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Monitoring of progressive collapse of skeletal structures

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Abstract. The authors propose an idea of monitoring the state of skeletal structures of high importance (e.g. roof structures over large-area buildings) with the aim of identification of slowly-developing plastic zones. This is formulated as an inverse problem within the framework of the Virtual Distortion Method, which was used previously to identify stiffness/mass modifications in similar manner. Permanent plastic strains developed in a truss element can be modeled by an initial strain (virtual distortion) introduced to the structure. The formation of subsequent plastic zones in the structure is assumed to be slow. Consequently, the design variable (plastic strain) is time-independent, which makes the inverse analysis efficient. This article presents problem formulation and numerical algorithm for identification of the plastic strains in truss structures. The identification relies on gradient-based optimization. A numerical example is included to demonstrate the efficiency of the algorithm.

1. Introduction

There are two types of skeletal structures of high importance – bridges and roof structures over large-area buildings. The former are designed with high safety factors, which makes them work in the elastic regime. The latter do not have such reserves of capacity and sometimes exhibit plastic behavior under extraordinary loadings. There are documented cases of progressive collapses of truss roof structures, which were slow, not sudden i.e. it took several months from the formation of first plastic hinge to the final disaster. A good example of such accident is the collapse of the exhibition centre in 2006, Katowice, Poland, with many casualties. Motivated by the mentioned fact, the authors propose an idea of monitoring the state of such large-area roof structures with the aim of identification of slowly-developing plastic zones. This task can be formulated as an inverse problem within the framework of the Virtual Distortion Method [1, 2], which was used previously to identify stiffness/mass modifications in skeletal structures. Permanent plastic strains developed in a truss/frame element can be modeled by an initial strain (distortion) introduced to the structure. As the formation of subsequent plastic zones in the structure is assumed to be slow, the design variable (plastic strain) is time-independent, which makes the inverse analysis efficient. The proposed idea of solving the identification problem of progressive collapse may be further extended to identification of other defects, e.g. loosening of bolts in a structural joint, using similarities between relevant constitutive laws.

2. Plastic strains as damage indicator

2.1. Baseline method - the Virtual Distortion Method

As mentioned in the Introduction, the Virtual Distortion Method (VDM) constitutes the framework of our analysis. The method belongs to fast reanalysis methods [3] and has been successfully used in various types of engineering analyses, including optimal remodelling [4, 5], progressive collapse [6], adaptive impact absorption [7, 8] and structural health monitoring (SHM) [9]-[14]. The VDM can be easily applied for linear systems. In order to account for non-linearities, one should use a piece-wise linear approximation of the original curve.

Let us briefly look at the VDM fundamentals using the truss structure model to explain how the method works. The basic idea of the VDM is to introduce some pseudo-strains (called virtual distortions) or equivalent pseudo-forces to the structure in order to model some modifications of the structure e.g. the local change of stiffness or mass in some element. For effective numerical calculation of these structural modifications using virtual distortions, one should build a set of interrelations between truss members in the form of the influence matrix. In statics, by introducing a virtual distortion to the truss structure of some redundancy, one gets a pre-stressed state in all members of the structure, which constitutes a vector of the 2D ($n \times n$, where n is the number of elements) influence matrix. Analogously in dynamics, one can introduce an impact distortion and consider resulting variations of strains in other members over some pre-defined time, which results in a 3D ($n \times n \times t$, where t is the number of time steps) influence matrix.

If we denote the influence matrix by $D_{\alpha\beta}$, the response of the modified structure can be expressed as a linear combination of the initial responses $\overset{\text{L}}{\varepsilon}_\alpha$, $\overset{\text{L}}{\sigma}_\alpha$ and the residual parts $\overset{\text{R}}{\varepsilon}_\alpha$, $\overset{\text{R}}{\sigma}_\alpha$ responsible for modeling of the modification, i.e.:

$$\varepsilon_\alpha = \overset{\text{L}}{\varepsilon}_\alpha + \overset{\text{R}}{\varepsilon}_\alpha = \overset{\text{L}}{\varepsilon}_\alpha + D_{\alpha\beta}\overset{0}{\varepsilon}_\beta \,, \qquad \sigma_\alpha = \overset{\text{L}}{\sigma}_\alpha + \overset{\text{R}}{\sigma}_\alpha = \overset{\text{L}}{\sigma}_\alpha + E_{\underline{\alpha}}\left(D_{\underline{\alpha}\beta} - \delta_{\underline{\alpha}\beta}\right)\overset{0}{\varepsilon}_\beta, \tag{1}$$

where the residual parts are defined as:

$$\overset{\text{R}}{\varepsilon}_\alpha = D_{\alpha\beta}\overset{0}{\varepsilon}_\beta \,, \qquad \overset{\text{R}}{\sigma}_\alpha = E_{\underline{\alpha}}\left(D_{\underline{\alpha}\beta} - \delta_{\underline{\alpha}\beta}\right)\overset{0}{\varepsilon}_\beta. \tag{2}$$

In the above formulas as well as in the next equations there is no summation over the underlined indices. The generalized forces for the truss structure subject to modifications and the one modelled by virtual distortions can be written in the following form:

$$\overline{N}_\alpha = E^\star_{\underline{\alpha}} a^\star_{\underline{\alpha}} \varepsilon_{\underline{\alpha}} \,, \qquad N_\alpha = E_{\underline{\alpha}} A_{\underline{\alpha}} \left(\varepsilon_{\underline{\alpha}} - \overset{0}{\varepsilon}_{\underline{\alpha}}\right). \tag{3}$$

In the equations (3) $\overline{N}_\alpha$ and N_α denote internal forces of the modified and modelled structure, respectively. The structural modification is understood here as a change of Young's modulus and cross-section area of an α-element. The modified quantities are marked by asterisks. Enforcing the equality of strains and generalized forces for the modified and modelled structures, the vector of stiffness modification parameters can be expressed as follows:

$$\mu_\alpha = \frac{k^\star_{\underline{\alpha}}}{k_{\underline{\alpha}}} = \frac{\varepsilon_{\underline{\alpha}} - \overset{0}{\varepsilon}_{\underline{\alpha}}}{\varepsilon_{\underline{\alpha}}}, \tag{4}$$

where $k^\star_\alpha = E^\star_{\underline{\alpha}} A^\star_{\underline{\alpha}}$ and $k_\alpha = E_{\underline{\alpha}} A_{\underline{\alpha}}$ are axial stiffnesses of the modified and original structural α-element, respectively. Let us notice that the vector μ_α depends nonlinearly on virtual distortions $\overset{0}{\varepsilon}_\alpha$, due to the relation (1)a.

Relationships (1) and (4) constitute a set of equations, which allow for computation of virtual distortions $\overset{0}{\varepsilon}_\alpha$ as well as updated strain responses ε_α for a given vector of stiffness modification parameters μ_α. In order to determine the virtual distortions $\overset{0}{\varepsilon}_\alpha$, the following set of equations have to be solved:

$$\left[\delta_{\alpha\beta} - \left(1_{\underline{\alpha}} - \mu_{\underline{\alpha}}\right) D_{\underline{\alpha}\beta}\right] \overset{0}{\varepsilon}_\beta = \left(1_{\underline{\alpha}} - \mu_{\underline{\alpha}}\right) \overset{\mathrm{L}}{\varepsilon}_{\underline{\alpha}}, \tag{5}$$

where 1_α denotes the vector of unit components. The updated strains and stresses for the modified structure can be calculated using formulas (1) without recalculating the stiffness matrix.

2.2. Design variables - plastic strains

In the SHM context, the VDM has been applied to follow changes of stiffness/mass [1], [11] - [14], which is the most frequently analysed symptom of damage in structures. In this paper authors propose the analysis of plastic strain development leading to progressive collapse. This could be an alternative indicator of a hazardous state of the monitored structure.

