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Active suspension control of 1D continuum under a travelling load

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Abstract

In this paper, a semi-active control method is proposed and applied to the vibration control of a 1D elastic continuum induced by a load travelling over it. The magnitude of the moving force has been assumed to be constant by neglect of the inertia forces. Full analytical, continuous solution is based on the power series method and is given in an arbitrary time interval. The time-marching scheme allows us to continue a solution in successive layers with initial conditions taken from the end of previous stages. The semi-active open loop control strategy is proposed. Shapes of damping functions are defined as a form of piecewise constant function. The designed control in bang bang form is suboptimal and it outperforms the passive case. The effectiveness of the switching algorithm was verified by numerical results.

1 Introduction

Problems of a load travelling along structures, such as strings, beams or plates at a higher range of speed, are of particular interest to practising engineers. A higher speed range means the speed at which successive passages of a moving load through the structure significantly increase amplitudes of displacements, up to infinity in the case of critical speed values. In the case of a string the considered speed can be within the range of 0.3 to 1.0 of the wave speed. Analytical and numerical solutions are applied to problems with a single or multi-point contact, such as train-track or vehicle-bridge interaction, pantograph collectors in railways, magnetic levitation railways, guideways in robotic technology, etc. Railway bridges have suffered a decrease in service life due to loading induced by heavy trucks travelling at high speed. As a result, many bridges are approaching the end of their useful life and will require extensive repair or replacement unless other ways are found to reduce stresses and strains due to these loads and to sustain the safety of the bridges. Structures with external control of parameters can resist a load in the efficient way. Classical passive control are replaced by new, active or semi-active control systems. Old, weak structures can be reinforced by supplementary supports with magneto- or electro-rheological dampers controlled externally (Figure 1). Active or semi-active control of structural vibrations plays an important role in the case of dynamic influence of external standing or travelling loads. Active methods of control are, unfortunately, energy-consuming and complicated in practical applications. Moreover, a poor control system can supply energy in the antiphase and in extreme cases can damage the structure. We will focus our research on semi-active systems composed of dampers, which require lower energetic effort.

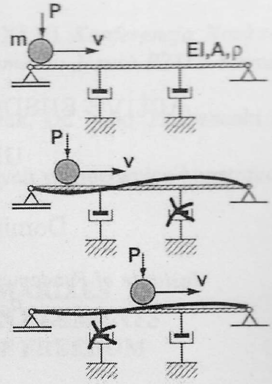

Figure 1: The idea of semi-active control of a beam deflection under a travelling load.

Numerous active and semi-active vibration control methods are widespread and some of them have been put into practice recently. Most of them are based on sky-hook or ground-hook concepts. These approaches are used for semi-active control of the moving oscillator problem. Also some theoretical approaches, based on the method of optimal Lyapunov functions, was applied into semi-active control of structures. Most of founded semi-active methods leads to feedback controls determined by state-space measures. In case of continuous system such an observer design is often too much complicated. The alternate method is an open loop control. It is particularly of use in problems where the excitation is determined.

In this paper we present the analytical solution of a semi-active control of vibrations in a string subjected to a travelling load. The string is supported by a set of viscous dampers. The method allows us to solve the problem analytically and express the continuous solution in a form useful for further analysis of the influence of damping functions on displacements and its derivatives. Whole time domain is split into time intervals. Full analytical solution in time interval in a form of power series is given. Semi-active control strategy is based on the previous numerical investigation. In this approach authors don't pay the attention to optimal solutions in sense of minimizing the performance index with respect to all admissible controls. They try to present cases where active dampers may outperform passive ones. The goal is to design efficient control so that the practical realization is the easiest way possible.

Preliminary investigation of the destination problem was published in [5].

