A methodology for a robust inverse identification of model parameters for porous sound absorbing materials

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Abstract

A methodology of inverse identification of parameters for the Johnson-Champoux-Allard-Lafarge model of porous sound absorbing materials (also with Pride and Lafarge enhancements) is advocated. The inverse identification is based on the measurements of surface acoustic impedance of porous samples. For a single sample of porous material set on a rigid backing wall such measurements provide two specific curves in the considered frequency range, namely, the real and imaginary parts of acoustic impedance. More data suitable for inverse identification can be gathered from additional measurements where the surface acoustic impedance is determined for the same sample yet with an air gap between the sample and the backing wall. As matter of fact, such measurements should be carried out for a few cases where the air gap varies in thickness. Eventually, a set of impedance curves is gained suitable for inverse simultaneous identification of model parameters. In the paper analytical solutions are given for both measurement configurations, namely, for a layer of porous material set on the rigid wall, and for the porous layer separated from the rigid wall by an air gap. These solutions are used by the identification procedure which minimises the difference between the experimental curves and the curves computed from the analytical solutions where the porous layer is modelled using some version of the mentioned poro-acoustic model. The minimisation is carried out with respect to the model parameters, however, not directly, since for this purpose the corresponding dimensionless parameters are introduced. Formulas for the dimensionless parameters are given with respect to the model parameters, and then conversely, for the model parameters with respect to the dimensionless ones. In the formulas two normalising frequencies are introduced which can be considered: one – as characteristic for viscous effects, and the other – as typical for thermal effects. It is claimed that they are not additional parameters, and can be set quite arbitrarily, however, reasonable values must be assumed to allow for very fast and robust identification with initial values for all dimensionless parameters set to 1. This feature is quite important in view of the fact that the choice of initial values for the actual model parameters is rather essential and can be often very problematic. The whole procedure is illustrated with a numerical example and by tests based on laboratory measurements of porous ceramic samples.

1 Introduction

Many models were proposed for the problem of sound absorption in porous media. The simplest models are purely empirical and involve only a couple of parameters. Of that sort are the models of Dealny and Bazley [1] with important generalizations proposed by Miki [2, 3]. They are valid for fibrous absorbent materials of very high porosity (originally they were proposed and validated for fibrous materials with porosity close to unity). The main material parameter is the flow resistivity of fibrous material. More complicated models are less restrictive, yet they involve more parameters related to the average porous geometry. Attenborough [4] proposed a model for rigid fibrous absorbents and granular materials using five parameters, namely: porosity, flow resistivity, tortuosity, steady flow shape factor, and dynamic shape factor. More recently, new empirical models for fibrous materials were proposed by Voronina [5], and for granular media by Voronina and Horoshenkov [6, 7].

A general semi-phenomenological model for sound absorbing porous media with rigid frame was originally formulated by Johnson et al. [8, 9] and substantially extended later by Champoux, Allard and Lafarge et al. [10, 11] to include thermal losses in porous medium. Another important improvements were introduced by Pride et al. [12]. It is a versatile model based on a set of independently measurable porous material parameters. In its standard version [13], which is often referenced as the Johnson-Champoux-Allard-Lafarge model (JCAL) [14], it uses six such parameters, namely: the total open porosity, the high frequency limit of the tortuosity (i.e., the classic parameter of tortuosity), the static viscous permeability (originally, the static air flow resistivity), the static thermal permeability, and two characteristic lengths – the viscous and the thermal one. Like in most of the other models of porous media there are also additional parameters represented by some well-known (and easily determinable) properties of fluid in pores (typically, the air). The importance of this model is also confirmed by the fact that together with its parameters it is fully incorporated by the Biot-Allard model for poroelastic media [13], which is essentially based on the Biot's equations of poroelasticity instead of the Helmholtz equation for time-harmonic acoustics. The poroelastic model for sound absorbing media must be utilised when the vibrations of solid frame cannot be neglected, for example, in case of soft porous media or in active systems involving porous materials [15, 16, 17, 18, 19].

The essential parameters of JCAL and Biot-Allard models which result from the micro-geometry of solid frame are in fact some sort of macroscopic, average geometric characteristics of porous medium. Although, most of them can be measured directly it is often problematic and requires instrumentation specific for each parameter. This is the main reason for development of inverse methods of parametric identification based on acoustical measurements carried out in one type of equipment.

Parametric identification of porous media based on acoustical measurements was investigated by few authors. Braccesi and Bracciali [20] applied a least squares regression based on measured reflection coefficient values of sound absorbing porous specimens to estimate reliable values for flow resistivity and structure factor used as parameters by an old model proposed by Zwikker and Kosten [21]. Alba et al. [22] applied an inverse identification method to obtain porosity, fibre diameter and density of fibrous sound absorbing materials using the Voronina model [23, 5]. Acoustical measurements in the impedance or standing wave tube were used to identify the parameters of JCAL model by Sellen et al. [24], Atalla and Panneton [25], and Zielinski [26]. The present paper is a continuation and extension of this last work.

2 Acoustic absorption and surface impedance of two-layered and single-layer media

The parametric identification of a porous material will be based on the confrontation of the measured acoustic impedances of a layer of porous medium with various gaps between the layer and the rigid wall, and the adequate impedances computed from the model with parameters adjusted by an optimization procedure. The acoustic absorption coefficient for such two-layered media will be also investigated. Figure 1 depicts the relevant configuration of a two-layered medium with total thickness ℓ. It is composed of: layer 1 with thickness $\ell_1 = \xi\ell$, where $\xi \in [0, 1]$, and layer 2 with thickness $\ell_2 = (1 - \xi)\ell$. The porosity and density of materials (media) used for each of the layers will be denoted respectively by ϕ_1 and ϱ_1 for medium 1, and by ϕ_2 and ϱ_2 for medium 2. It case of the layer 1 the medium is fluid (air) so as a matter of fact its porosity is 1.