Within the framework of the VDM one can not only use pseudo-strains. The plastic strains with real physical interpretation can be also modeled using the influence matrix $D_{\alpha\beta}$ [6]. One condition must be met however – the constitutive material law has to be piece-wise linear. In the engineering approach, the law is usually bi-linear e.g. elastic-plastic with one section modeling the elastic zone, the other section modeling the yielding zone. Fig. 1 presents a bilinear, elastic-

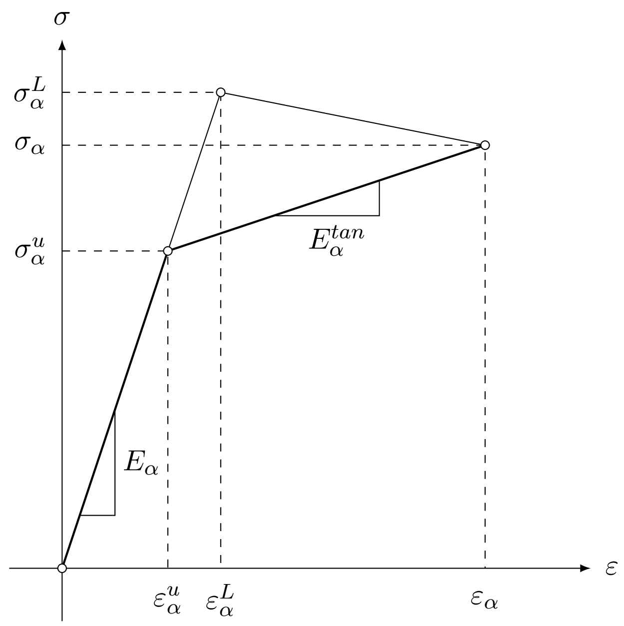

Figure 1. Model of elastic-plastic material.

plastic material model used for analysis of progressive collapse. The yield stress is marked by σ_α^u and the corresponding strain by ε_α^u. E_α^{tan} denotes here the tangent modulus. The point $\left(\overset{\mathrm{L}}{\varepsilon}_\alpha, \overset{\mathrm{L}}{\sigma}_\alpha\right)$ refers to the structural response calculated within the elastic regime only. The yield condition can be expressed by the following expression:

$$\sigma_\alpha - \sigma_\alpha^u = E_{\underline{\alpha}}^{tan} \left(\varepsilon_{\underline{\alpha}} - \overset{0}{\beta}_{\underline{\alpha}}\right), \tag{6}$$

where the virtual distortions $\overset{0}{\beta}_\alpha$ refer to plastic strains. The updated, total strains ε_α and the virtual distortions $\overset{0}{\beta}_\alpha$ are computed using Eqs. (1), which take now the following forms:

$$\varepsilon_\alpha = \overset{\mathrm{L}}{\varepsilon}_\alpha + D_{\alpha\beta}\overset{0}{\beta}_\beta\,, \qquad \sigma_\alpha = \overset{\mathrm{L}}{\sigma}_\alpha + E_{\underline{\alpha}}\left(D_{\underline{\alpha}\beta} - \delta_{\underline{\alpha}\beta}\right)\overset{0}{\beta}_\beta. \tag{7}$$

The virtual distortions $\overset{0}{\beta}_\beta$ can be determined using the Eqs. (7a) and (6). It leads to the formula:

$$\left[\delta_{\alpha\beta} - \left(1_{\underline{\alpha}} - \gamma_{\underline{\alpha}}\right) D_{\underline{\alpha}\beta}\right]\overset{0}{\beta}_\beta = \left(1_{\underline{\alpha}} - \gamma_{\underline{\alpha}}\right)\left(\overset{\mathrm{L}}{\varepsilon}_{\underline{\alpha}} - \varepsilon^u_{\underline{\alpha}}\right), \tag{8}$$

where γ_α is a dimensionless coefficient:

$$\gamma_\alpha = \begin{cases} \frac{E^{tan}_{\underline{\alpha}}}{E_{\underline{\alpha}}}, & |\overset{\mathrm{L}}{\sigma}_\alpha| > |\sigma^u_\alpha| \\ 0, & |\overset{\mathrm{L}}{\sigma}_\alpha| < |\sigma^u_\alpha| \end{cases}. \tag{9}$$

2.3. Problem formulation

Local plastic deformation causes strain and stresses redistribution in structural elements. A sensor-equipped structure allows for monitoring of those states and identification of plastic strains. Based on the measured quantities $\overset{\mathrm{M}}{\varepsilon}_\alpha$, this can be performed in an optimization process by minimization of an objective function expressed in terms of the virtual distortions $\overset{0}{\beta}_\alpha$.