2 Formulation and solution of the problem

The system under consideration is shown in Figure 2. Continuum is simply supported by a set of control dampers. The moving load is passing upon the string at a constant velocity. The mass accompanying the travelling load is small compared with mass of the continuum and is neglected. Reactions of dampers are proportional to the velocity of displacements in given points. The transverse vibration of the considered system is governed by the partial differential equation

$$\Gamma(u(x,t)) + \mu\,\frac{\partial^2 u(x,t)}{\partial t^2} = -\sum_{i=1}^{Z} b_i(t)\,\frac{\partial u(a_i,t)}{\partial t}\,\delta(x-a_i) + P\,\delta(x-vt). \tag{1}$$

where Γ is the stiffness operator (for a Bernoulli-Euler beam $\Gamma = EJ\,\partial^4/\partial x^4$), μ is the constant mass density per unit length, P is the concentrated force passing the string at the constant velocity v, $b_i(t)$ is the ith damping coefficient as function of time, $u(x,t)$ is a transverse deflection of the string at the point (x,t), Z is the number of viscous supports, a_i is the ith fixed point of a damper and δ is the Dirac delta. The boundary and initial conditions are as follows:

$$u(0,t)=0, \quad u(l,t)=0, \quad u(x,0)=0, \quad \dot{u}(x,0)=0\,. \tag{2}$$

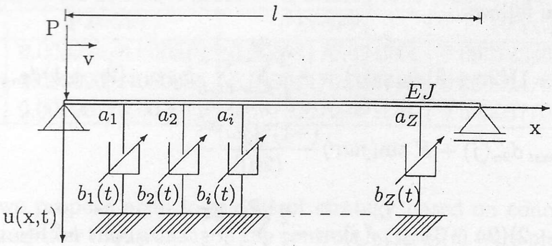

Figure 2: Euler-Bernoulli beam system supported with set of active dampers.

The response of the continuum in Eq. (1) with conditions (2) is represented as a series expansion of the sine eigenfunctions given by

$$u(x,t) = \frac{2}{l}\sum_{j=1}^{\infty} V(j,t)\sin\frac{j\pi x}{l}\ , \qquad V(j,t) = \int_0^l u(x,t)\sin\frac{j\pi x}{l}\mathrm{d}x\ . \tag{3}$$

Each term of Eqn. (1) is multiplied by $\sin\frac{j\pi x}{l}$ and then integrated with respect to x in the interval $[0,l]$

$$\int_0^l \Gamma(u(x,t))\sin\frac{j\pi x}{l}\mathrm{d}x + \mu\ddot{V}(j,t) = -\int_0^l \sum_{i=1}^{Z} b_i(t)\frac{\partial u(a_i,t)}{\partial t}\sin\frac{j\pi x}{l}\delta(x-a_i)\mathrm{d}x + P\sin\frac{j\pi vt}{l}\ . \tag{4}$$

This yields a system of ordinary differential equations

$$\mu\ddot{V}(j,t) + \frac{2}{l}\sum_{i=1}^{Z}\sum_{k=1}^{\infty} b_i(t)\dot{V}(k,t)\sin\frac{k\pi a_i}{l}\sin\frac{j\pi a_i}{l} + \int_0^l \Gamma(u(x,t))\sin\frac{j\pi x}{l}\mathrm{d}x = P\sin\frac{j\pi vt}{l} \tag{5}$$

To obtain the analytical solution we assume piecewise constant controls $b_i(t)$ such that

$$b_i(t) \colon \left[0, \frac{l}{v}\right] \to [b_{min}, b_{max}]\ , \qquad b_i(t) = \begin{cases} b_{ip}, & \forall t \in (t_{p-1}, t_p], \quad p = 1 \ldots s \\ 0, & t = 0 \end{cases}\ , \tag{6}$$

where s denotes number of time intervals. The solution being looked for, is the general solution, where integration constants can be simply represented in every time interval $(t_{p-1}, t_p]$ by initials $C1_j = V(j, t_{p-1})$, $C2_j = \dot{V}(j, t_{p-1})$ taken from the ending values of previous one. It is an easy way to combine the interval solutions to a global one. The solving procedure, which is presented, is based on the power-series method. Denoting t_{p-1} by τ, the solution for $t \in (t_{p-1}, t_p]$ is supposed to take a form

$$V(j,t) = \sum_{n=0}^{\infty} d_n(j)(t-\tau)^n\ , \tag{7}$$

Substitution of (7) into (5), after some basic algebraic transitions, yields system of recurrence equations as follows

