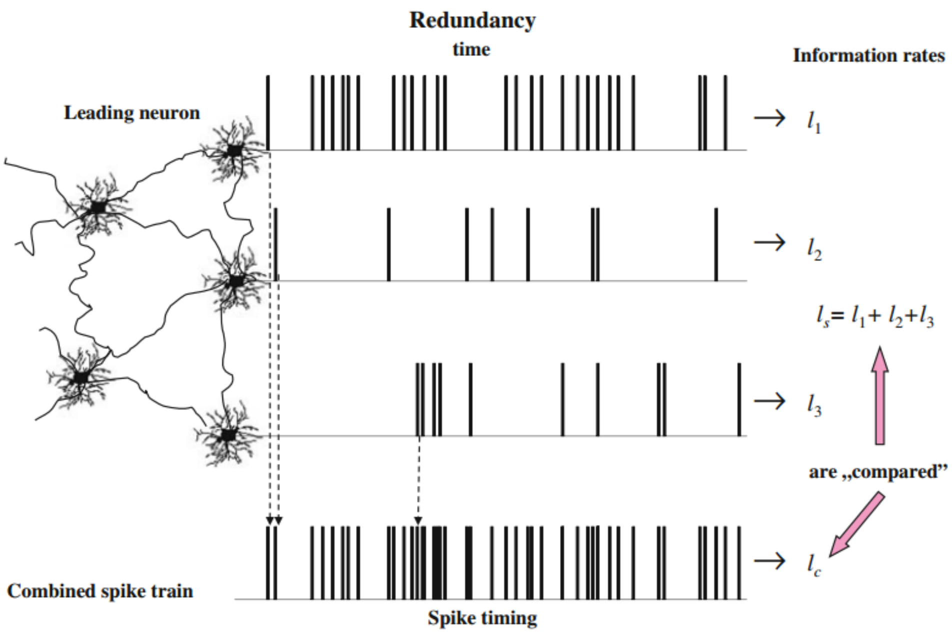
Recordings were obtained from Lister Hooded rats weighing 300-400 g at the time of surgery

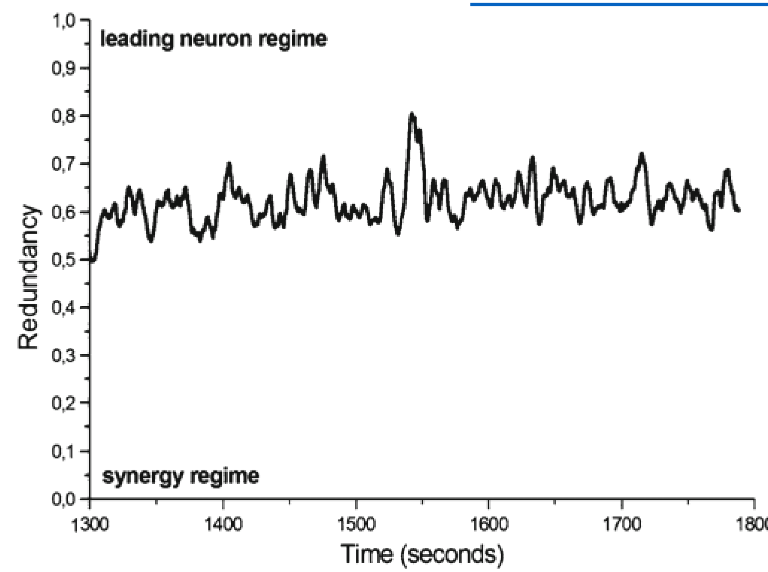
The brain was sliced coronally into 100-µm thick sections, which were mounted and sustained to aid visualization of the electrode track and trip

tetrodes were implanted in the **primary visual cortex** **13 neurons** **6 neurons**

Redundancy - experimental results
measuring neurons population collaborations

Sanchez-Vives Lab





Redundancy concept
Reich D. S., Mechler F., Victor J. D., ***Science*** 2001

$$R = \frac{l_S - l_C}{l_S - \max_{1 \le i \le k}\{l_i\}}$$

Szczepanski J., Arnold M., Wajnryb E., Amigó J. M., Sanchez-Vives M. V., ***Biological Cybernetics,*** 2011