The free surface of two-layered medium (i.e., at $x = \ell$) is subjected to the excitation by a plane harmonic acoustic wave which penetrates the layers and is reflected by the interface between them (at $x = \xi\ell$) and by the rigid wall termination (at $x = 0$). The interference pattern of those waves can be found by solving the unidimensional Helmholtz equations of harmonic acoustics (with the angular frequency ω) for both coupled layers. The pressure and velocity fields are therefore defined within the layer 1, i.e., for $x \in [0, \xi\ell]$, as

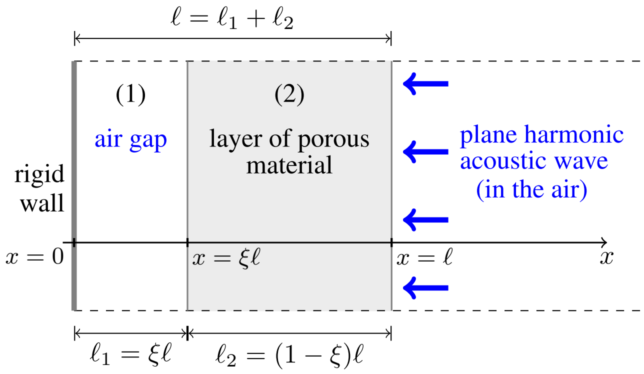

Figure 1: Two-layered medium: a layer of porous material with an air gap to rigid wall

follows

$$p^{(1)}(x) = A_1^{(1)} \mathrm{e}^{-\mathrm{i}k_1 x} + A_2^{(1)} \mathrm{e}^{\mathrm{i}k_1 x} , \qquad v^{(1)}(x) = \frac{k_1}{\omega \varrho_1} \left(A_1^{(1)} \mathrm{e}^{-\mathrm{i}k_1 x} - A_2^{(1)} \mathrm{e}^{\mathrm{i}k_1 x} \right) , \tag{1}$$

where k_1 is the wavenumber in medium 1 (i is the imaginary unit), while $A_1^{(1)}$ and $A_2^{(1)}$ are the (unknown) pressure amplitudes of incident and reflected waves in layer 1. Similarly, these fields inside the layer 2, i.e., for $x \in [\xi\ell, \ell]$, are defined by formulas:

$$p^{(2)}(x) = A_1^{(2)} \mathrm{e}^{-\mathrm{i}k_2 x} + A_2^{(2)} \mathrm{e}^{\mathrm{i}k_2 x} , \qquad v^{(2)}(x) = \frac{k_2}{\omega \varrho_2} \left(A_1^{(2)} \mathrm{e}^{-\mathrm{i}k_2 x} - A_2^{(2)} \mathrm{e}^{\mathrm{i}k_2 x} \right) , \tag{2}$$

where k_2 is the wavenumber in medium 2, while $A_1^{(2)}$ and $A_2^{(2)}$ are the pressure amplitudes of incident and reflected waves in layer 2.

The unknown amplitudes are derived by applying the boundary and interface conditions, namely: the zero normal velocity at the rigid wall, i.e., at $x = 0$: $v^{(1)}(0) = 0$; the pressure and velocity flux continuity and at the interface between two layers, i.e., at $x = \xi\ell$: $p^{(1)}(\xi\ell) = p^{(2)}(\xi\ell)$ and $\phi_1 v^{(1)}(\xi\ell) = \phi_2 v^{(2)}(\xi\ell)$; and finally, the pressure boundary condition at the free surface, i.e., at $x = \ell$: $p^{(2)}(\ell) = \hat{p}$, where $\hat{p}$ is the acoustic pressure amplitude of the incident plane harmonic wave penetrating the two-layered medium. Eventually, the following results are obtained:

$$A_1^{(1)} = A_2^{(1)} = \hat{p} \frac{2k_2\phi_1\varrho_1 \mathrm{e}^{\mathrm{i}(\xi k_1 + (1+\xi)k_2)\ell}}{A_0}, \tag{3}$$

$$A_1^{(2)} = \hat{p} \frac{(k_2\phi_1\varrho_1 + k_1\phi_2\varrho_2)\mathrm{e}^{\mathrm{i}(1+2\xi)k_2\ell} + (k_2\phi_1\varrho_1 - k_1\phi_2\varrho_2)\mathrm{e}^{\mathrm{i}(2\xi k_1 + (1+2\xi)k_2)\ell}}{A_0}, \tag{4}$$

$$A_2^{(2)} = \hat{p} \frac{(k_2\phi_1\varrho_1 + k_1\phi_2\varrho_2)\mathrm{e}^{\mathrm{i}(2\xi k_1 + k_2)\ell} + (k_2\phi_1\varrho_1 - k_1\phi_2\varrho_2)\mathrm{e}^{\mathrm{i}k_2\ell}}{A_0}, \tag{5}$$

where the denominator

$$A_0 = (k_2\phi_1\varrho_1 + k_1\phi_2\varrho_2)\left(\mathrm{e}^{2\mathrm{i}\xi k_2\ell} + \mathrm{e}^{2\mathrm{i}(\xi k_1 + k_2)\ell}\right) + (k_2\phi_1\varrho_1 - k_1\phi_2\varrho_2)\left(\mathrm{e}^{2\mathrm{i}k_2\ell} + \mathrm{e}^{2\mathrm{i}\xi(k_1+k_2)\ell}\right). \tag{6}$$

Now, the surface acoustic impedance at the free surface of two-layered medium, i.e., at $x = \ell$, can be computed as follows:

$$Z = \frac{p^{(2)}(\ell)}{-\phi_2 v^{(2)}(\ell)} = \frac{\hat{p}}{-\phi_2 v^{(2)}(\ell)} = \frac{\omega\varrho_2}{\phi_2 k_2}\left[-\frac{A_1^{(2)}}{\hat{p}} \mathrm{e}^{-\mathrm{i}k_2\ell} + \frac{A_2^{(2)}}{\hat{p}} \mathrm{e}^{\mathrm{i}k_2\ell} \right]^{-1}, \tag{7}$$

where the coefficients $A_1^{(2)}/\hat{p}$ and $A_2^{(2)}/\hat{p}$ are calculated from the formulas derived above, and it should be noticed that the surface impedance $Z(\omega)$ is a complex-valued frequency-dependent characteristics which actually does not depend on the excitation pressure amplitude $\hat{p}$.