For the identification of plastic zones let us assume the elastic-ideally plastic material model. This implies limitations on stresses: $|\sigma_\alpha| \leq |\sigma^u_\alpha|$, which can be taken into account in the objective function as an additional penalty term:

$$F = \sum_{\alpha=1}^{n}\left(\varepsilon_\alpha - \overset{\mathrm{M}}{\varepsilon}_\alpha\right)^2 + \sum_{\alpha=1}^{n}\left(c\left(\sigma_\alpha - \sigma^u_\alpha\right)\right)^2\Big|_{|\sigma_\alpha|>|\sigma^u_\alpha|}, \tag{10}$$

where c is a constant coefficient. The penalty term is calculated only for elements in which stresses exceed the yield stresses. The design variable $\overset{0}{\beta}_\alpha$ is iteratively updated using the steepest descent method:

$$\overset{0}{\beta}{}^{(i+1)}_\alpha = \overset{0}{\beta}{}^{(i)}_\alpha - \Delta F^{(i)}\frac{\nabla_\alpha F^{(i)}}{\left(\nabla_\beta F^{(i)}\right)\left(\nabla_\beta F^{(i)}\right)}, \tag{11}$$

where the gradient of the objective function reads:

$$\nabla_\alpha F^{(i)} = \frac{\partial F^{(i)}}{\partial \overset{0}{\beta}{}^{(i)}_\alpha} = 2\left(\varepsilon^{(i)}_\delta - \overset{\mathrm{M}}{\varepsilon}_\delta\right)D_{\delta\alpha} + 2c\left(\sigma^{(i)}_\beta - \sigma^u_\beta\right)\frac{\partial\sigma^{(i)}_\beta}{\partial\overset{0}{\beta}{}^{(i)}_\alpha}\Big|_{|\sigma_\alpha|>|\sigma^u_\alpha|}. \tag{12}$$

The partial gradient of stresses with respect to design variable $\overset{0}{\beta}_\alpha$ in Eq. (12) can be determined by the differentiation of Eq. (1) with respect to $\overset{0}{\beta}_\alpha$:

$$\frac{\partial\sigma^{(i)}_\beta}{\partial\overset{0}{\beta}{}^{(i)}_\alpha} = E_{\underline{\beta}}\left(D_{\underline{\beta}\alpha} - \delta_{\underline{\beta}\alpha}\right). \tag{13}$$

The partial gradient is constant for each iteration. The gradient of the objective function (12) has non-zero components only for the plastic elements.

2.4. Numerical algorithm

The approach for identification of plastic zones presented in the previous subsection has been implemented in Java. The numerical algorithm can be divided into 3 main steps:

(i) Initial data and assumptions:
- initial structure subjected to given load,
- selection of monitored elements (in a particular case – all elements),
- numerical model of the initial structure (with or without known plastic deformations) used to simulate the measured responses $\overset{\mathrm{M}}{\varepsilon}_\alpha$,
- assumption of the material model e.g. elastic-ideally plastic,
- initial plastic strains $\overset{0}{\beta}_\alpha$ (zeros in most cases),
- update of total strains ε_α and stresses σ_α, Eqs. (7).

(ii) Preliminary computations:
- influence matrix $D_{\alpha\beta}$ corresponding to the monitored elements,
- initial strains $\overset{\mathrm{L}}{\varepsilon}_\alpha$ and initial virtual distortions $\overset{0}{\beta}_\alpha$ according to Eq. (8),
- gradient of stresses, Eq. (13).

(iii) Iterative computations:
- objective function, Eq. (10),
- gradient of the objective function, Eq. (12),
- update of plastic strains (virtual distortions), Eq. (11),
- termination condition, e.g. $\frac{F^i}{F^1} < 10^{-5}$.