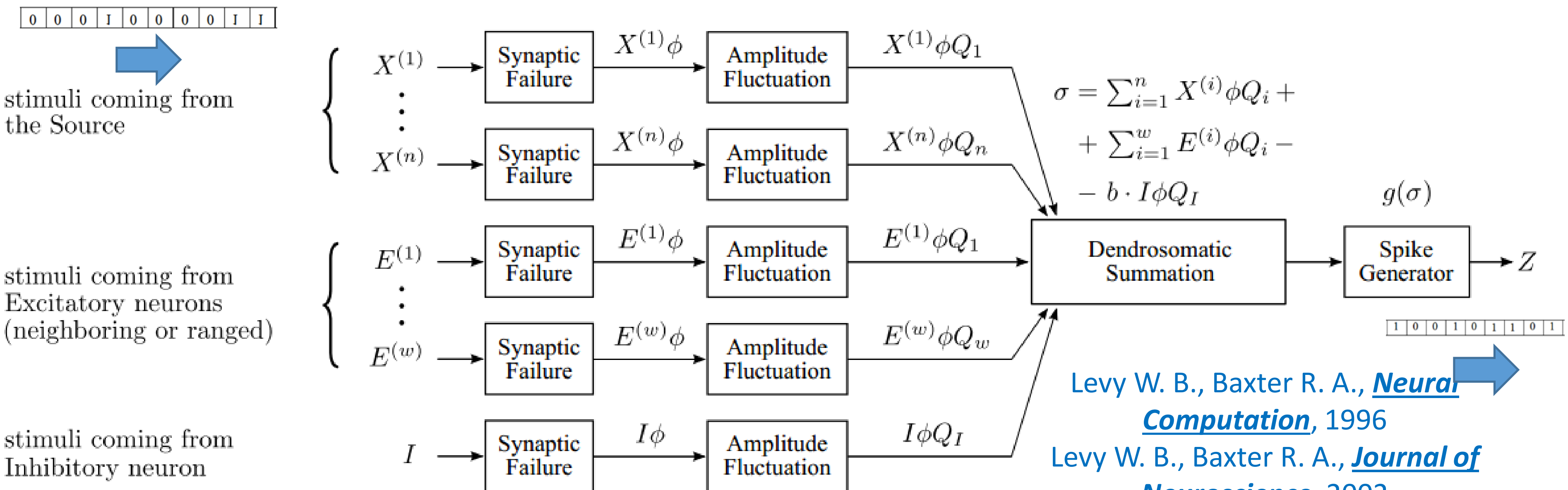
l_i-the information rate of neuron N_i
l_S- the sum of information rates for each cell separately
l_C- the information rate of the combined spike train

BRAIN –inspired networks

Model of neuron proposed by Levy-Baxter

probabilistic approach

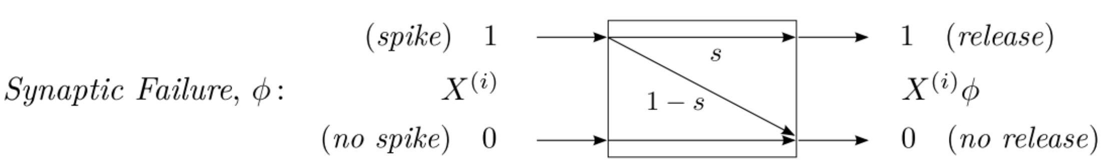
incorporates all *essential qualitative* mechanisms



Levy W. B., Baxter R. A., ***Neural Computation***, 1996

Levy W. B., Baxter R. A., ***Journal of Neuroscience***, 2002

Paprocki B., Szczepanski J., ***Neurocomputing***, 2013



Model of neuron: Levy-Baxter idea

Activation Threshold Formula

$$\sigma = \sum_{i=1}^{n} X^{(i)} \phi\, Q_i + \sum_{v=1}^{w} E^{(v)} \phi\, Q_v - b \cdot l \phi Q_l$$