In the proposed approach the first layer will be the air gap so that $\phi_1 = 1$, while $\varrho_1 = \varrho_{\mathrm{f}}$ and $k_1 = \omega/c_{\mathrm{f}}$, where ϱ_{f} and c_{f} are the density of air and the sound speed in air, respectively. They are the well-known constants easily and precisely determinable for any conditions of ambient pressure and temperature. The second layer will be a layer of porous material with open porosity $\phi_2 = \phi \in (0,1)$ filled with air. Moreover, for this layer: $\varrho_2 = \varrho(\omega)$ and $k_2 = \omega/c(\omega)$, where the effective density $\varrho(\omega)$ and the effective speed of sound $c(\omega)$ for porous material are not real-valued constants, yet they are rather complex frequency-dependent characteristics.

In the case when there is no air gap, and the porous layer is set directly on the rigid wall, that is for $\xi = 0$, $\ell_1 = 0$, and $\ell_2 = \ell$, the whole problem is simplified and the surface acoustic impedance reads as follows

$$Z = \frac{\varrho c}{\phi} \frac{\exp(2\mathrm{i}\omega\ell/c) + 1}{\exp(2\mathrm{i}\omega\ell/c) - 1} = -\mathrm{i}\frac{\varrho c}{\phi} \cot\big(\omega\ell/c\big). \tag{8}$$

When the surface acoustic impedance of two-layered or single-layer medium is known, the relevant reflection coefficient can be calculated [13, 18]:

$$R(\omega) = \frac{Z(\omega) - Z_{\mathrm{f}}}{Z(\omega) + Z_{\mathrm{f}}}, \tag{9}$$

where $Z_{\mathrm{f}} = \varrho_{\mathrm{f}} c_{\mathrm{f}}$ is the characteristic impedance of fluid (air) in pores and outside the porous medium. Finally, knowing the reflection coefficient, the acoustic absorption coefficient is determined as follows:

$$A(\omega) = 1 - |R(\omega)|^2. \tag{10}$$

3 Model parameters for sound absorbing rigid porous materials

The formulas for acoustic impedance and absorption derived in the previous Section require the effective density $\varrho(\omega)$ and speed of sound $c(\omega)$ of porous material. The effective speed of sound in porous medium can be calculated from the following formula (similar to the classical one valid for a homogeneous isotropic medium):

$$c(\omega) = \sqrt{K(\omega)/\varrho(\omega)}, \tag{11}$$

where $K(\omega)$ is the effective bulk modulus of porous medium. Such approach assumes that the porous medium exhibits macroscopically isotropic behaviour, and for the considered frequency range of interest it can be homogenized into a dispersive medium of equivalent fluid.

The approach is used by many models of sound absorbing rigid porous materials, including the so-called Johnson-Champoux-Allard model. It is an advanced semi-phenomenological model which provides the following formulas for the effective density and bulk modulus of homogenized porous medium with open porosity and rigid (motionless) skeleton, namely:

$$\varrho(\omega) = \varrho_{\mathrm{f}}\alpha(\omega), \qquad K(\omega) = \frac{P_0}{1 - \dfrac{\gamma_{\mathrm{f}} - 1}{\gamma_{\mathrm{f}}\alpha'(\omega)}}. \tag{12}$$

Here, P_0 is the ambient mean pressure, whereas ϱ_{f} and γ_{f} are the density and the ratio of specific heats for the fluid (air) in pores. Moreover, $\alpha(\omega)$ is the so-called dynamic (viscous) permeability of porous medium; it is a complex-valued frequency-dependent characteristics, with the real part always greater than 1, which illustrates the fact that the effective density of virtual fluid equivalent to porous medium is greater the actual density of fluid in pores. Finally, $\alpha'(\omega)$ is the thermal analogue of viscous dynamic tortuosity.

According to the Johnson-Champoux-Allard model, the tortuosity functions are calculated using the following formulas

$$\alpha(\omega) = \alpha_\infty + \frac{\nu_{\mathrm{f}}}{\mathrm{i}\omega}\frac{\phi}{k_0}\sqrt{\frac{\mathrm{i}\omega}{\nu_{\mathrm{f}}}\left(\frac{2\alpha_\infty k_0}{\Lambda\phi}\right)^2 + 1}, \qquad \alpha'(\omega) = 1 + \frac{\nu'_{\mathrm{f}}}{\mathrm{i}\omega}\frac{\phi}{k'_0}\sqrt{\frac{\mathrm{i}\omega}{\nu'_{\mathrm{f}}}\left(\frac{2k'_0}{\Lambda'\phi}\right)^2 + 1}. \tag{13}$$

Here, ν_f is the kinematic viscosity of pore-fluid (air), whereas $\nu'_\mathrm{f} = \nu_\mathrm{f}/\mathrm{Pr}_\mathrm{f}$ with Pr_f being the Prandtl number of pore-fluid (air). The remaining six parameters depend on the micro-geometry of porous medium, and as a matter of fact, they describe it in an averaging way; they are: ϕ – the total porosity, α_∞ – the tortuosity parameter (and at the same time, the high-frequency limit of the viscous tortuosity function), k_0 – the (viscous, static) permeability, k'_0 – the thermal analogue of permeability, and finally, Λ and Λ' – two characteristic lengths representative for viscous and thermal effects, respectively. When the six parameters of 'averaged geometry' are determined the dynamic tortuosities (13) can be easily calculated for any frequency from the admissible range, since the necessary parameters of air are known. Then, the effective characteristics of density, bulk modulus, and eventually, the effective speed of sound can be computed and used for the formulas for acoustic impedance and absorption of a porous layer set directly to the rigid wall or with an air gap between.

