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The Material Anisotropy Influence on Modelling of Rutting Test with Application of Linear Viscoelasticity Constitutive Equations

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Abstract

In the paper a general approach to modelling of anisotropic linear viscoelastic material properties is presented. Rychlewski's (1983) spectral decomposition theorem is used and one dimensional relaxation functions of linear viscoelasticity model is adopted to eigenvalues of stiffness tensor and named Kelvin relaxation functions. Proposed model was implemented in the FEM system Abaqus on the example of transversely isotropic and isotropic material. On the basis of experimental data available in literature, models were calibrated and verified. Constitutive relations were used in the complex boundary value problem modelling standard rutting test used in road sector to assess the resistance of asphalt mixtures to rutting.

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Keywords: constitutive modeling; viscoelasticity; finite element method; anisotropic materials; asphalt concretes

1. Introduction

This paper considers modelling of rutting test with anisotropic viscoelastic materials on the example of transversely isotropic material. The constitutive model was implemented in the finite element method system Abaqus via user subroutine UMAT [1]. Subsequently, material parameters for the model were derived from experimental data presented in the paper [2] both for transversely isotropic material and for isotropic material. A complex initial boundary value problem of rutting test [3] was modelled with certain reasonable simplifications, i.e. contact of a tire with a specimen was modelled as uniform pressure load on the area of a contact zone. Obtained results indicate necessity for taking account of transverse isotropy in mineral asphalt mixes in laboratory tests and numerical simulations.

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2. Constitutive equations of viscoelasticity in convolution form

Constitutive equations of linear viscoelasticity could be formulated in a different equivalent form, which is a consequence of the theory of linear operators and the theory of distribution [4]. Due to the fact that finite element method system Abaqus will be used, in this paper constitutive equations are formulated as follows

$$\boldsymbol{\sigma}(t) = \int_{0^-}^{t} \tilde{\mathbf{S}}(t-\tau)\cdot\frac{\partial\boldsymbol{\varepsilon}(\tau)}{\partial\tau}d\tau, \quad \boldsymbol{\varepsilon}(t) = \int_{0^-}^{t} \tilde{\mathbf{C}}(t-\tau)\cdot\frac{\partial\boldsymbol{\sigma}(\tau)}{\partial\tau}d\tau, \tag{1}$$

where derivatives and integrals are meant in a sense of the theory of distributions and linear operators. In (1) $\tilde{\mathbf{S}}$ and $\tilde{\mathbf{C}}$ are respectively relaxation and creep operators, which are analogous to the Hooke's tensors for anisotropic materials $\boldsymbol{\sigma} = \mathbf{S}\cdot\boldsymbol{\varepsilon}$, $\boldsymbol{\varepsilon} = \mathbf{C}\cdot\boldsymbol{\sigma}$, where $\mathbf{S}$ is the fourth order stiffness tensor and $\mathbf{C}$ is the fourth order compliance tensor.

Two relations expressed by (1) are equivalent, but we will focus on equation $(1)_1$ and materials with transverse isotropy symmetry. According to the spectral decomposition theorem [5] an operator of relaxation can be expressed in a following form

$$\tilde{\mathbf{S}}^{triso}(t) = \tilde{K}_1^{triso}(t)\mathbf{P}_1^{triso} + \tilde{K}_2^{triso}(t)\mathbf{P}_2^{triso} + \tilde{K}_3^{triso}(t)\mathbf{P}_3^{triso} + \tilde{K}_4^{triso}(t)\mathbf{P}_4^{triso}, \tag{2}$$

where functions $K_i^{triso}(t)$ are respectively eigenfunctions of relaxation, which were proposed to be named Kelvin relaxation functions in [6] and the fourth order tensors $\mathbf{P}_i^{triso}$ are orthogonal projectors (product of eigentensors of $\mathbf{S}$ and $\mathbf{C}$) in case of transversely isotropic material model. Projectors are defined as follows ([7]):

$$\mathbf{P}_1^{triso} = \mathbf{w}_I \otimes \mathbf{w}_I, \quad \mathbf{P}_2^{triso} = \mathbf{w}_{II} \otimes \mathbf{w}_{II}, \tag{3}$$

$$\mathbf{P}_3^{triso} = (\mathbf{I}-\mathbf{M})\lozenge(\mathbf{I}-\mathbf{M}) - \frac{1}{2}(\mathbf{I}-\mathbf{M})\otimes(\mathbf{I}-\mathbf{M}), \quad \mathbf{P}_4^{triso} = \mathbf{M}\lozenge(\mathbf{I}-\mathbf{M}) + (\mathbf{I}-\mathbf{M})\lozenge\mathbf{M}, \tag{4}$$

where $\mathbf{M} = \mathbf{m}\otimes\mathbf{m}$ and the operator $\lozenge$ is defined in Cartesian basis as

$$(\mathbf{A}\lozenge\mathbf{B})_{ijkl} = \frac{1}{2}\left(A_{ik}B_{jl} + A_{il}B_{jk}\right). \tag{5}$$