Associated **Inhibitor**

Inhibitory Impact

Activation potential *compared with activation threshold* g

Inner inputs from **exons**

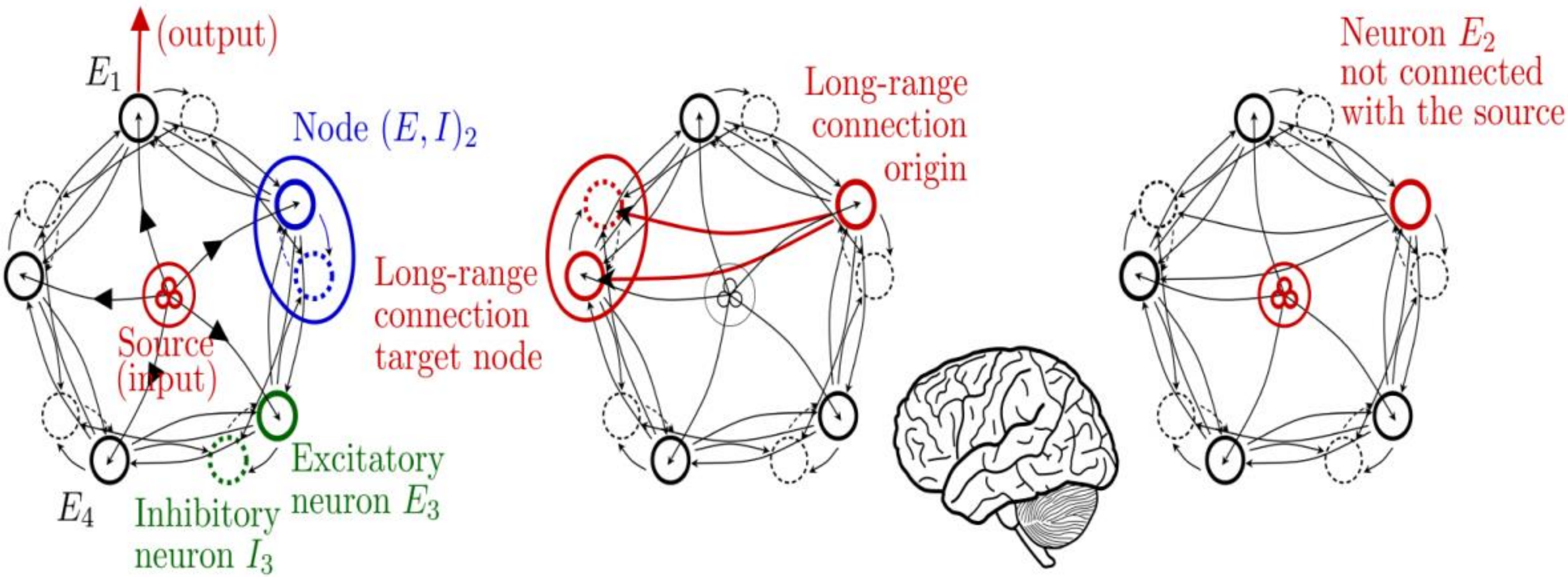
Source input (for excons only)

Amplitude fluctuations (noise)

Synaptic failure/Synaptic noise

Levy W. B., Baxter R. A., ***Neural Computation***, 1996
Levy W. B., Baxter R. A., ***Journal of Neuroscience***, 2002

Brain-inspired networks architecture

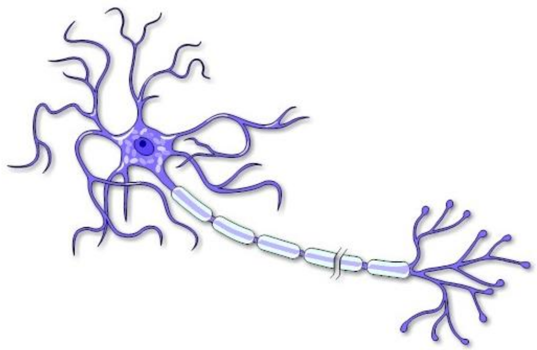


Paprocki B., Szczepanski J., ***Biosystems***, 2011
Paprocki B., Szczepanski J., ***Brain Research***, 2013
Paprocki B., Szczepanski J., ***Neurocomputing***, 2013