3. Numerical example

Figure 2 shows an example of a truss structure of the degree of redundancy 4 subjected to the load $P = 1725\,kN$. The steel elements have pipe cross-sectional area with the wall thickness of $1\,cm$ and the outer diameters either $10\,cm$ (elements no. 3, 7, 8, 11, 12) or $30\,cm$ (remaining elements). The basic length of the module is equal to $L = 10\,m$.

The material model is assumed as elastic-ideally plastic (Young's modulus $E = 210\,GPa$, tangent modulus $E^{tan} = 0$). The yield stress equal to $\sigma^u_\alpha = 210$ MPa is assumed for all elements.

The applied load P causes plastic deformations in 4 elements no. 3, 7, 9, 11. The results obtained using the Newton-Raphson method are shown in Table (1) in the column (4) for strains and column (5) for stresses. The influence matrix $D_{\alpha\beta}$ as well as the reference responses $\overset{\mathrm{L}}{\varepsilon}_\alpha$ (column (1)) and $\overset{\mathrm{L}}{\sigma}_\alpha$ (column (2)) were computed for elastic range (initial structure). Evidently, the stresses in elements no. 3 and 9 exceed the yield stress. The aim of the optimization procedure is to determine the virtual distortions $\overset{0}{\varepsilon}_\alpha$, which should simulate the plastic strains and keep the stresses within the permissible limits determined by the adopted material model.

The objective function is assumed according to Eq. (10) with constant $c = 210\,\frac{1}{\mathrm{GPa}}$. The measured strains are simulated numerically. Because of the static character of the applied load, it is assumed that strain sensors are required in each structural member. The number of iterations is set to 500. The identification algorithm presented in Section 2.4 is used. Figure 3 illustrates the decrease of the objective function in subsequent iteration steps according to the formula:

$$n = \log_{10} \frac{F^{(i)}}{F^{(1)}}. \tag{14}$$

Supplied with the initial information on the measured responses $\overset{\mathrm{M}}{\varepsilon}_\alpha$, the VDM-based algorithm successfully identified all 4 plastic strains (column (6) in Table (1)) generated in the structure

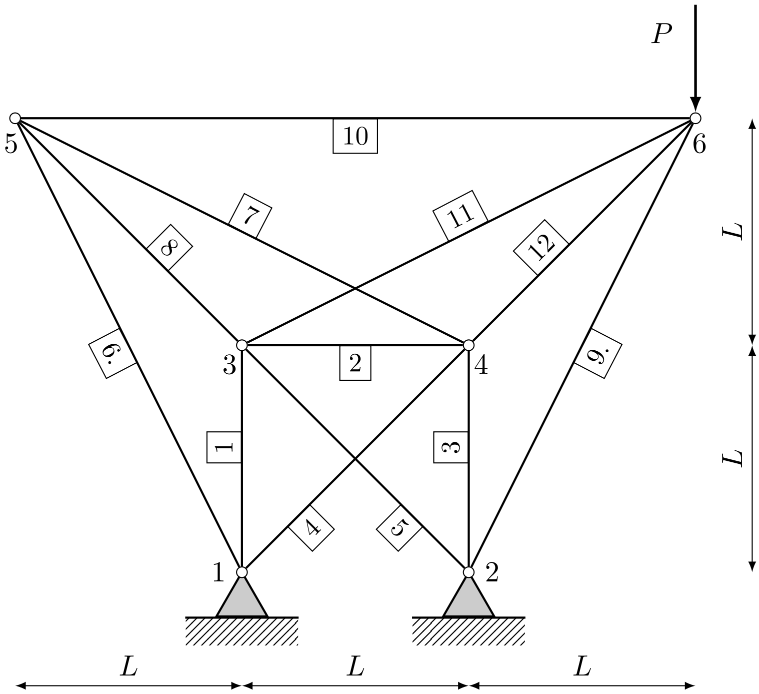

Figure 2. Tested truss structure.

Table 1. Listing of the results for the initial, modified and modelled structure.

