Finite displacement dynamic model of twin beams with controllable damper

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Abstract

Lateral vibrations of two parallel cantilever beams joined at their free ends by a viscoelastic member are studied experimentally and theoretically. A dynamic model of the structure is proposed. It fits the experimental data well and allows to estimate the shear modulus and damping coefficient of the member. The model can be useful for the development of semi-active control strategies for double-beam systems with controllable damping members.

Keywords: sandwich beam, double-beam system, shear deformation, magnetorheological elastomer, semi-active damping

1. Introduction

Systems of two parallel elastic beams can be found in various devices and structures – examples include aircraft wing spars, double-beam cranes, bridge spans, or linear guideways in plotters. Recently semi-active methods of vibration suppression for such systems have been developed. In Ref. [1] the controlled delamination of a two-layer beam is employed for the releasing of strain energy accumulated in the deformed structure. The research Ref. [2] deals with semi-active damping of two beams joined by elastomer composite with iron particles, whose stiffness and dissipative properties increase when it is exposed to magnetic field. A simple switching strategy allows to reduce vibrations more effectively than in the case of the elastomer permanently activated. A similar system is experimentally investigated in Ref. [3], however in this study an elastic hermetic container filled with granules and subjected to underpressure acts as the damping member.

The scope of the research is to provide a dynamic model for systems analyzed in two former papers.

2. Experiment

The scheme of the investigated system is depicted in Fig. 1. Two aluminum beams of length $L = 700$ mm, width $b = 25$ mm and thickness $d = 2$ mm are mounted in parallel in a clamped configuration and joined at their free ends by a damping member made of MS-polymer adhesive. When the beams are deflected, the member undergoes shear deformation, which is the main source of elastic and dissipative forces in the system. The mass of the member amounts to $2M = 18$ g, length $2a = 33$ mm, height $2h = 15$ mm, and width equals to b. In order to keep the gap between the beams constant over their lengths, two lightweight rolls were placed at distances of $x_1 = 230$ mm and $x_2 = 470$ mm from the support. Three laser displacement sensors were aimed at the system at distances 230, 460 and 695 mm. An initial deflection of the system was applied, and after releasing of the structure, the vanishing free vibrations were recorded.

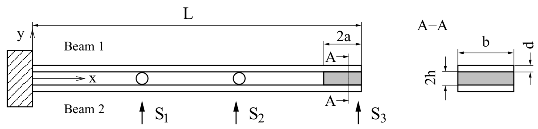

Figure 1: Scheme of the analyzed system.

3. Model

Lateral vibrations in x-y plane are studied, and $w_{1,2}(x,t)$ denote displacements of beams in direction y. The beams are considered linear Bernoulli-Euler cantilevers, subjected to both internal and external viscous damping. Gravity acts perpendicularly to x-y plane, so its influence can be neglected.

The anti-buckling rolls are treated as linear springs of stiffness $K_r = 20000$ N m^{-1}, which generate transverse forces

$$F_{r_{1,2}} = -K_r(w_1(x_{1,2},t) - w_2(x_{1,2},t)). \quad (1)$$

By assumption, the member is made of the Kelvin-Voigt material characterized by shear stress-strain relation $\tau = G\varphi + G^*\dot{\varphi}$, where G [Pa] is the Kirchhoff modulus and G^* [Pa s] is the damping coefficient. The member is modelled as a two-link diagonal truss, exerting forces to both beams as depicted in Fig. 2. It is short enough to neglect its rotary inertia, but the mass is included.

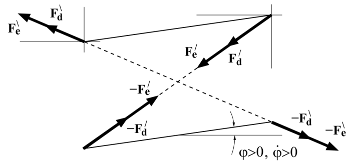

Figure 2: Forces generated by the truss and acting on the beams.

*The research has been supported within the framework of the project UMO-2013/11/D/ST8/03437 funded by the National Science Centre.

It is assumed that the end parts of both beams (i.e. over the damping element) remain almost straight, their slopes of deflection are approximately equal to φ, and the value of φ is small, so $\sin(\varphi) \simeq \varphi$ and $\cos(\varphi) \simeq 1$. Then the components of elastic forces in the assumed coordinates amount to

$$F^{/}_{e,x} = -\frac{Gb}{2}\left(\widetilde{w} + 2a\varphi\right), \quad F^{/}_{e,y} = -\frac{Gbh}{2a}\left(\widetilde{w} + 2a\varphi\right),$$
$$F^{\backslash}_{e,x} = \frac{Gb}{2}\left(\widetilde{w} - 2a\varphi\right), \quad F^{\backslash}_{e,y} = -\frac{Gbh}{2a}\left(\widetilde{w} - 2a\varphi\right), \quad (2)$$

where

$$\widetilde{w} = w_1(L-a,t) - w_2(L-a,t),$$
$$\varphi = \frac{1}{2}\left(\frac{\partial w_1}{\partial x}(L-a,t) + \frac{\partial w_2}{\partial x}(L-a,t)\right) \quad (3)$$

The components of dissipative forces are given by analogous formulas, in which G is replaced with G^*, and $\widetilde{w}$, φ – with their time derivatives.