Brain model - we consider three 5-node architectures powered with 3-dimensional source of information

Parameters of the brain-inspired networks

communication channel



Source Parameters

Firing rate f_R

Entropy

Correlation

Neuron parameters

Synaptic failure/synaptic **noise s** *random variable*

Activation threshold g

Number of synapses n

Amplitude fluctuations Q_i *random variables*

Inhibitor strength b

Network parameters

Size/Delay r *radius of circle*

Long-range connections (architectures)

Number of nodes/neurons

Numerical simulation/ Estimations details

- Synapses number $n = 3$
- Firing-rate of the source $0 \le f_R \le 1$ in step of 0.05
- Synaptic success $0 \le s \le 1$ in step of 0.05
- Amplitude fluctuation g ∈{0.2, 0.3, 0.5, 0.7, 0.9, 1.0, 1.2, 1.6}
- Inhibition strength b ∈ {0.0, 0.25, 0.5, 0.75, 1.0, 1.5}
- Sequences lengths **1 000 000 bits**, this assures high accuracy

to reach **high accuracy** we consider very **long sequences**

We chose to consider architectures consisting of 5 nodes

Brain-inspired networks - efficiency criteria

Mutual_Information (between Inp, Out) / Energy

We assume that **most energy is consumed for generating spikes**

- For **excitatory** neurons E *without accesss* to the source

$$\frac{MI}{\vartheta}=\frac{MI(s,f_r,b,g)}{s\cdot(b\,f_I+\sum_w f_w)}$$

- For **excitatory** neurons E with access to the source X

$$\frac{MI}{\vartheta}=\frac{MI(s,f_r,b,g)}{s\cdot(n f_r+b\,f_I+\sum_w f_w)}$$

- For **inhibitory** neurons I

$$\frac{MI}{\vartheta}=\frac{MI(s,f_r,b,g)}{s\cdot\sum_w f_w}\mathrm{M}$$

Paprocki B., Szczepanski J., ***Biosystems***, 2011
Paprocki B., Szczepanski J., ***Brain Research***, 2013
Paprocki B., Szczepanski J., ***Neurocomputing***, 2013

Papers has been supported by Polish National Science Centre Grant NN519 646540

f_I - **firing rate** of the inhibitory neuron, from the same node as a given neuron E
f_w - **firing rate** of the *w*th excitatory neuron, preceeding a given neuron

Brain-inspired networks questions

What is the role of synaptic failure/synaptic noise in the network?

What is the role of inhibitory neurons in the network?
How the inhibitors influence on the Mutual Information-Energy and Mutual Information efficiency?