α	Initial (elastic) structure		Modified (plastic) structure (Newton-Raphson)			Modelled structure (VDM)		
	$\overset{L}{\varepsilon}_\alpha$ [x0.001]	$\overset{L}{\sigma}_\alpha$ [MPa]	ε_α^{pl} [x0.001]	ε_α [x0.001]	σ_α [MPa]	$\overset{0}{\beta}_\alpha$ [x0.001]	ε_α [x0.001]	σ_α [MPa]
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	0.4138	86.91	0.0	0.5155	108.26	0.0	0.5155	108.25
2	0.0500	10.49	0.0	0.1060	22.27	0.0	0.1060	22.27
3	-1.0698	-224.65	-1.2354	-2.2354	-210.00	-1.2353	-2.2354	-210.00
4	0.3103	6.52	0.0	0.0394	8.28	0.0	0.0394	8.28
5	-0.6132	-128.78	0.0	-0.8464	-177.73	0.0	-0.8464	-177.73
6	0.3001	63.02	0.0	0.4005	84.11	0.0	0.4005	84.11
7	-0.9174	-192.65	-0.4648	-1.4648	-210.00	-0.4646	-1.4648	-210.00
8	0.8739	183.52	0.0	0.9835	206.53	0.0	0.9835	206.51
9	-1.1601	-243.62	-1.7082	-2.7082	-210.00	-1.7080	-2.7082	-210.00
10	0.2615	54.92	0.0	0.3179	66.76	0.0	0.3179	66.76
11	-0.6429	-135.01	-0.1845	-1.1845	-210.00	-0.1836	-1.1845	-210.00
12	0.0670	14.07	0.0	-0.6548	-137.50	0.0	-0.6548	-137.47

due to the load P. The performace of the inverse VDM analysis can be compared with the direct Newton-Raphson results listed in Table (1). The values of the quantities in columns (3)-(6), (4)-(7), (5)-(8) are almost identical. One should note that the identification is feasible until the

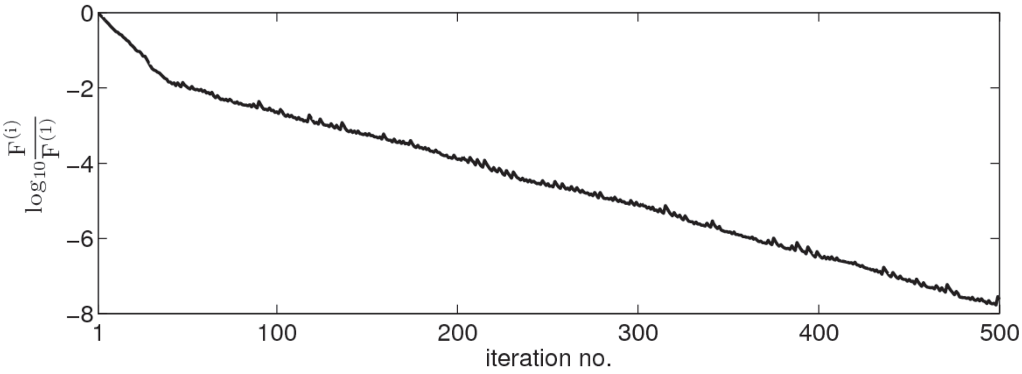

Figure 3. Decrease of the objective function in subsequent iterations.

moment of total collapse of the structure i.e. its transformation into a kinematic mechanism. In the analyzed example any other plastic location except for those in elements no. 3, 7, 9, 11 would provoke such a mechanism.

4. Conclusions

The paper presents an idea of identification of progressive collapse as an inevitable symptom of the deteriorating condition of a skeletal structure. Plastic strains in elements are certainly a reliable indicator for the owner of the structure to undertake immediate action e.g. inspection on site.

The problem is formulated within the framework of the Virtual Distortion Method. The solution is obtained via gradient-based optimization. The procedure assumes that the structure is instrumented with sensors collecting its responses, which are necessary for the identification algorithm. In the presented numerical study, these measured responses are simulated numerically with the Newton-Raphson method.

The solution of the VDM-based inverse problem of identification of plastic strains is demonstrated for static load in this paper. This corresponds to standard modeling of damage as stiffness degradation for static problems (cf. [11]). Further research will be focused on extending the approach for dynamic excitations, analogously to [1, 14]. Experimental verification is also planned.

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