Tensor $\mathbf{M}$ is commonly called parametric tensor and is obtained as a tensor product of unit vector $\mathbf{m}$ which is normal to the plane of isotropy. Projectors $\mathbf{P}_1^{triso}$ and $\mathbf{P}_2^{triso}$ require further clarification, tensors $\mathbf{w}_I$ and $\mathbf{w}_{II}$ are eigentensors (equivalents to eigenvectors in a spectral decomposition of the second order tensors) defined as

$$\mathbf{w}_K = w_1^K\mathbf{M} + w_2^K(\mathbf{I}-\mathbf{M}), \tag{6}$$

where components w_i^K have to comply with additional conditions, i.e. eigentensors $\mathbf{w}_K$ have to be unit length and orthogonal to each other in a sense of the second order tensors norm and scalar product respectively [7]. Fulfilling these conditions results in one independent component, e.g. w_1^I called stiffness distributor being the fifth material constant together with four Kelvin moduli in case of elasticity and transversely isotropic material.

In case of isotropy a relaxation operator has standard form

$$\tilde{\mathbf{S}}^{iso}(t) = \tilde{K}_1^{iso}(t)\mathbf{P}_1^{iso} + \tilde{K}_2^{iso}(t)\mathbf{P}_2^{iso}, \tag{7}$$

where $\tilde{K}_1^{iso}(t)$ and $\tilde{K}_2^{iso}(t)$ are Kelvin relaxation functions responsible for change in volume and shape, respectively. Projectors are defined as

$$\mathbf{P}_1^{iso} = \frac{1}{3}\mathbf{I}\otimes\mathbf{I}, \quad \mathbf{P}_2^{iso} = \mathbf{1} - \frac{1}{3}\mathbf{I}\otimes\mathbf{I}, \tag{8}$$

where $\mathbf{1} = \mathbf{I}\Diamond\mathbf{I}$, cf. (5).

Double contraction of a tensor product of projectors with stress and strain tensor leads to the viscoelastic relation between eigenstates only

$$\boldsymbol{\sigma}_i(t) = \int_{0^-}^{t} \tilde{K}_i(t-\tau)\frac{\partial \boldsymbol{\varepsilon}_i(\tau)}{\partial \tau}d\tau \tag{9}$$

Tensors $\boldsymbol{\sigma}_i$ (also $\boldsymbol{\varepsilon}_i$) in equation (9) are orthogonal to each other (for different indices) in a sense of scalar product of second order symmetric tensors. An index convention is not used in (9), and consequently proposed constitutive equation has a following form

$$\boldsymbol{\sigma}(t) = \sum_i \boldsymbol{\sigma}_i(t) = \sum_i \int_{0^-}^{t} \tilde{K}_i(t-\tau)\frac{\partial \boldsymbol{\varepsilon}_i(\tau)}{\partial \tau}d\tau\ . \tag{10}$$

Since Kelvin relaxation functions map linearly eigenstrains to eigenstresses one dimensional linear viscoelastic models such as Burgers, Zener and Prony Series can be adopted. In this paper the Prony's Series model [8] is used to describe response of every Kelvin relaxation function

$$\tilde{K}_i(t) = K_{i0}\left(1 - \sum_j^N e_{ij}\left(1 - \exp\left(\frac{-t}{\tau_{ij}}\right)\right)\right)H(t), \tag{11}$$

where K_{i0} , e_{ij} , τ_{ij} , N , $H(t)$ denotes instantaneous Kelvin modulus, separation parameters, relaxation times, number of Maxwell model branches in the Prony Series model and Heaviside distribution respectively.

Proposed above constitutive equations were implemented in the finite element method system Abaqus via user subroutine UMAT.