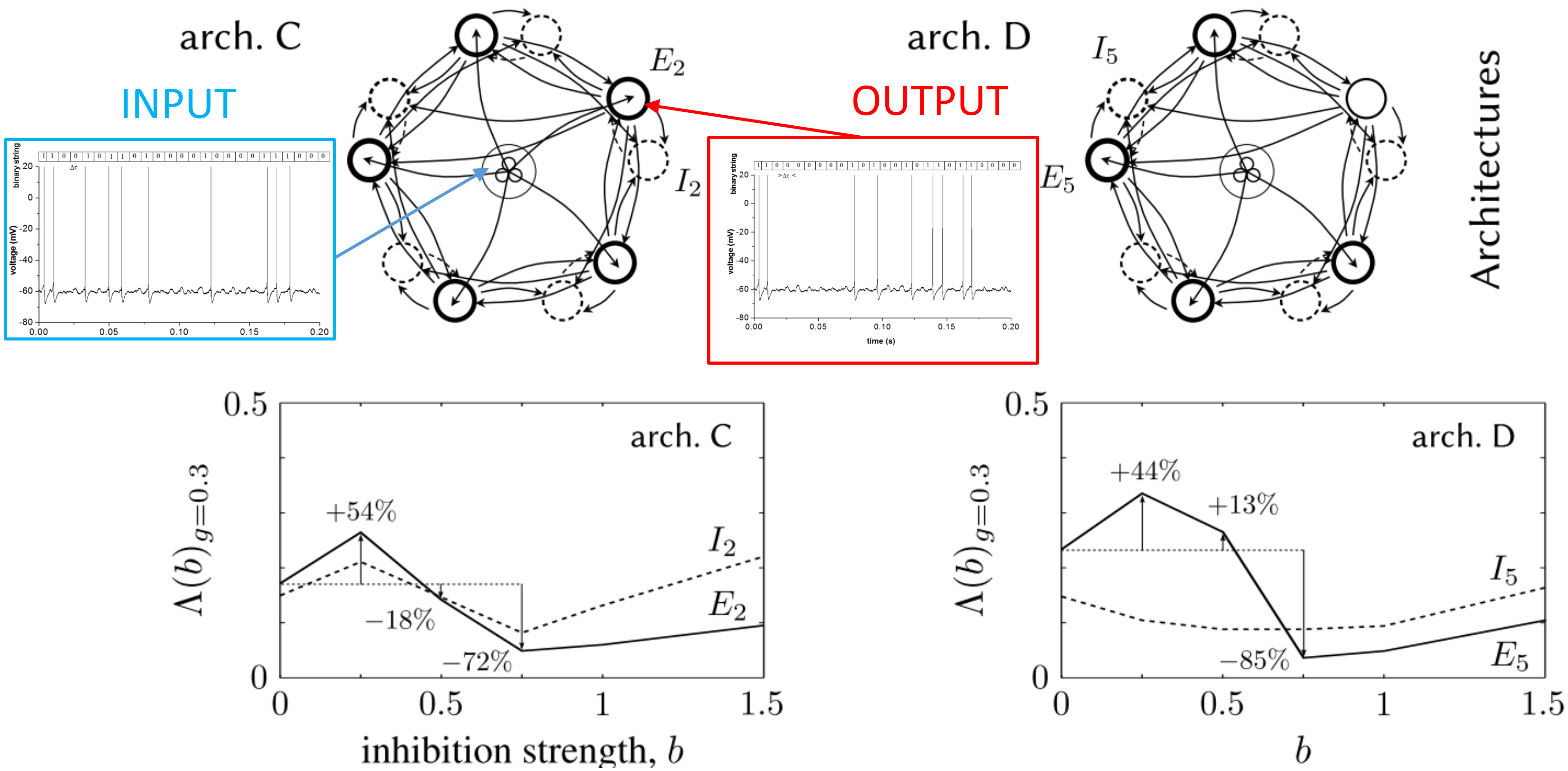
How the long-range connections affect the Mutual Information-Energy and Mutual Information efficiency?

How the size of the network, i.e. delay effects influence on the Mutual Information-Energy and Mutual Information efficiency?

Brain-inspired networks

Paprocki B., Szczepanski J., ***Brain Research***, 2013

Inhibitory effects (inhibitory neuron strength)