$$
\begin{aligned}
\mu\,(2n+1)(2n+2)d_{2n+2}(j) &= -\frac{2}{l}\sum_{i=1}^{Z}\sum_{k=1}^{\infty} b_{ip}\,\alpha_{ijk}(2n+1)d_{2n+1}(k)+\\
&- \Gamma_{const}\,d_{2n}(j) + P\,\sin(j\omega\tau)\frac{(-1)^n(j\omega)^{2n}}{(2n)!}\,,\\
\mu\,(2n+2)(2n+3)d_{2n+3}(j) &= -\frac{2}{l}\sum_{i=1}^{Z}\sum_{k=1}^{\infty} b_{ip}\,\alpha_{ijk}(2n+2)d_{2n+2}(k)+\\
&- \Gamma_{const}\,d_{2n+1}(j) + P\,\cos(j\omega\tau)\frac{(-1)^n(j\omega)^{2n+1}}{(2n+1)!}\,,
\end{aligned}
\tag{8}
$$

where Γ_{const} is determined by stiffness operator of the continuum. Following notations were introduced:

$$
\frac{\pi v}{l} = \omega, \qquad \sin\frac{j\pi a_i}{l}\,\sin\frac{k\pi a_i}{l} = \alpha_{ijk}\ .
$$

System (8) is expected to be calculated form $n = 0$ assuming $d_0(j) = V(j,\tau)$ and $d_1(j) = \dot{V}(j,\tau)$.

3 Control design

System formulated in previous section is classified to be bilinear. Numerous techniques, which stem primarily from calculus of variation, have been derived for the optimal control solution for such a systems. Pontryagin's maximum principle uses Hamilton's equations. Using this principle and considering the discrete model of continuum it can be derived, that the optimal active suspension is controlled in bang-bang form. Discrete system of continuum supported with a set of controlled dampers can be represented in general by the equation

$$
\dot{y}(t) = A\,y(t) + B\,y(t)\,\Lambda(t) + f(t)\ , \tag{9}
$$

where $y(t) \in \mathbb{R}^n$ is generalized state vector, $\Lambda(t)$ is control, A and B are matrices, f(t) is excitation. Assuming $\Lambda(t) \in [-1,1]^m$ and quadratic cost functional, such that

$$
J = \frac{1}{2}\int_0^T \langle y(t), Qy(t)\rangle\ \mathrm{d}t\ , \tag{10}
$$

Hamiltonian of the system (9), that can be written as following

$$
H(t,y,\Lambda,\eta) = \langle\eta(t), Ay(t)\rangle + \langle\eta(t), By(t)\rangle\Lambda + \langle\eta(t), f(t)\rangle - \frac{1}{2}\langle y(t), Qy(t)\rangle\ , \tag{11}
$$

takes maximum value, when control equals

$$
\Lambda(t) = \mathrm{sign}\langle\eta(t), By(t)\rangle\ . \tag{12}
$$

Here, $\eta(t)$ satisfies differential equation

$$
\dot{\eta}(t) = -\frac{\partial H}{\partial y}\ . \tag{13}
$$

Due to implicit form of controls, the optimal problem cannot be solved directly. Difficulties additionally grow up in case of the continuum that is performed by multidimensional discrete system.

Table 1: Payoff values for different speed of travelling load (active/passive suspension).

Velocity	PAYOFF$_1$	PAYOFF$_2$	PAYOFF$_3$
0.1c	0.000334/0.000349	0.023941/0.024264	0.000223/0.000223
0.4c	0.000023/0.000033	0.045071/0.045711	0.000064/0.000064
0.9c	0.000007/0.000011	0.087920/0.091351	0.000037/0.000040