The dynamics of the system is governed by two classical equations of lateral beam vibrations, with additional point forces generated by the springs and the truss. The following equation describes the motion of beam 1

$$\left(\rho A + M\delta_{L-a}\right)\frac{\partial^2 w_1}{\partial t^2} + EI\frac{\partial^4 w_1}{\partial x^4} + E^*I\frac{\partial^5 w_1}{\partial x^4 \partial t} + c\frac{\partial w_1}{\partial t} +$$
$$- F_{r_1}\delta_{x_1} - F_{r_2}\delta_{x_2} - P\frac{\partial^2 w_1}{\partial x^2} +$$
$$- F^{/}_{e,y}\delta_L - F^{\backslash}_{e,y}\delta_{L-2a} - F^{/}_{d,y}\delta_L - F^{\backslash}_{d,y}\delta_{L-2a} = 0, \quad (4)$$

where $A = bd$ is the cross-sectional area of the beam, $I = bd^3/12$ – the second moment of an area, E^* denotes the internal damping coefficient of aluminum, and c is the external damping coefficient of air. The value $P = F^{/}_{e,x} + F^{\backslash}_{e,x} + F^{/}_{d,x} + F^{\backslash}_{d,x}$ denotes the sum of forces generated by the truss and acting on the beam in direction x. It is assumed that all axial forces are concentrated at the beam tip, because part of the beam over the truss is a short, so its slope is small.

A dynamic equation of beam 2 is analogous. The sign at the axial force changes to „+", because if one beam is being compressed, the other one is stretched. Obviously the „−" sign at transverse forces is replaced with „+". The points, where the forces generated by the truss act, are swapped.

4. Verification

A continuous problem was discretized using the Galerkin procedure based on the cantilever beam eigenfunctions. The number of 8 base vectors was enough to provide a satisfactory accuracy of the approximate solution. The resultant set of ordinary equations was solved using the *Fehlberg* method.

Firstly, the stiffness and dissipative properties of the single aluminum beam were estimated basing on free vibrations induced in the first mode. The obtained values are $EI = 1.01$ N m^2, $E^*I = 0.00026$ N m^2 s and $c = 0.01283$ Pa s. Afterwards, the model of a double-beam system was fitted to the experimental data. This data was acquired from the free vibration trial, with an initial tip deflection of 49 mm, and lasting 6.3 seconds. The fitting quality was measured by Pearson correlation coefficient between empirical and theoretical time responses, averaged over sensors S_1, S_2 and S_3. The values of G and G^* which maximize the quality criterion were chosen as estimators. A simple optimization technique based on the systematic search yielded to $G = 365$ kPa and $G^* = 6.683$ kPa s. The optimal value of mean correlation coefficient equal to 0.988 indicates that the model fits the experimental data well.

Figure 3 presents the comparison of empirical and theoretical time series of transverse displacements at the position of sensor S_3, and over the whole time of the trial. In Fig. 4 the deflection of the beam is shown over its entire length. The dots denote the actually measured displacement during the selected moments of the first cycle. The first cycle was chosen because of the highest deflection, which yielded to the strongest and easily observed S-shaped deformations of the beam.

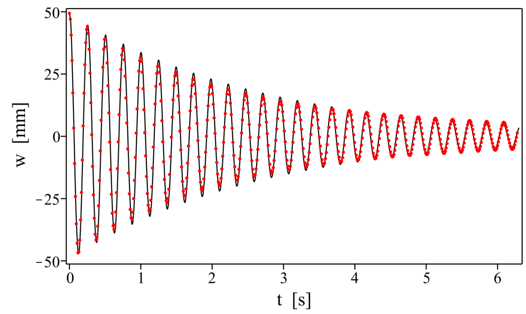

Figure 3: Empirical (dots) and theoretical (line) free vibrations at position of sensor S_3.

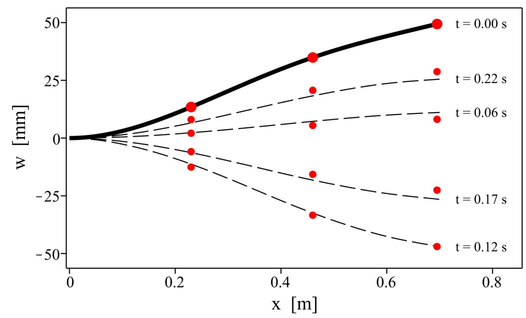

Figure 4: Empirical (dots) and theoretical (lines) beam deflection during the first cycle of free vibrations.

5. Conclusions

A dynamic model of the system of twin cantilever beams connected at their free ends by an elastomer damping member is proposed. The model fits the experimental data well. The Kirchhoff modulus and shear damping coefficient of the elastomer are identified. The proposed model can be a basis for the development of optimal control strategies for double-beam systems with adaptive damping members. It may be also useful for establishing such geometrical and physical parameters of these systems supposed to provide the highest efficiency of vibration suppression.

References

[1] Mroz, A., Orlowska, A., Holnicki-Szulc, J., Semi-active damping of vibrations. Prestress Accumulation-Release strategy development, *Shock and Vibration*, 17, pp. 123-136, 2010.

[2] Dyniewicz, B., Bajkowski, J. M., Bajer, C. I., Semi-active control of a sandwich beam partially filled with magnetorheological elastomer, *Mechanical Systems and Signal Processing*, doi: 10.1016/j.ymssp.2015.01.032, 2015.

[3] Bajkowski, J. M., Dyniewicz, B., Bajer, C. I., Damping properties of a beam with vacuum-packed granular damper, *Journal of Sound and Vibration*, 341, pp. 74-85, 2015.