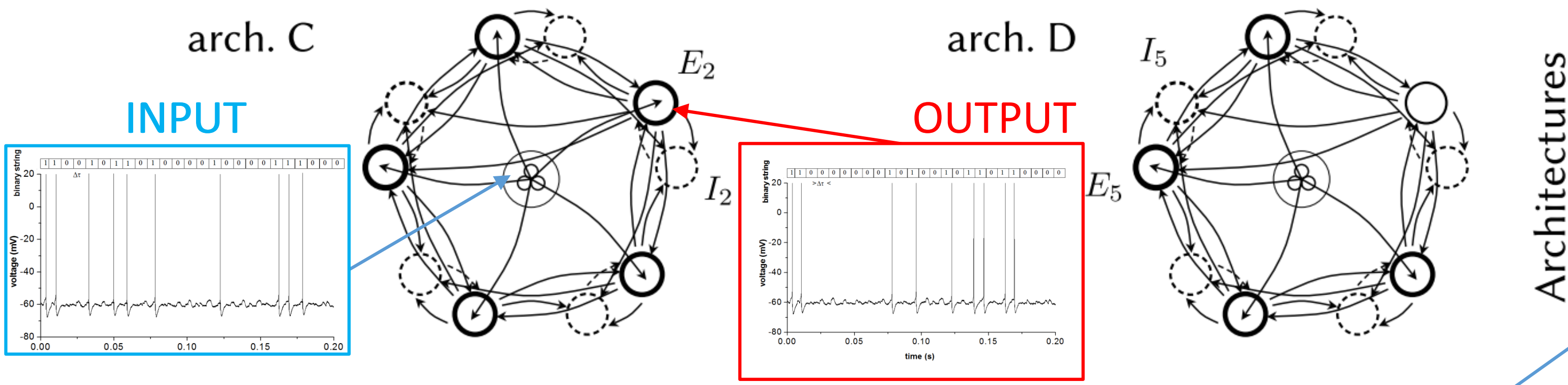


Effectiveness increases even by 50 percent

Brain-inspired networks

Paprocki B., Szczepanski J., ***Brain Research***, 2013

Long-range connections effects

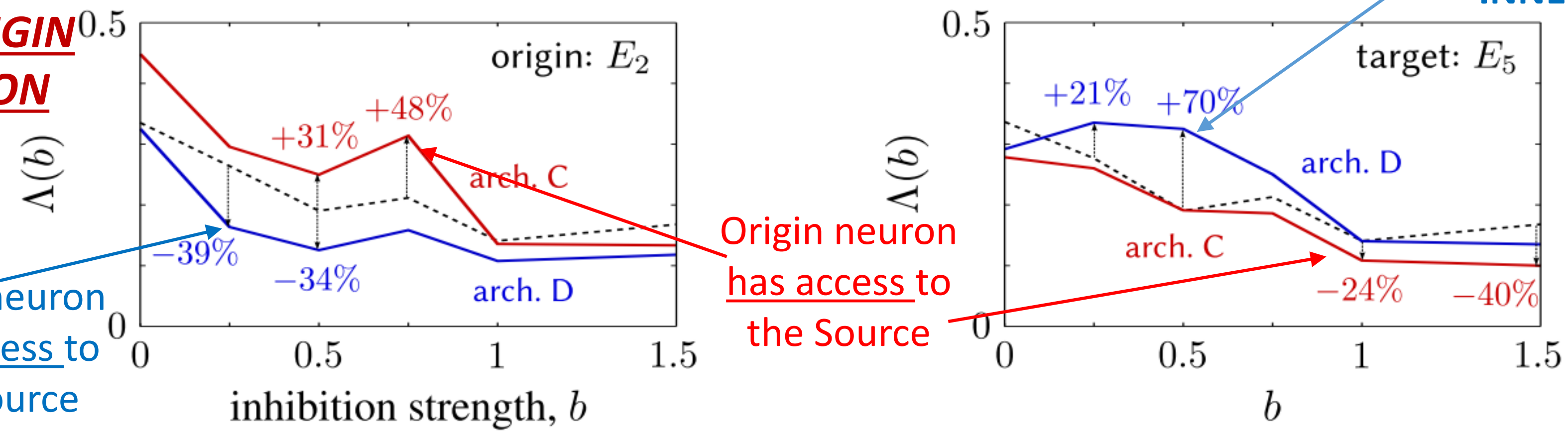


FOR TARGET NEURON

Origin neuron has no access to the Source

INNER CON.