4 Parametric identification of sound absorbing rigid porous media

4.1 Identification procedure

The real and imaginary parts of the functions of surface impedance provided by the analytical model described in previous Sections may be used to identify the geometric parameters by adjusting them in order to make the analytical impedance curves fit with the ones found from the relevant measurements of a porous sample to be identified. Nevertheless, such approach will rather not be very robust, and so usually not successful, when dealing directly with the original model parameters listed above. Instead, the following set of dimensionless parameters is proposed [26]:

$$p_1 = \alpha_\infty - 1, \qquad p_2 = \frac{\nu_\mathrm{f}}{\omega_*}\frac{\phi}{k_0}, \qquad p_3 = \frac{\nu'_\mathrm{f}}{\omega'_*}\frac{\phi}{k'_0},$$
$$p_4 = \frac{\omega_*}{\nu_\mathrm{f}}\left(\frac{2\alpha_\infty k_0}{\Lambda\phi}\right)^2, \qquad p_5 = \frac{\omega'_*}{\nu'_\mathrm{f}}\left(\frac{2k'_0}{\Lambda'\phi}\right)^2. \tag{14}$$

Here, two additional quantities are introduced, namely $\omega_* = 2\pi f_*$ and $\omega'_* = 2\pi f'_*$, where f_* and f'_* are some sort of reference (or scaling) frequencies, for viscous and thermal effects, respectively. They allow to properly scale the dimensionless parameters. The main purpose is that for the same initial value (typically 1) used for these parameters by the optimization procedure, when it is completed the identified values of dimensionless parameters will be of similar order. It is important to notice that the scaling frequencies are not additional parameters since they can be chosen rather arbitrarily, and for various choices the same results should be obtained [26]. Nevertheless, reasonable values for these frequencies must be used (for example, $f_* = 3\,\mathrm{kHz}$ and $f'_* = 1\,\mathrm{kHz}$), and moreover, the inequality $f_* > f'_*$ should be asserted, which is in accordance with the fact that the thermal effects are more sound at lower frequencies and the viscous ones at higher frequencies.

Now, the tortuosity functions can be expressed with respect to the dimensionless parameters, namely:

$$\alpha(\omega) = 1 + p_1 + \frac{\omega_*}{\mathrm{i}\omega}p_2\sqrt{\frac{\mathrm{i}\omega}{\omega_*}p_4 + 1}, \qquad \alpha'(\omega) = 1 + \frac{\omega'_*}{\mathrm{i}\omega}p_3\sqrt{\frac{\mathrm{i}\omega}{\omega'_*}p_5 + 1}. \tag{15}$$

And thus, eventually, the formulas for surface impedance (7) or (8) can be expressed with respect to these dimensionless parameters. However, yet another dimensionless parameter should be added to the set of five (14), namely: $p_0 = \phi$, since the porosity appears independently (from k_0 and k'_0) in the formulas for surface impedance (7) or (8). When the dimensionless parameters are known the original model parameters can be determined by the following inverse formulas:

$$\begin{gathered}\phi = p_0, \qquad \alpha_\infty = 1 + p_1, \qquad k_0 = \frac{\nu_\mathrm{f}}{\omega_*}\frac{p_0}{p_2}, \qquad k'_0 = \frac{\nu'_\mathrm{f}}{\omega'_*}\frac{p_0}{p_3}, \\ \Lambda = \frac{2+2p_1}{p_2}\sqrt{\frac{\nu_\mathrm{f}}{\omega_* p_4}}, \qquad \Lambda' = \frac{2}{p_3}\sqrt{\frac{\nu'_\mathrm{f}}{\omega'_* p_5}}.\end{gathered} \tag{16}$$

Now, the identification procedure should be stated up as follows:

- In some frequency range the surface impedance of a sample of porous material of known thickness is measured in the impedance tube. As a matter of fact a few measurements are carried out for the same sample set directly to a rigid termination in the tube and with air gaps of various thickness between the sample and the rigid termination.
- An objective function is defined as the sum of squared measures of difference between the measured curves (i.e. the real and imaginary parts of surface impedances) and their analytical analogues computed from the model discussed above, with some assumed values for the dimensionless parameters $p_0, p_1, \dots, p_5$.
- The objective function is minimized with respect to the six dimensionless parameters: $p_0, p_1, \dots, p_5$. When the minimization procedure is completed, the identified values of dimensionless parameters are used to calculate the geometric parameters of the rigid porous model.

This procedure will be illustrated below by one numerical example and a final test using experimental measurements of porous ceramic samples.

4.2 Numerical example

A numerical test was carried out to verify the identification procedure. First, for some realistic original porous material parameters listed in Table 1 the impedance curves were calculated for a layer of thickness 20 mm in the frequecny range from 500 Hz to 6 kHz. The impedances were computed for three cases: without air gap, with an air gap 10 mm-thick, and with an air gap 20 mm-thick. Then, some random noise was added to the impedance curves and such noisy signals were used as the artificial measurement curves by the identification procedure which started with all dimensionless parameters equal to 1. The reference frequencies used for scaling/weighting the parameters were: $f_* = 4\,\mathrm{kHz}$ and $f'_* = 1\,\mathrm{kHz}$.