In this paper we propose open loop control strategy based on concept presented in Figure 1. The assumption that was made the controls $b_1(t), b_2(t)$ were piecewise constant, they belonged to a closed set $\mathbb{B}$ and they are expressed in bang-bang form. The goal is to design efficient control so that the practical realization is the easiest way possible. In this purpose for simplicity we take into account controls that are bang-bang and only one switching time for every is assumed so that

$$b_1(t) = b_{max}U_1(t) - b_{max}U_1(t-\tau_1), \quad b_2(t) = b_{max}U_1(t-\tau_2), \tag{14}$$

where $U_1(t)$ is unit step function and $b_{max} = sup(\mathbb{B})$. So in fact damper no.1 is first switch *on* then in time $t = \tau_1$ turns into *off* mode. Situation for damper no.2 is reverse. Below we define cost integrands such that they can determine travel comfort (cases 1, 3) or structural damage (case 2).

$$\begin{aligned} &(1) \quad \text{PAYOFF}_1 = \int_0^{l/v} |u(vt,t)|\, dt \\ &(2) \quad \text{PAYOFF}_2 = RMS(\dot{u}(vt,t)) = \left(\frac{v}{l}\int_0^{l/v} (\dot{u}(vt,t))^2\, dt\right)^{1/2} \\ &(3) \quad \text{PAYOFF}_3 = \int_0^{l/v} |\ddot{u}(vt,t)|\, dt \end{aligned} \tag{15}$$

The task is to find pairs (τ_1, τ_2) such that minimize costs

$$(\tau_1, \tau_2) = \underset{\tau_1,\tau_2\in[0,l/v]}{\arg\min}\ \text{PAYOFF}(u(t), b_1(t), b_2(t)), \tag{16}$$

where $b_1(t), b_2(t)$ are defined as before.

4 Numerical results

In numerical results we consider continuum that is represented by Bernoulli-Euler beam with parameters: $l = 2m$, $\mu = 0.78kg/m$, $EJ = 10^4 Nm^2$. Two active dampers are fixed to the beam at positions $a_1 = 0.25l, a_2 = 0.75l$. Force of magnitude $P = 100N$ is travelling with velocities $v = 0.1c, 0.4c, 0.9c$, where c denotes so called critical speed and $c = \frac{\pi}{l}(\frac{EJ}{\mu})^{1/2}$. By the passive case we mean constant damping $b_1(t) = b_{max}, b_2(t) = b_{max}, \forall t \in [0, l/v]$. In computations we assumed $b_{max} = 3 \cdot 10^4$ in all cases. Results of Payoffs for optimal active and passive cases are presented in Table 1. In case of $v = 0.9c$ extremal trajectory for $u(t)$ with its control is shown in Figure 3. The best performance of proposed strategy is observed in case of the highest speed of travelling load. For cases, where we minimize PAYOFF$_2$, PAYOFF$_3$ we expect much better performance by applying controls with more than one switching. Velocities and accelerations incorporated into these costs include high-frequency harmonics that can be reduced by high-frequency

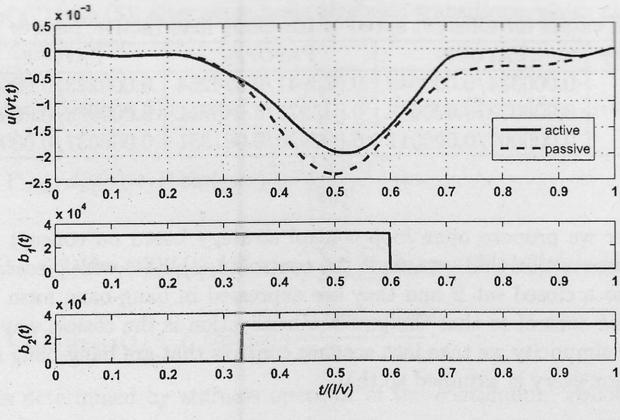

Figure 3: Extremal deflection trajectory and controls in case of $v = 0.9c$.

switching controls. Because of the significantly higher complexity of the optimisation problem, computing of such controls may be difficult. Appropriate gradient methods may, however, be useful [4].

5 Conclusions

In this paper the analytical solution of the response of a semi-active controlled 1D continuum has been presented. The technique has been applied to exemplary control of Euler-Bernoulli beam. The open-loop, bang-bang control strategy has been proposed and its performance has been verified for three different cost integrands. Control strategy is simple for a practical design. Further optimisation is the ongoing research topic of the authors.

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