FOR ORIGIN NEURON



Origin neuron has access to the Source

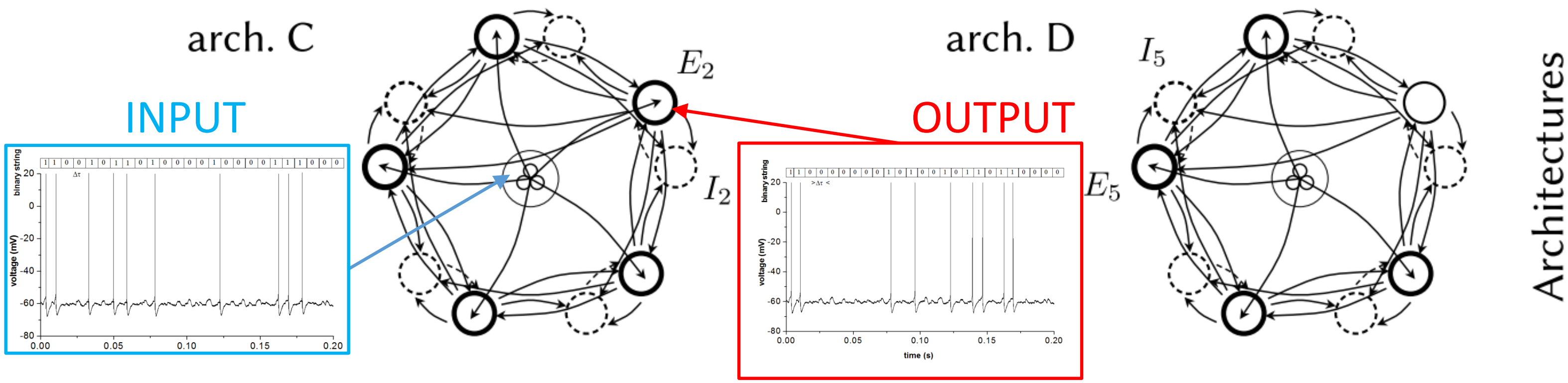
Origin neuron has access to the Source

Effectiveness increases even by 70 percent

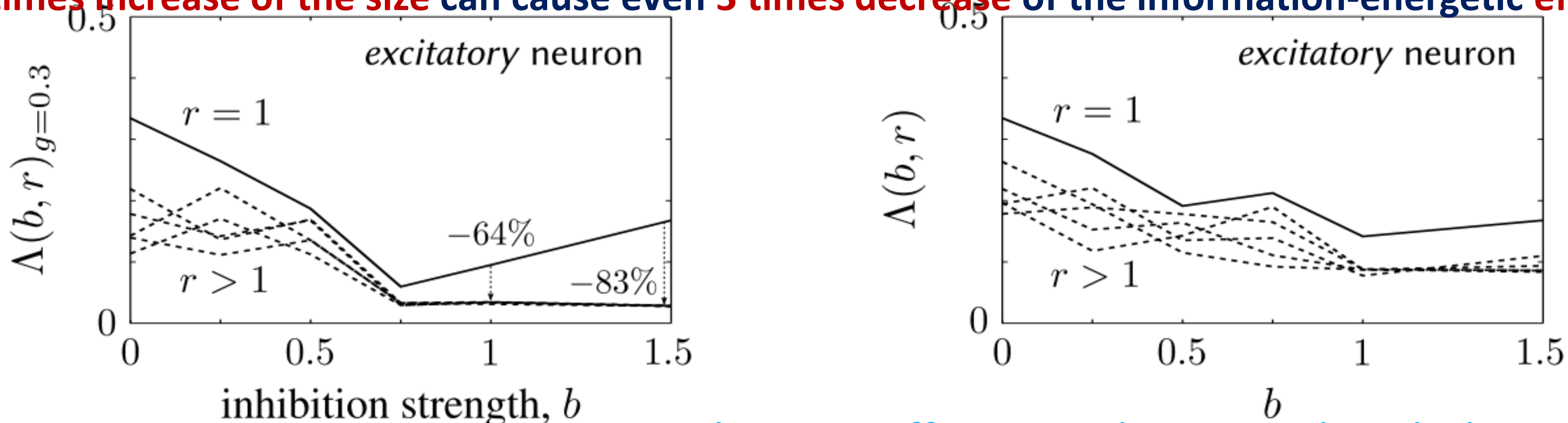
Brain-inspired networks

Paprocki B., Szczepanski J., ***Brain Research***, 2013

Size effects (Delay effects)