Parameter	Symbol	Unit	Original value	Identified value	Error [%]
total porosity	ϕ	%	95.00	95.70	0.74
tortuosity	α_∞	–	1.400	1.368	2.32
viscous permeability	k_0	$10^{-9}\,\mathrm{m}^2$	0.600	0.607	1.10
thermal permeability	k'_0	$10^{-9}\,\mathrm{m}^2$	4.000	4.082	2.04
viscous length	Λ	$10^{-6}\,\mathrm{m}$	70.00	65.78	6.04
thermal length	Λ'	$10^{-6}\,\mathrm{m}$	200.0	203.4	1.69

Table 1: Original and identified values of geometric parameters and relative errors

Figures 2 and 3 present the noisy signals simulating the measurements and the impedances found after the identification procedure was accomplished. The ‘found’ curves are actually almost exactly the same as the original signals before the random noise was applied, which will be confirmed below when showing the results of acoustic absorption.

The geometric parameters were computed from the identified values of the dimensionless parameters. They are listed in Table 1 together with the original parameters and the relative errors which are also shown in

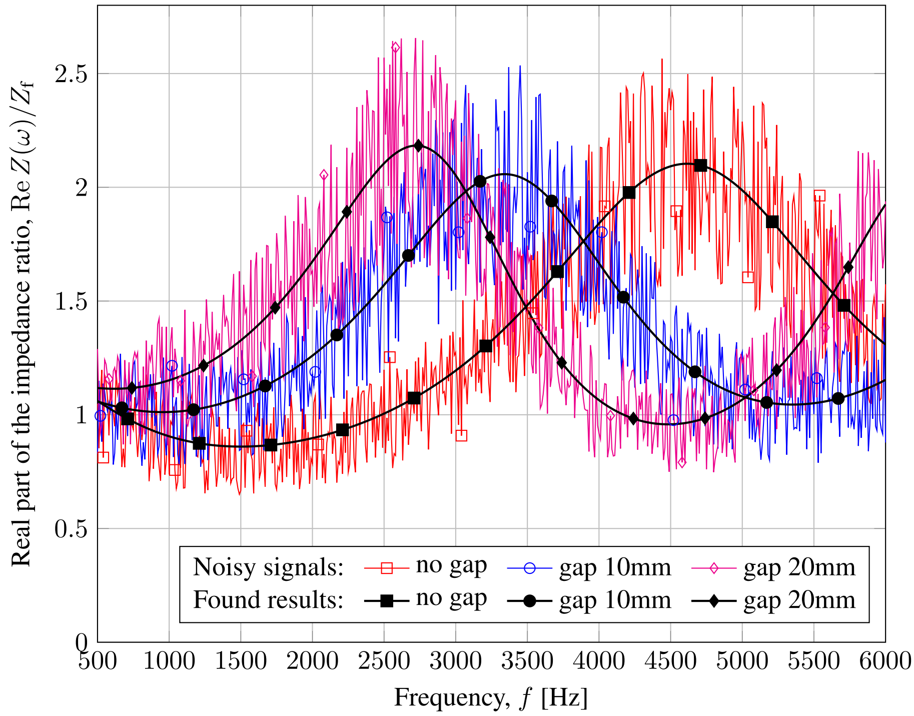

Figure 2: Real part of the impedance ratio for the 25 mm-thick porous layer in three configurations: set directly to the rigid wall, or with air gaps of 10 mm or 20 mm between the layer and the wall

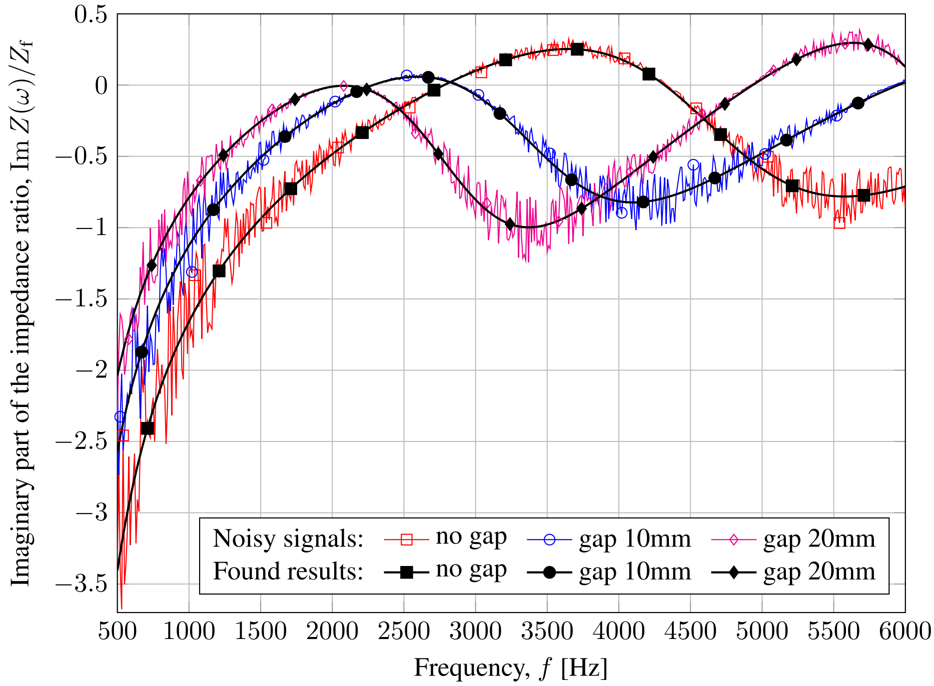

Figure 3: Imaginary part of the impedance ratio (see Figure 2)

Figure 4. These errors are rather small, below 10%, and they are typical results for this sort of numerical test (i.e., similar errors were found for another trials with similar level of random noise). It is important to notice that the error in porosity estimation is close to 0.