3. Calibration of models

In the paper by Zhang et al. [2] experimental results of creep tests for a number of mineral asphalt mixes in axial compression and indirect tension tests (Brazilian test [9]) are presented. Benefiting from the Wolfram's *Mathematica*® system and a non-linear optimization procedure implemented there in parameters for both transversely isotropic and isotropic material were established.

3.1. Transversely isotropic material

Calibration of transversely isotropic model was split into two parts. On the basis of the axial compression creep test, in which axial shortening and change in circumference were measured, the first and second Kelvin relaxation functions parameters and the stiffness distributor was established. In the next step the third Kelvin relaxation function parameters were determined with fixed parameters of the first and second Kelvin relaxation functions comparing the analytical solution [9] of the indirect tension test with experimental data. Unfortunately there is no data available to establish parameters for the fourth Kelvin relaxation function. However, the fourth eigenstate is responsible for shearing like the third eigenstate; therefore, in simulation of rutting test the same parameters were assumed for the fourth Kelvin relaxation function, as established for the third Kelvin relaxation function. Parameters obtained from a non-linear regression procedure for transversely isotropic material are presented in Table 1.

Table 1. Parameters for transversely isotropic model.

	w_1^I	K_{i0} [MPa]	e_{i1}	τ_{i1} [s]	e_{i2}	τ_{i2} [s]
$\tilde{K}_1(t)$	0.5981	1550.0	0.3452	337.661	0.6405	163.294
$\tilde{K}_2(t)$	0.5981	5243.7	0.7884	1.4941	0.2116	64.022
$\tilde{K}_3(t)$	-	1666.7	0.7285	1.3245	0.2484	17.9127

3.2. Isotropic material

Calibration of isotropic model was based on experimental data from the axial compression test since there are only two Kelvin relaxation functions in case of isotropy and both interact in one dimensional compression. In Table 2 parameters obtained from a non-linear optimization procedure for 2 elements of the Prony Series are presented.

Table 2. Parameters for isotropic model.

	K_{i0} [MPa]	e_{i1}	τ_{i1} [s]	e_{i2}	τ_{i2} [s]
$\tilde{K}_1(t)$	51020.4	0.6224	25.8437	1.15e-8	46.468
$\tilde{K}_2(t)$	5330.5	0.7802	1.3577	0.2145	58.4764

4. Numerical model of rutting test

It is necessary to introduce reasonable simplifications in a numerical model of rutting test since an actual test requires from 10 to 30 thousands cycles of wheel moves. In our study only effect of anisotropy is in interest that is why only several wheel moves will be investigated. Another simplification is neglecting contact interaction between a tire and a specimen, instead of complicated contact procedure constant pressure moving across a specimen in a wheel-like manner was modelled. In rutting test a specimen is a plate of dimensions 500x180x100 mm in a steel mold, cf. Fig. 1.

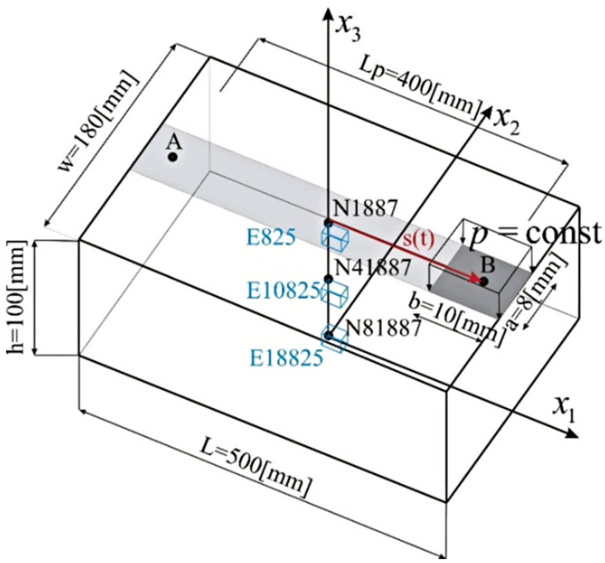

Fig. 1. Characteristic dimensions and load application in rutting test modelling.

Process of loading was split into two phases: phase 1 – load is in point B and ramps linearly from 0 to 0.6 MPa in a period of one second, phase 2 – load 0.6 MPa moves between points A and B in a function of time given as

$$s(t) = \frac{s_{AB}}{2}\cos\left(2\pi\left(t - t_0\right)\right), \tag{12}$$

where s_{AB} is a distance between points A and B and t_0 is time of initial loading equal to 1 s. Load function was programmed as an user subroutine DLOAD in Fortran language [1].