2 times increase of the size can cause even 3 times decrease of the information-energetic efficiency

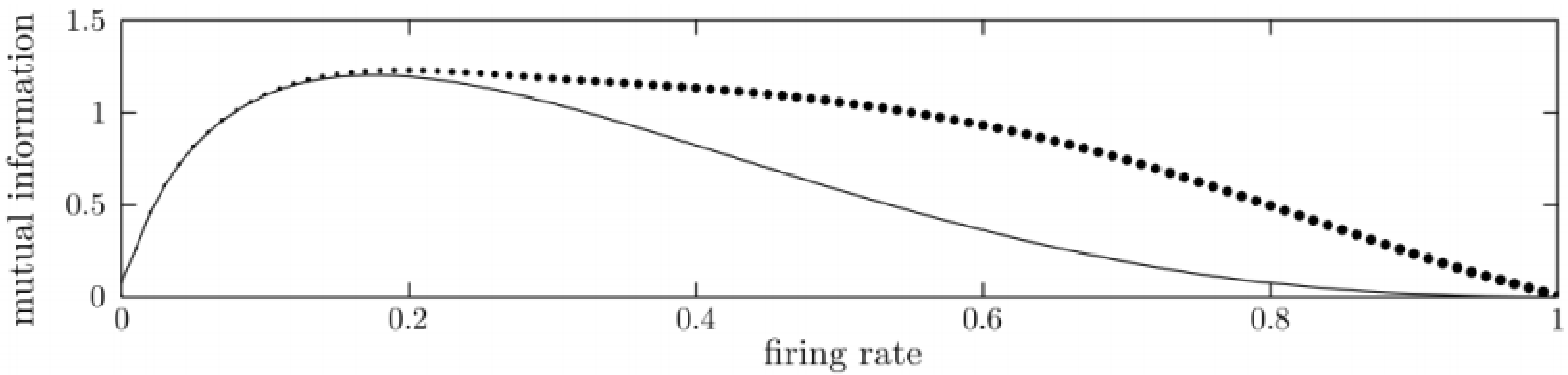


The most effective is the network with the smallest size

Feed-forward networks

Paprocki B., Szczepanski J., ***Biosystems***, 2011

Synaptic failure/Synaptic noise s



Solid line – Mutual Information with zero noise $s = 1$
Dotted line - maximal Mutual Information values
Size of a dot is proportional to $1 - s$(noise), indicating the bigger the dot
The corresponding Mutual Information value is achieved at lower s.

The most effective is the network with the smallest size

Conclusions

Related to the experiments

- We apply the **Method of Estimation of Information Transmission Rate** (ITR) by neurons - this allows to characterize quantitatively the ITR in different brain states or brain areas
- Relative complexity curves **discriminate** neuronal responses under different experimental conditions
- ***In vivo*** sources transmit more information than ***in vitro*** source, as expected!!!, but we are able to characterize the transmission rates **quantitatively** (we observed the increase even by factor of 2)
- Information transmission by nearby neurons occurs in **the mid-regime** (Redundancy and Relative Mutual Information) – just to assure a kind of Reliability ?
- If the source is ergodic the entropy can be read off from the saturation levels, it is related to the **choice of parameters**)
- The choice of coding affects the results - the obtained information depends on the coding used

Conclusions

Related to the brain-inspired networks

- All brain-inspired networks components (inhibitory, longe-range connections, size/delay) significantly **improve** the Information-Energetic efficiency

  - **Inhibitory neurons** improve the Information-Energetic transmission efficiency even by **50 percent**
  - **Longe-range connections** improve the Information-Energetic transmission efficiency even by **70 percent**
  - **Size/delay** affects essentially on the transmission efficiency. The most effective is the network with the smallest size (**2 times increase of the size can cause even 3 times decrease of the information-energetic efficiency**)

Biological organisms/Biological communication systems ***evolve***

to <u>optimize the Information-Energetic efficiency</u>