Figure 5 shows the curves of the acoustic absorption computed for the original parameters (without noise) and the identified parameters for all three cases of porous material configuration. It is clearly visible that the

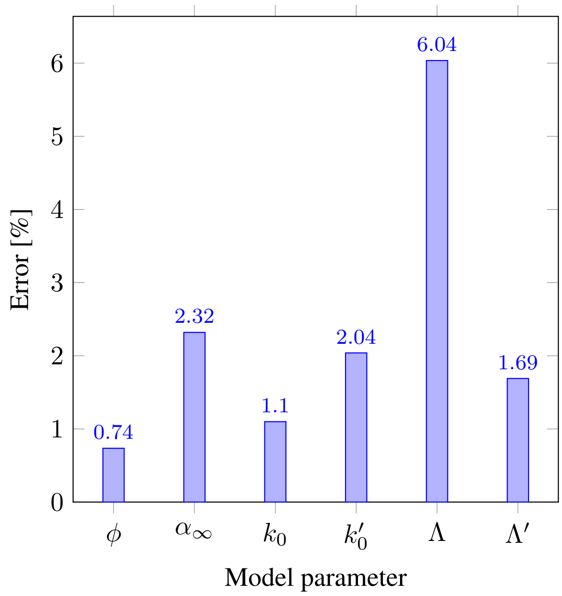

Figure 4: Errors of the identified model parameters with respect to their original values

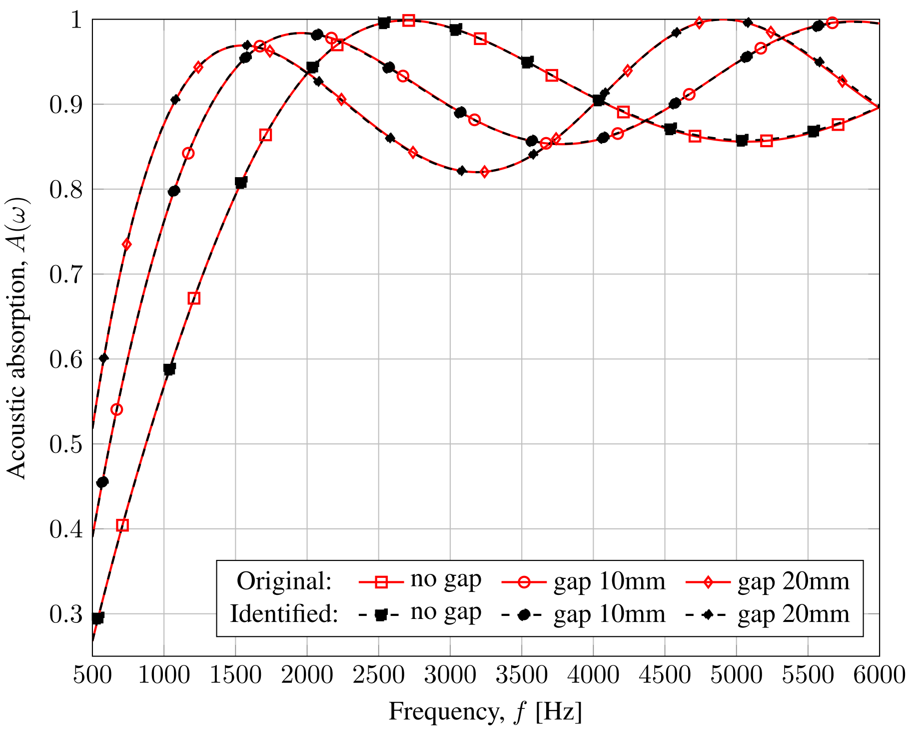

Figure 5: Acoustic absorption of the 25 mm-thick layer (in three configurations: set to the rigid wall or with air gaps between the layer and the wall) – the results computed for the original and identified values of model parameters

corresponding curves are nearly identical which gives some notion about the sensitivity of small variations of some geometric parameters to the results of acoustic absorption and impedance.

4.3 Experimental validation of the identification procedure

The identification procedure was eventually applied for high-porosity alumina foam [27]. It has been reported recently that such foams exhibit very good sound absorbing properties [28]. From two specimens of such foam manufactured separately – one with thickness app. 24 mm, the other with thickness app. 18 mm – two cylindrical samples were cut out with the diameter of 29 mm to be well-fitted inside the impedance tube. Both samples were measured in the tube for their surface acoustic impedance and absorption using the so-called two-microphone transfer function method [29, 30, 31, 32]. Each of the samples were tested in five configurations: first, set directly to the rigid piston termination in the tube, and then with air gaps between the sample and the rigid termination so that the total thickness of such two-layered sound absorbing was 30, 40, and 50 mm.

The impedance curves measured for the thicker sample were used by the parametric identification algorithm. Their real and imaginary parts are shown in Figures 6 and 7 together with the corresponding curves calculated after the identification from the analytical model using the identified parameters. The geometric parameters identified by the squared-error minimization procedure are listed in Table 2 and their corresponding dimensionless parameters – obtained for the reference frequencies $f_* = 4\,\mathrm{kHz}$ and $f'_* = 1\,\mathrm{kHz}$ – are shown in Figure 8. It should be noticed that the identified porosity of about 90% is in excellent accordance with the

Parameter	Symbol	Unit	Identified value
total porosity	ϕ	%	89.63
tortuosity	α_∞	–	1.203
viscous permeability	k_0	$10^{-9}\,\mathrm{m}^2$	2.269
thermal permeability	k'_0	$10^{-9}\,\mathrm{m}^2$	5.350
viscous length	Λ	$10^{-6}\,\mathrm{m}$	50.43
thermal length	Λ'	$10^{-6}\,\mathrm{m}$	339.3