Boundary conditions were imposed at the bottom and side surfaces, constraining all displacements. The whole model consists of 9000 fully integrated bilinear hexahedron elements with the edges of the length 10 mm.

5. Results

In Fig. 2 plot of displacement u_3 along axis x_3 at node 1887 in the middle of the top surface of the specimen (cf. Fig. 1) is presented both for transversely isotropic and isotropic material model. It is easily observed that material model calibrated on the assumption of isotropy is stiffer than on the assumption of transverse isotropy.

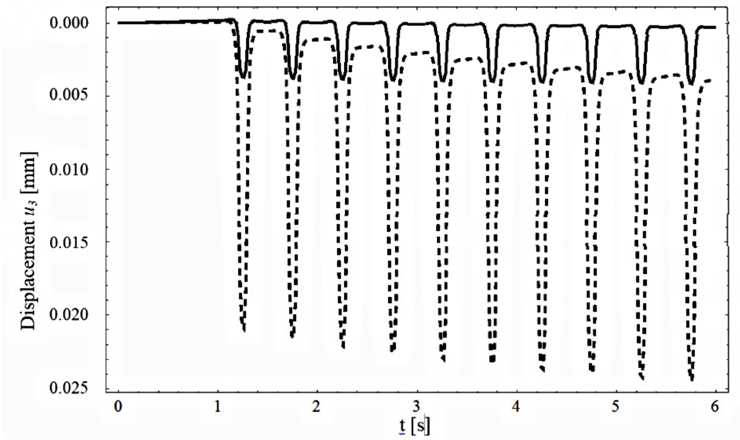

Fig. 2. Displacement u_3 at node 1887 (cf. Fig. 1) for isotropy (continuous) and transverse isotropy (dashed).

In Fig. 3 reduced stress in element number 825 (cf. Fig. 1) according to Huber-Mises-Hencky yield criterion is presented. Despite the fact that element 825 is in surface layer reduced stress in case of transversely isotropic material is twice as higher as in case of isotropic material.

The reason of this phenomenon may be the influence of boundary conditions simulating a steel mold constraint. In rutting test reduced stress is insignificantly small but increase in value when modelling with transversely isotropic material may play a crucial role in other situations, when viscoplasticity and damage phenomena are present.

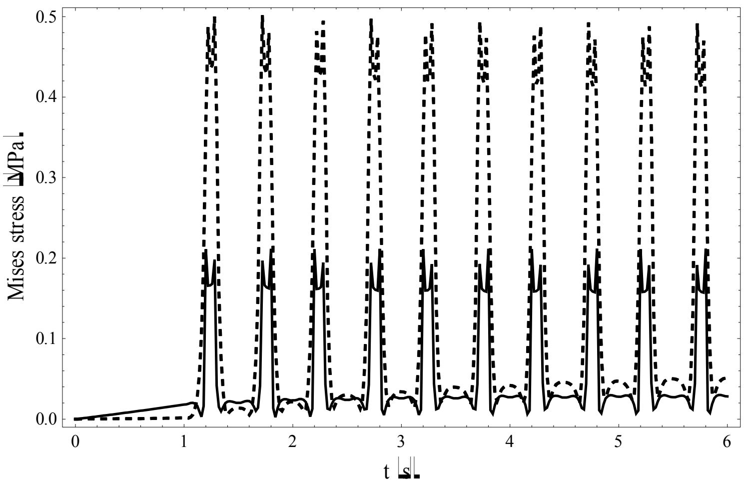

Fig. 3. Huber-Mises stress in element 825 (cf. Fig. 1) for isotropy (continuous) and transverse isotropy (dashed).

6. Conclusions

In conclusion, anisotropy, especially transverse isotropy, in rutting test and more generally mineral asphalt mixes modelling has a significant effect on obtained results and should be considered both in a numerical modelling and laboratory tests. Normally in road laboratories asphalt mixtures are treated as isotropic materials, although their placing processes lead to a clear distinction of one of the directions. Many of the researchers ignore the effects of anisotropy of the material, assuming that it is negligible. In this work it is shown that the effect is significant, and the difficulty of formulating a theoretical model of viscoelasticity for transversely isotropic materials can be overcome by the use of the spectral decomposition theorem and introduction of Kelvin relaxation functions.

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