Table 2: Identified values of geometric parameters for porous ceramic sample

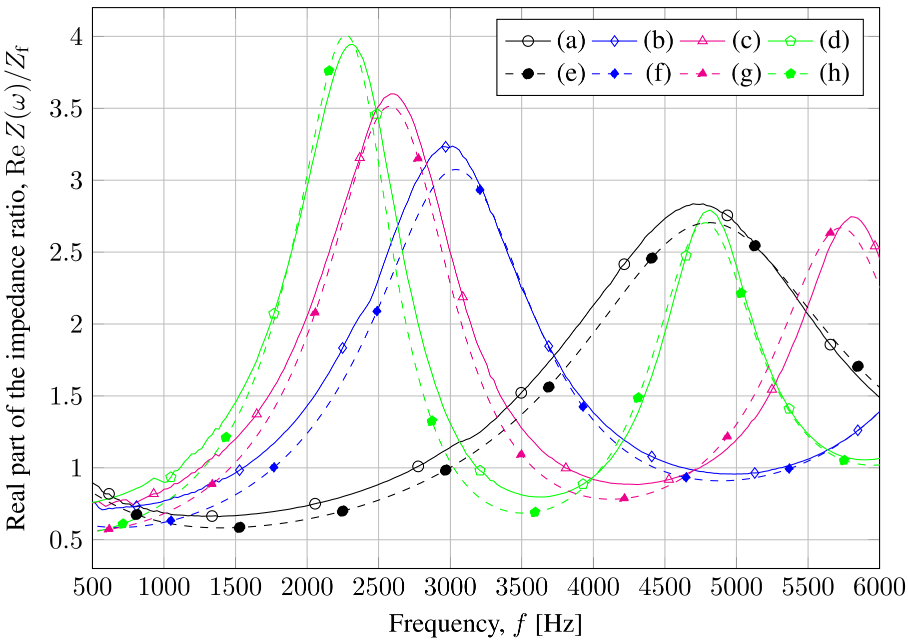

Figure 6: Real part of the impedance ratio of 24 mm-thick porous ceramic sample with or without air gap to the rigid wall, and the total thickness: (a,e) equal to ceramic sample (no gap), (b,f) 40 mm, (c,g) 50 mm, (d,h) 60 mm. The results of the experimental testing (a,b,c,d) and modelling (e,f,g,h) using the identified parameters

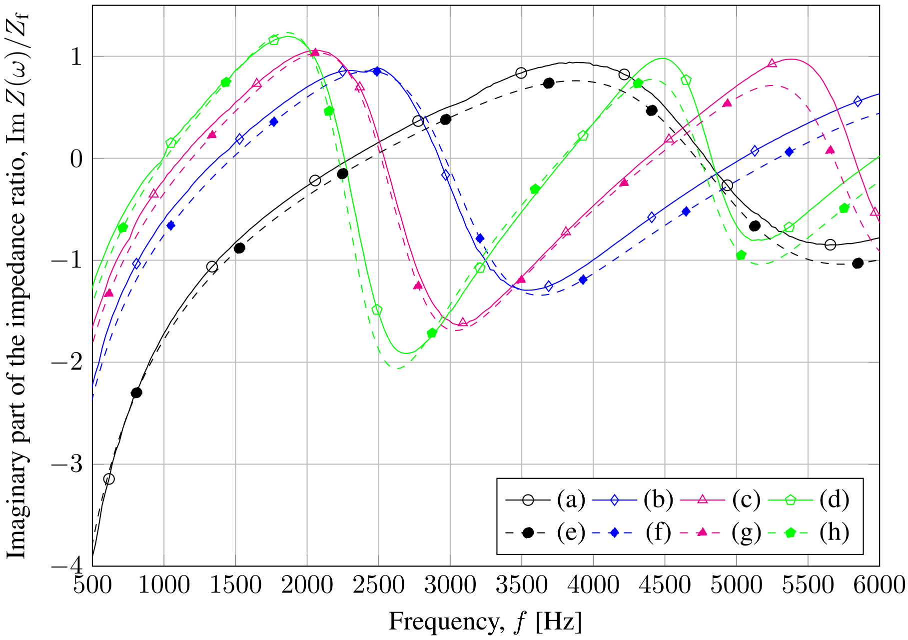

Figure 7: Imaginary part of the impedance ratio of 24 mm-thick porous ceramic sample (see caption to Figure 6)

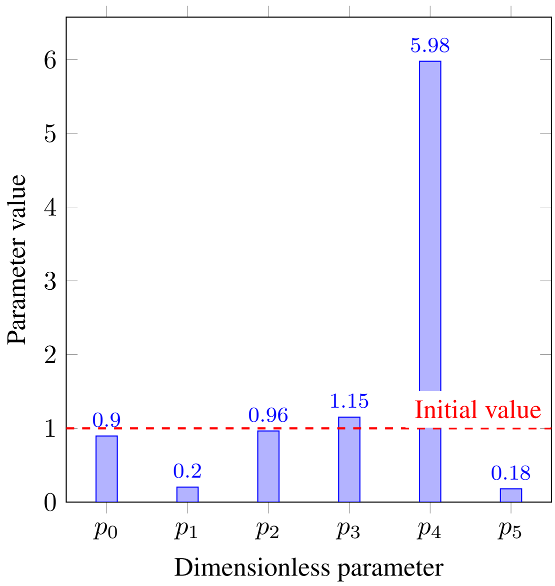

Figure 8: Identified values of dimensionless parameters

value declared by the foam manufacturer. Finally, the acoustic absorption curves measured in the impedance tube and computed from the model are presented in Figure 9 for the identified porous sample with thickness 24 mm, and in Figure 10 for another sample of presumably the same porous ceramic yet with thickness 18 mm. In this latter case the discrepancies between the measured and modelled results are bigger (probably because of a poor quality of one face of this sample), yet the general agreement between the corresponding curves is rather good, which essentially validates the identification.

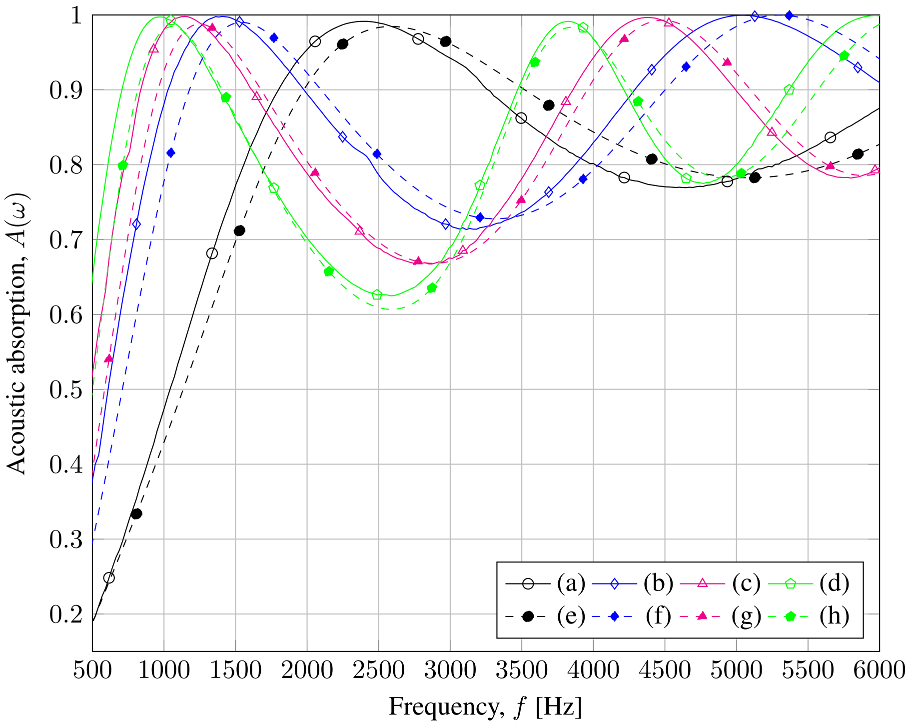

Figure 9: Acoustic absorption for the 24 mm-thick porous ceramic sample in various configurations (see caption to Figure 6). The results of the experimental testing (a,b,c,d) and modelling (e,f,g,h) using the identified parameters

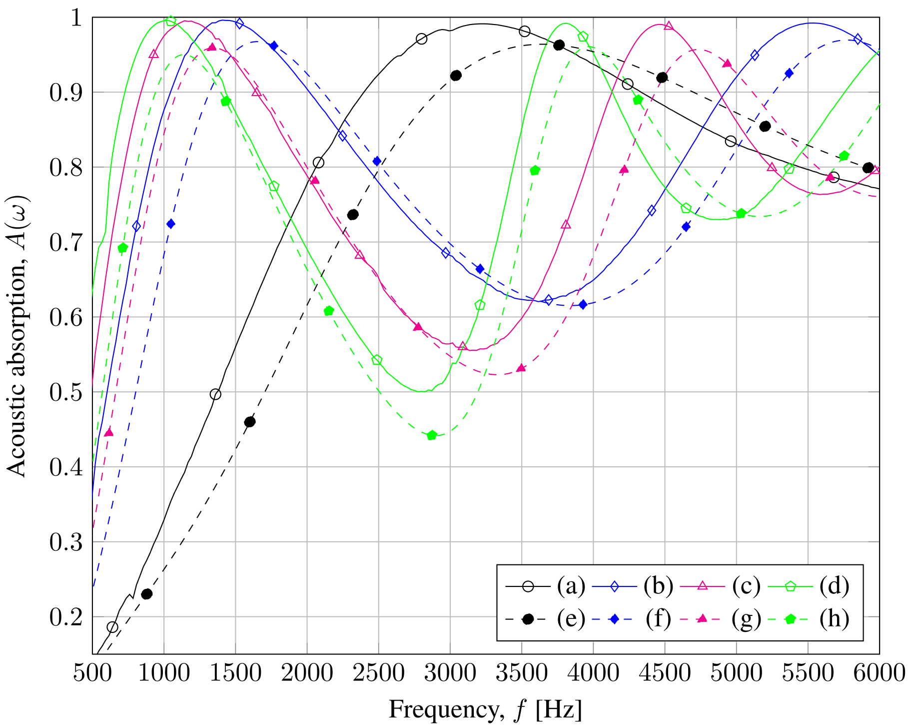

Figure 10: Acoustic absorption for the 18 mm-thick porous ceramic sample in various configurations (as in the case of the thicker sample – see caption to Figure 6). The results of the experimental testing (a,b,c,d) and modelling (e,f,g,h) using the parameters identified by the procedure based on the impedance measurements of the thicker sample

5 Conclusions

- There are two main features which should affirm robustness of identification methodology: (1) the optimization procedure is carried out with respect to suitably normalized dimensionless parameters; (2) it uses more experimental data gathered from additional measurements of the same sample yet with a known air gaps between the sample and the backing wall.
- Two reference frequencies are not additional parameters and for various pairs of these scaling factor the same final results should be obtained, that is: although the sets of dimensionless parameters will be different yet they will provide the same model parameters. Nevertheless, the values for reference frequencies must be chosen reasonably.
- A 'better' choice of reference frequencies will entail less iterations of the optimization procedure. However, unreasonable choices may generate false results. As a matter of fact, the validity of minima found by the optimization procedure may be judged as correct (i.e., global not local) when it is achieved by a few calculations with various (reasonable) pairs of scaling frequencies.
- There is no need to use specific initial values for the sought parameters: the standard suggested initial value for all dimensionless parameters is unity.

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